3rd year pathway presentation at 17:00

Recap quiz

1) What is the dual of

$$\begin{aligned} \max \quad & \underline{c}^T \underline{x} \\ \text{sub to} \quad & A\underline{x} \leq \underline{b} \\ & \underline{x} \geq \underline{0} \end{aligned}$$

2)

	Primal	Dual
Goal	max	
# Variables	n	
# unrestricted variables	n'	
# constraints	m	
# equality constraints	m'	

3) Weak duality theorem says:

If $\underline{x}$ is a ________________ to LP above
and $\underline{y}$ is a ________________ to its dual

then ______ $\geq$ ______

Example of using complementary slackness

Example 9.1. Consider the linear program:

$$\begin{aligned} \text{maximise} \quad & 2x_1 - x_2 + 8x_3 \\ \text{subject to} \quad & 2x_3 \leq 1 \\ & 2x_1 - 4x_2 + 6x_3 \leq 3 \\ & -x_1 + 3x_2 + 4x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \tag{9.5}$$

Show that $x_1 = 17/2$, $x_2 = 7/2$, $x_3 = 0$ is an optimal solution to this program.

Modelling Revisited (non-linear objective)

Example 9.2. A factory makes 2 different parts (say, part X and part Y). Their plant has 4 separate processes in place: there are two older processes (say, process 1 and 2) that produce parts X and Y directly, as well as two different integrated processes for producing both X and Y simultaneously. The 4 processes can be run simultaneously, but require labour, raw metal, and electricity. The hourly inputs and outputs for each process are as follows:

	Outputs		Inputs		
Process	X	Y	Metal	Electricity	Labour
1	4	0	100 kg	800 kWh	16 hrs
2	0	1	70 kg	600 kWh	16 hrs
3	3	1	120 kg	2000 kWh	50 hrs
4	6	3	270 kg	4000 kWh	48 hrs

Piecewise linear concave/convex objectives

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limited resources:
6000 kg metal
100000 kWh electric
1000 hours labour

Revenue
Y sells for £1800 per unit
X sells for £1000 per unit

As before p_i = # hours of process i
m, e, l = amount of metal/electric/labour used
x, y = amount of X, Y produced.

$$
\begin{aligned}
\text{maximise} \quad & 1000x + 1800y \\
\text{subject to} \quad & x = 4p_1 + 3p_3 + 6p_4 \\
& y = p_2 + p_3 + 3p_4 \\
& m = 100p_1 + 70p_2 + 120p_3 + 270p_4 \\
& e = 800p_1 + 600p_2 + 2000p_3 + 4000p_4 \\
& l = 16p_1 + 16p_2 + 50p_3 + 48p_4 \\
& m \leq 6000 \\
& e \leq 100000 \\
& l \leq 1000 \\
& p_1, p_2, p_3, p_4 \geq 0 \\
& x, y \text{ unrestricted} \\
& m, e, l \text{ unrestricted}
\end{aligned}
$$

Piecewise linear concave/convex objectives

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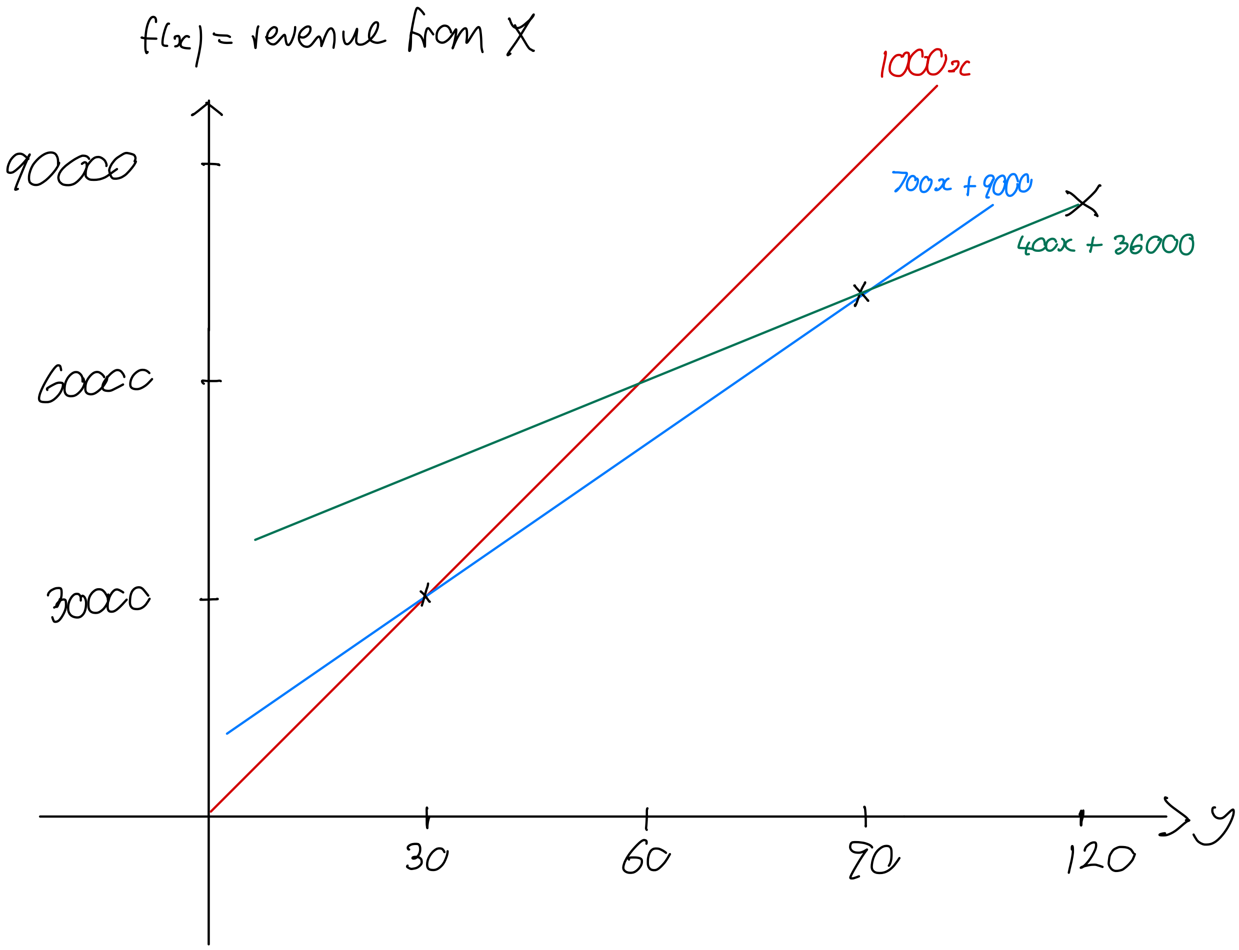
limited resources:
6000 kg metal
100 000 kWh electric
1000 hours labour

Revenue
Y sells for £1800 per unit
X sells for £1000 per unit first 30
£700 next 60
£400 remaining



$$
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& e \leq 100000 \\
& l \leq 1000 \\
& p_1, p_2, p_3, p_4 \geq 0 \\
& x, y \text{ unrestricted} \\
& m, e, l \text{ unrestricted}
\end{array}
$$

Application : sensitivity analysis

sensitivity analysis. (Example from earlier)

Primal

$$\begin{aligned} \text{maximise} \quad & 2x_1 - x_2 + 8x_3 \\ \text{subject to} \quad & 2x_3 \le 1 \\ & 2x_1 - 4x_2 + 6x_3 \le 3 \\ & -x_1 + 3x_2 + 4x_3 \le 2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

optimal solution $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 17/2 \\ 7/2 \\ 0 \end{pmatrix}$

Dual

$$\begin{aligned} \text{minimise} \quad & y_1 + 3y_2 + 2y_3 \\ \text{sub to} \quad & 0y_1 + 2y_2 - y_3 \geq 2 \\ & 0y_1 - 4y_2 + 3y_3 \geq -1 \\ & 2y_1 + 6y_2 + 4y_3 \geq 8 \\ & y_1, y_2, y_3 \geq 0. \end{aligned}$$

optimal solution $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 5/2 \\ 3 \end{pmatrix}$