Some models:

- The basic survival model:
- The withdrawal dead model:
- The permanent disability model:
- The healthy-sick; (or the sick death model):

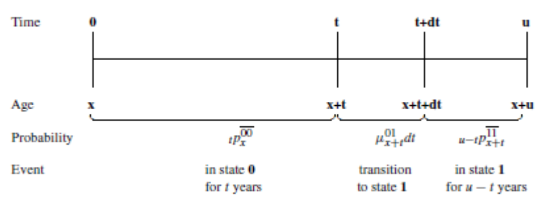
Occupancy probability:

$$ {}_tp_x^{\overline{ii}} = \exp\left(-\int_0^t \sum_{j=0, j\neq i}^{m} \mu_{x+s}^{ij} ds \right) $$

For all other probabilities follow the path
In permanent disability model:

$$ {}_up_x^{01} = \int_0^u {}_tp_x^{\overline{00}} \mu_{x+t}^{01} \; {}_{u-t}p_{x+t}^{\overline{11}} dt $$

Why?



General - Kolmogorov foward equations
Discrete:

$$ {}_{t+h}p_x^{ij} = \; {}_tp_x^{ij} \; {}_hp_{x+t}^{jj} + \sum_{k=0, k\neq j}^{m} {}_tp_x^{ik} \; {}_hp_{x+t}^{kj} $$

with:

$$ {}_hp_{x+t}^{jj} = 1 - h \sum_{k=0, k\neq j}^{m} \mu_{x+t}^{jk} + o\left(h\right) $$

$$ {}_hp_{x+t}^{kj} = h\mu_{x+t}^{kj} + o\left(h\right) $$

Continuous:

$$ \frac{d}{dt} \; {}_tp_x^{ij} = \sum_{k=0, k\neq j}^{m} \left({}_tp_x^{ik} \mu_{x+t}^{kj} - \; {}_tp_x^{ij} \mu_{x+t}^{jk} \right) $$

$$ {}_0p_x^{ii} = 1 $$

$$ {}_0p_x^{ij} = 0 $$

State dependent annuities

The value of an annuity of 1 per year payable continuously while the life is in some state j (which may be equal to i) given that the life is currently in state i

$$\bar{a}_x^{ij} = \int_0^\infty e^{-\delta t} \ {}_tp_x^{ij} dt$$

If the annuity is payable at the start of each year, from the current time, conditional on the life being in state j, given that the life is currently in state i.

$$\ddot{a}_x^{ij} = \sum_{k=0}^{\infty} v^k \ {}_kp_x^{ij}$$

If $i \neq j$ ${}_0p_x^{ij} = 0$

$$\ddot{a}_{x:\overline{n}|}^{ij} = \sum_{k=0}^{n-1} v^k \ {}_kp_x^{ij}$$

$$\bar{a}_{x:\overline{n}|}^{ij} = \bar{a}_x^{ij} - e^{-\delta n} \sum_{k=0}^{m} {}_np_x^{ik} \ \bar{a}_{x+n}^{kj}$$

For example: alive-dead model:$\bar{a}_{x:\overline{n}|}^{00} = \bar{a}_x^{00} - e^{-\delta n} \ {}_np_x^{00} \ \bar{a}_{x+n}^{00}$

Sickness-death model:$\bar{a}_{x:\overline{n}|}^{00} = \bar{a}_x^{00} - e^{-\delta n} \ {}_np_x^{00} \ \bar{a}_{x+n}^{00} - e^{-\delta n} \ {}_np_x^{01} \ \bar{a}_{x+n}^{10}$

State dependent insurance benefits

A unit benefit is payable immediately on each future transfer into state j, given that the life is currently in state i (which may be equal to j).

$$\bar{A}_x^{ij} = \int_0^\infty \sum_{k=0, k\neq j}^{\infty} e^{-\delta t} \ {}_tp_x^{ik} \ \mu_{x+t}^{kj} dt$$

Term insurance benefit:

$$\begin{aligned} \bar{A}_{x:\overline{n}|}^{ij} &= \int_0^n \sum_{k=0, k\neq j}^{m} e^{-\delta t} \ {}_tp_x^{ik} \ \mu_{x+t}^{kj} dt \\ &= \bar{A}_x^{ij} - e^{-\delta n} \sum_{k=0}^{m} {}_np_x^{ik} \bar{A}_{x+n}^{kj} \end{aligned}$$

Alive- dead model:

$$\bar{A}_{x:\overline{n}|}^{00} = \bar{A}_x^{00} - e^{-\delta n} \ {}_np_x^{00} \bar{A}_{x+n}^{00}$$

Sickness-death model for $i = 0$ and $j = 1$:

$$\bar{A}_{x:\overline{n}|}^{01} = \bar{A}_x^{01} - e^{-\delta n} \ {}_np_x^{00} \bar{A}_{x+n}^{01} - e^{-\delta n} \ {}_np_x^{01} \bar{A}_{x+n}^{11}$$