1. The population of City of London follows Makeham Survival Model with $A = 0.00019$, $B = 0.0000029$ and $C = 1.132$. With Excel construct a life table for (x) taking values from 20 to 115. Interest rate is 0.4 per annum. Make sure you derive p_x, q_x, $\ddot{a}_x$, A_x, ${}_{10}E_x$, $\ddot{a}_{x:\overline{10}|}$. Please explain the meaning of all these terms and write down all the formulae used in the Excel file.

Hint: ${}_{10}E_x = v^{10}\ {}_{10}p_x$ and $\ddot{a}_{x:\overline{10}|} = \ddot{a}_x -{}_{10} E_x \ddot{a}_{x+10}$

2. An insurer issues a whole life insurance (annual) to Shawn Alvarez aged 30, with sum insured £100,000 (payable at the end of the year of death). Premiums are payable annually in advance for 10 years, or until earlier death. Commission is 20% of first year premium, and 5% of all premiums after. Find the gross annual premium that Shawn Alvarez needs to pay for this policy using the lifetable found in question 1.

$$L_0^g = Sv^{K_{30}+1} + 0.2P + 0.05P\left(a_{\overline{min(K_30+1,10)}|}\right) - P\ddot{a}_{\overline{min(K_30+1,10)}|}$$

Equivalence principle:

$$\begin{aligned} E\left(L_0^g\right) &= 0 \\ SA_{30} + 0.2P + 0.05P\left(\ddot{a}_{30:\overline{10}|} - 1\right) - P\ddot{a}_{30:\overline{10}|} &= 0 \\ SA_{30} - P\left(0.95\ddot{a}_{30:\overline{10}|} - 0.15\right) &= 0 \end{aligned}$$

$$P = \frac{100,000A_{30}}{0.95\ddot{a}_{30:\overline{10}|} - 0.15}$$

$A_{30} = 0.14598632$
$\ddot{a}_{30:\overline{10}|} = 8.422240232$

$$P = 1859.43$$

3. Calculate the gross premium policy value at time $t = 3$ for Mr. Alvarez' policy. Note that after 3, the policy earns income for maximum 7 years.

$$L_3^g = Sv^{K_{33}+1} + 0.05P\left(\ddot{a}_{\overline{min(K_33+1,10)}|}\right) - P\ddot{a}_{\overline{min(K_33+1,10)}|}$$

$$\begin{aligned} E\left(L_3^g\right) &= SA_{33} + 0.05P\left(\ddot{a}_{33:\overline{7}|}\right) - P\ddot{a}_{30:\overline{7}|} \\ E\left(L_3^g\right) &= SA_{33} - P\left(0.95\ddot{a}_{33:\overline{7}|}\right) = 5320.20 \end{aligned}$$

4 and 5 Calculate the net premium policy value at time 3, using the same policy basis (the same survival model) but now $i = 0.02$. Explain why your answer is different than in Question 3.

Hint - make sure you calculate first the net premium under this policy basis and then calculate the reserves.

If we were to keep the interest rate constant:

$$P = \frac{SA_{30}}{\ddot{a}_{30:\overline{10}|}} = 1733.31$$

$$E\left(L_3^g\right) = SA_{33} - P\left(\ddot{a}_{33:\overline{7}|}\right) = 5526.67$$

As we are working with net premiums (lower than gross premiums), the reserves need to be higher in order to cover for the expenses.

Now if we decrease the interest rate:

$$P = \frac{SA_{30}}{\ddot{a}_{30:\overline{10}|}} = 4025.58$$

- the net premium is higher when the interest rate was higher because the life insurance benefit is higher at lower interest rate, while the term annuity is higher. While the net premium is higher this will have a negative effect on the reserves, but at the same time there will be a positive effect from A_{33} and negative effect coming from $\ddot{a}_{33:\overline{7}|}$. The positive effect dominates.

$$\begin{aligned} E\left(L_3^g\right) &= SA_{33} - P\left(\ddot{a}_{33:\overline{7}|}\right) \\ &= 12472.42 \end{aligned}$$

There are two changes that we need to take account of:

- gross policy value versus net policy value at the same interest rate
- change of interest rate effect on all actuarial functions

Please see the diagram in Excel.