Week 9 Ruin Theory (Cont.)

Impact of parameters

Further assumptions

① Poisson parameter for the number of claims is 1

change time unit $E(N(t)) = ~~\ldots~~ 12$ in 1 year, ① in 1 month

② the expected value of an individual claim is 1

change Monetary unit $m_1 = £1000$

$= £'1$ unit: $£' = £1000$

③ Individual claims have an exponential distribution

Reason: e^{-Ru} and $\psi(u)$ can be calculated

when $F(x) = 1 - e^{-x}$ mean $= 1$

$$\psi(u) = \frac{1}{1+\theta} e^{-\frac{\theta u}{1+\theta}}$$

$\psi(u,t)$

features: ① $t \uparrow$, $\psi(u,t) \uparrow$ $\psi(u,t)$ is increasing function of t

② For small values of t, $\psi(u,t)$ increases very quickly.

③ For large values of t, $\psi(u,t)$ increases less quickly, and approaches asymptotically the value of $\psi(u)$

$t \to \infty$ $\psi(u,t) \to \psi(u)$

---

u initial surplus

① $u \uparrow$, $\psi(u,t) \downarrow$

② $\psi(u)$ is an non-increasing function of u.

$X \sim$ Exponential, $\psi(u)$ decreasing function of u.

$$\frac{d}{du}\psi(u) = \frac{-\theta}{1+\theta}\psi(u) < 0$$

θ premium loading factor

$\theta \uparrow$, $\psi(u,t) \downarrow$

$$\frac{d}{d\theta}\psi(u) = -\frac{1}{1+\theta}\psi(u) - \frac{u}{(1+\theta)^2}\psi(u) < 0$$

Slide 9 - 12 Poisson parameter → read by yourself

$\lambda \uparrow \downarrow$, ~~$\psi(u,t)$~~ $\psi(u)$ not change

time ruin happen may change

unit standardisation

$\lambda \uparrow$, $\begin{cases} S \uparrow & \text{out} \\ c = (1+\theta)\lambda m, \uparrow & \text{in} \end{cases}$

ψ not change

λ double, $\begin{cases} S \text{ double} \\ c \text{ double} \end{cases}$, volatility $\uparrow$, ruin may ~~may~~ happen earlier

Concluding remarks:

① If $\theta = 0$, then $\psi(u) = 1$ irrespective of the value of u.

For any form of $F(x)$

$c = \lambda m_1 = E(S)$ $\quad S_t \leq E(S_t)$

no safty buffer θ $\quad S_t > E(S_t) \rightarrow$ ruin happens

If $\theta < 0$, then $\psi(u) = 1$

premium too low

lower than $E(S)$

② Standardisation of unit

£1 = 100 pence

$\psi(u)$ when $F(x) = 1 - e^{-\alpha x}$ 1 pound

$\Updownarrow$

$\psi(\alpha u)$ when $F(x) = 1 - e^{-x}$ 100 pence $\alpha = 100$

Example Question slide 15

$u = £2$ million

$N \sim$ Poisson (50) $\lambda = 50$

$X \sim$ Gamma $(150, \frac{1}{4})$ $\alpha = 150,\ \beta = \frac{1}{4}$

$\theta = 30\%$

changes of parameters $\rightarrow$ $\psi(u)$ ultimate ruin

i. $\theta = 30\% \rightarrow 28\%$

$\theta \downarrow$, $c \downarrow$ income

X, N same out flow $\psi(u) \uparrow$

ii. $X \sim \text{Gamma}(150, \frac{1}{4}) \rightarrow X \sim \text{Gamma}(150, \frac{1}{2})$

mean $= \frac{\alpha}{\beta}$ var $= \frac{\alpha}{\beta^2}$

	mean	var
$150, \frac{1}{4}$	600	2400
$150, \frac{1}{2}$	300	600

Claims are smaller on average ← mean

less uncertain ← var

$\downarrow \psi(u)$

iii $\lambda = 50 \rightarrow 60$

$\psi(u)$ the same

$\uparrow \lambda$, $\rightarrow N$ claim occur more often X unchanged

$C = (1+\theta)\lambda m$, $C \uparrow$ premium increase proportionally $\psi(u)$: same

time of ruin will be earlier $\uparrow$ volatility