c) $\quad R^c{}_{dab} = \partial_a \Gamma^c{}_{bd} - \partial_b \Gamma^c{}_{ad} + \Gamma^c{}_{ae}\Gamma^e{}_{bd} - \Gamma^c{}_{be}\Gamma^e{}_{ad}$

$$R^x{}_{yxy} = \partial_x \Gamma^x{}_{yy} - \partial_y \Gamma^x{}_{yx} + \Gamma^x{}_{xe}\Gamma^e{}_{yy} - \Gamma^x{}_{ye}\Gamma^e{}_{xy}$$

- $\partial_x \Gamma^x{}_{yy} = -\frac{1}{2} e^{x-y}$

- $\partial_y \Gamma^x{}_{yx} = 0$

- $\Gamma^x{}_{xe}\Gamma^e{}_{yy} = \Gamma^x{}_{xy}\Gamma^y{}_{yy} = 0$

- $\Gamma^x{}_{ye}\Gamma^e{}_{xy} = \Gamma^x{}_{yx}\Gamma^x{}_{xy} + \Gamma^x{}_{yy}\Gamma^y{}_{xy} = \cancel{\frac{1}{2}}\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}e^{x-y}\right)\left(\frac{1}{2}\right)$

$$= \frac{1}{4}\left(1 - e^{x-y}\right)$$

$$\Rightarrow R^x{}_{yxy} = -\frac{1}{2}e^{x-y} - 0 + 0 - \frac{1}{4}\left(1 - e^{x-y}\right) = -\frac{1}{4}\left(1 + e^{x-y}\right)$$

$$R_{xyxy} = g_{xa} R^a{}_{yxy} = g_{xx} R^x{}_{yxy} = -\frac{1}{4}\left(e^y + e^x\right)$$

(4) $ds^2 = e^y dx^2 + e^x dy^2$

a) $\mathcal{L} = e^y \dot{x}^2 + e^x \dot{y}^2$

b) $\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^a}\right) - \frac{\partial \mathcal{L}}{\partial x^a} = 0 \iff \ddot{x}^a + \Gamma^a{}_{bc}\dot{x}^b \dot{x}^c = 0$

x) $\frac{\partial \mathcal{L}}{\partial \dot{x}} = 2e^y \dot{x} \rightarrow \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = 2e^y(\ddot{x} + \dot{y}\dot{x})$

$\frac{\partial \mathcal{L}}{\partial x} = e^x \dot{y}^2$

$\Rightarrow 2e^y(\ddot{x} + \dot{y}\dot{x}) - e^x \dot{y}^2 = 0 \Rightarrow \ddot{x} + \dot{y}\dot{x} - \frac{1}{2}e^{x-y}\dot{y}^2 = 0$

$\Rightarrow \Gamma^x{}_{xy} = \Gamma^x{}_{yx} = \frac{1}{2}, \quad \Gamma^x{}_{yy} = -\frac{1}{2}e^{x-y}$

y) $\frac{\partial \mathcal{L}}{\partial \dot{y}} = 2e^x \dot{y} \rightarrow \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = 2e^x(\ddot{y} + \dot{x}\dot{y})$

$\frac{\partial \mathcal{L}}{\partial y} = e^y \dot{x}^2 \Rightarrow 2e^x(\ddot{y} + \dot{x}\dot{y}) - e^y\dot{x}^2 = 0$

$\Rightarrow \ddot{y} + \dot{x}\dot{y} - \frac{1}{2}e^{y-x}\dot{x}^2 = 0$

$\Rightarrow \Gamma^y{}_{xy} = \Gamma^y{}_{yx} = \frac{1}{2}, \quad \Gamma^y{}_{xx} = -\frac{1}{2}e^{y-x}$

c) We compute the Ricci tensor from its definition:

$$
\begin{aligned}
R_{ac} &= g^{bd} R_{abcd} \\
&= K g^{bd} (g_{ac} g_{bd} - g_{ad} g_{cb}) \\
&= K (n g_{ac} - g_{ad} \delta^{d}{}_{c}) \\
&= K (n-1) g_{ac}
\end{aligned}
$$

in n- dimensions.

Hence, the Ricci scalar is:

$$R = g^{ac} R_{ac} = K n (n-1)$$

(6)

$$R_{ab} - \frac{1}{2} R g_{ab} + \lambda g_{ab} = 0$$

Taking the trace of this equation by contracting with g^{ab} we get:

$$R - \frac{n}{2} R + n\lambda = - \left(\frac{n-2}{2} \right) R + n\lambda = 0$$

$$\Rightarrow R = \frac{2n}{n-2} \lambda = 4\lambda \quad \text{for } n = 4$$

Substituting this result back into the equation,

$$R_{ab} - \frac{1}{2}(4\lambda) g_{ab} + \lambda g_{ab} =$$

$$= R_{ab} - \lambda g_{ab} = 0$$

$$\Rightarrow R_{ab} = \lambda g_{ab}$$

(7) Consider the Christoffel symbols:

$$\Gamma^a{}_{bc} = \frac{1}{2} g^{ad} (\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc})$$

Contracting a and b:

$$\Gamma^a{}_{ac} = \frac{1}{2} g^{ad} (\cancel{\partial_a g_{cd}} + \partial_c g_{ad} - \cancel{\partial_d g_{ac}})$$

$$= \frac{1}{2} g^{ad} \partial_c g_{ad}$$

because $g^{ad} = g^{da}$. Using the relation between the trace and the determinant of a matrix,

$$g^{ab} \partial_c g_{ab} = \frac{1}{g} \partial_c g \qquad \text{when } g = |\det g_{ab}|$$

$$\Rightarrow \Gamma^a{}_{ac} = \frac{1}{2|g|} \partial_c |g|$$

(8) $$ds^2 = -dt^2 + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

The non-vanishing Christoffels are calculated using the Euler-Lagrange equations. We find:

$$\Gamma^r{}_{rr} = \frac{Kr}{1-Kr^2}, \quad \Gamma^r{}_{\theta\theta} = -r(1-Kr^2), \quad \Gamma^r{}_{\phi\phi} = \sin^2\theta\, \Gamma^r{}_{\theta\theta}$$

$$\Gamma^\theta{}_{r\theta} = \Gamma^\theta{}_{\theta r} = \frac{1}{r}, \quad \Gamma^\theta{}_{\phi\phi} = -\sin\theta\cos\theta$$

$$\Gamma^\phi{}_{r\phi} = \Gamma^\phi{}_{\phi r} = \frac{1}{r}, \quad \Gamma^\phi{}_{\theta\phi} = \Gamma^\phi{}_{\phi\theta} = \cot\theta$$

The Ricci tensor is given by,

$$\begin{aligned} R_{ab} &= R^c{}_{acb} \\ &= \partial_c \Gamma^c{}_{ab} - \partial_a \Gamma^c{}_{cb} + \Gamma^d{}_{ab}\Gamma^c{}_{cd} - \Gamma^d{}_{ca}\Gamma^c{}_{db} \\ &= \partial_c \Gamma^c{}_{ab} - \partial_a\partial_b \ln\sqrt{|g|} + \Gamma^d{}_{ab}\partial_d \ln\sqrt{|g|} - \Gamma^d{}_{ca}\Gamma^c{}_{db} \end{aligned}$$

$$\det g = -\frac{r^4\sin^2\theta}{1-Kr^2} \Rightarrow \sqrt{|g|} = \frac{r^2\sin\theta}{\sqrt{1-Kr^2}}$$

$$\ln\sqrt{|g|} = 2\ln r + \ln\sin\theta - \frac{1}{2}\ln(1-Kr^2)$$

$$\Rightarrow \partial_r \ln\sqrt{|g|} = \frac{2}{r} + \frac{Kr}{1-Kr^2} = \frac{2-Kr^2}{r(1-Kr^2)}$$

$$\partial_\theta \ln\sqrt{|g|} = \cot\theta$$

Notice that $R_{ta} = 0$ for $a = t, r, \theta, \phi$ because the Christoffels that can contribute actually vanish.

$$R_{rr} = \partial_c \Gamma^c{}_{rr} - \partial_r \partial_r \ln\sqrt{|g|} + \Gamma^r{}_{rr} \partial_r \ln\sqrt{|g|} - \Gamma^c{}_{dr}\Gamma^d{}_{cr}$$

$$\partial_c \Gamma^c{}_{rr} = \partial_r \Gamma^r{}_{rr} + \cancelto{0}{\partial_\theta \Gamma^\theta{}_{rr}} + \cancelto{0}{\partial_\phi \Gamma^\phi{}_{rr}}$$

$$= \frac{K(1+Kr^2)}{(1-Kr^2)^2}$$

$$\partial_r \partial_r \ln\sqrt{|g|} = \frac{-2+5Kr^2-K^2r^4}{r^2(1-Kr^2)^2}$$

$$\Gamma^r{}_{rr} \partial_r \ln\sqrt{|g|} = \frac{K(2-Kr^2)}{(1-Kr^2)^2}$$

$$\Gamma^c{}_{dr}\Gamma^d{}_{cr} = (\Gamma^r{}_{rr})^2 + (\Gamma^\theta{}_{r\theta})^2 + (\Gamma^\phi{}_{r\phi})^2 = \frac{2}{r^2} + \frac{K^2r^2}{(1-Kr^2)^2}$$

$$\Rightarrow R_{rr} = \frac{2K}{1-Kr^2}$$

$$R_{r\theta} = \partial_c \Gamma^c{}_{r\theta} - \cancelto{0}{\partial_r \partial_\theta \ln\sqrt{|g|}} + \Gamma^c{}_{r\theta} \Gamma^d{}_{dc} - \Gamma^c{}_{rd}\Gamma^d{}_{\theta c}$$

$$= \partial_\theta \cancelto{0}{\Gamma^\theta{}_{r\theta}} + \Gamma^\theta{}_{r\theta} \partial_\theta \ln\sqrt{|g|} - \cancel{\Gamma^r{}_{rr}} \cancelto{0}{\Gamma^r{}_{\theta r}}$$

$$- \cancelto{0}{\Gamma^r{}_{r\theta}} \Gamma^\theta{}_{\theta r} - \Gamma^\phi{}_{r\phi}\Gamma^\phi{}_{\theta\phi}$$

$$= \frac{1}{r}\cot\theta - \frac{1}{r}\cot\theta = 0$$

Similarly, we find $R_{r\phi} = 0$

$$R_{\theta\theta} = \partial_c \Gamma^c{}_{\theta\theta} - \partial_\theta \partial_\theta \ln\sqrt{|g|} + \Gamma^c{}_{\theta\theta}\, \partial_c \ln\sqrt{|g|} - \Gamma^c{}_{\theta d}\, \Gamma^d{}_{\theta c}$$

$$= \partial_r \Gamma^r{}_{\theta\theta} - \partial_\theta \partial_\theta \ln\sqrt{|g|} + \Gamma^r{}_{\theta\theta}\, \partial_r \ln\sqrt{|g|}$$

$$-2\Gamma^r{}_{\theta\theta}\, \Gamma^\theta{}_{\theta r} - \Gamma^\phi{}_{\theta\phi}\, \Gamma^\phi{}_{\theta\phi}$$

$$= 2Kr^2$$

By symmetry

$$R_{\phi\phi} = R_{\theta\theta} \sin^2\theta$$

$$R_{\theta\phi} = 0$$

Having found the components of R_{ab}, we can find the Ricci scalar:

$$R = g^{ab} R_{ab} = 6K$$

and finally the Einstein tensor:

$$G_{ab} = R_{ab} - \frac{1}{2} R\, g_{ab}$$

$$G_{tt} = 3K$$

$$G_{rr} = \frac{K}{1-Kr^2}, \quad G_{\theta\theta} = -Kr^2, \quad G_{\phi\phi} = G_{\theta\theta} \sin^2\theta$$

from which we identify T_{ab}.

(9)

$$
\begin{aligned}
0 &= [\nabla_c, \nabla_d](T_{ab} X^a Y^b) \\
&= (\nabla_c \nabla_d - \nabla_d \nabla_c)(T_{ab} X^a Y^b) \\
&= \nabla_c \nabla_d (T_{ab} X^a Y^b) - (c \leftrightarrow d) \\
&= \nabla_c \left[(\nabla_d T_{ab}) X^a Y^b + T_{ab} Y^b \nabla_d X^a + T_{ab} X^a \nabla_d Y^b \right] \\
&\quad - (c \leftrightarrow d) \\
&= X^a Y^b \nabla_c \nabla_d T_{ab} \\
&\quad + (\nabla_d T_{ab})(Y^b \nabla_c X^a + X^a \nabla_c Y^b) \\
&\quad + Y^b (\nabla_c T_{ab})(\nabla_d X^a) + T_{ab} (\nabla_c Y^b)(\nabla_d X^a) \\
&\quad + X^a (\nabla_c T_{ab})(\nabla_d Y^b) + T_{ab} (\nabla_c X^a)(\nabla_d Y^b) \\
&\quad + T_{ab} Y^b \nabla_c \nabla_d X^a + T_{ab} X^a \nabla_c \nabla_d Y^b \\
&\quad - (c \leftrightarrow d)
\end{aligned}
$$

$$
\begin{aligned}
&= X^a Y^b \nabla_c \nabla_d T_{ab} + T_{ab}(Y^b \nabla_c \nabla_d X^a + X^a \nabla_c \nabla_d Y^b) \\
&\quad + (\nabla_d T_{ab})(Y^b \nabla_c X^a + X^a \nabla_c Y^b) \\
&\quad + (\nabla_c T_{ab})(Y^b \nabla_d X^a + X^a \nabla_d Y^b) \\
&\quad + T_{ab} \left[(\nabla_c X^a)(\nabla_d Y^b) + (\nabla_d X^a)(\nabla_c Y^b) \right] \\
&\quad - (c \leftrightarrow d)
\end{aligned}
$$

Note that all terms except those in the first line cancel. Hence, we are left with

$$
\begin{aligned}
0 &= [\nabla_c, \nabla_d](T_{ab} X^a Y^b) \\
&= X^a Y^b [\nabla_c, \nabla_d] T_{ab} \\
&\quad + T_{ab}\left(Y^b [\nabla_c, \nabla_d] X^a + X^a [\nabla_c, \nabla_d] Y^b \right) \\
&= X^a Y^b [\nabla_c, \nabla_d] T_{ab} \\
&\quad + T_{ab}\left(Y^b R^a{}_{ecd} X^e + X^a R^b{}_{ecd} Y^e \right) \\
&= X^a Y^b \Big([\nabla_c, \nabla_d] T_{ab} \\
&\qquad\qquad + R^e{}_{acd} T_{eb} + R^e{}_{bcd} T_{ae} \Big)
\end{aligned}
$$

Since X^a and Y^b are arbitrary, we find

$$[\nabla_c, \nabla_d] T_{ab} = - R^e{}_{acd} T_{eb} - R^e{}_{bcd} T_{ae}$$

as required.

(11) a)

Consider the static spherically symmetric metric as in the lectures:

$$ds^2 = -e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

We compute the Christoffels using the Euler-Lagrange equations. From the metric we get the following Lagrangian:

$$\mathcal{L} = -e^{2A(r)}\dot{t}^2 + e^{2B(r)}\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2)$$

The Euler-Lagrange equations are:

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^a}\right) - \frac{\partial \mathcal{L}}{\partial x^a} = 0 \qquad (*)$$

They are equivalent to the geodesic equation:

$$\ddot{x}^a + \Gamma^a{}_{bc}\,\dot{x}^b \dot{x}^c = 0$$

The components of $(*)$ are

t) $$\frac{\partial \mathcal{L}}{\partial \dot{t}} = -2e^{2A}\dot{t} \Rightarrow \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{t}}\right) = -2e^{2A}\left(\ddot{t} + 2A'\dot{r}\dot{t}\right)$$

$$\frac{\partial \mathcal{L}}{\partial r} = 0$$

$$\Rightarrow \quad \ddot{t} + 2A'\dot{r}\dot{t} = 0 \quad \Rightarrow \Gamma^t{}_{tr} = A'$$

r) $$\frac{\partial \mathcal{L}}{\partial \dot{r}} = 2e^{2B}\dot{r} \Rightarrow \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) = 2e^{2B}\left(\ddot{r} + 2B'\dot{r}^2\right)$$

$$\frac{\partial \mathcal{L}}{\partial r} = 2\left[-e^{2A}A'\dot{t}^2 + e^{2B}B'\dot{r}^2 + r\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right)\right]$$

$$\Rightarrow \quad \ddot{r} + B'\dot{r}^2 + e^{2(A-B)}A'\dot{t}^2 - re^{-2B}\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) = 0$$

$$\Rightarrow \quad \Gamma^r{}_{tt} = e^{2(A-B)}A', \qquad \Gamma^r{}_{rr} = B'$$

$$\Gamma^r{}_{\theta\theta} = -re^{-2B}, \qquad \Gamma^r{}_{\phi\phi} = -r\sin^2\theta\, e^{-2B}$$

$\theta)$ $$\frac{\partial \mathcal{L}}{\partial \dot\theta} = 2r^2\dot\theta \, , \quad \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) = 2r^2\left(\ddot\theta + \frac{2}{r}\dot r\dot\theta\right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 2r^2 \sin\theta\cos\theta\,\dot\phi^2$$

$$\Rightarrow \quad \ddot\theta + \frac{2}{r}\dot r\dot\theta - \sin\theta\cos\theta\,\dot\phi^2 = 0$$

$$\Rightarrow \quad \Gamma^\theta{}_{r\theta} = \frac{1}{r} \, , \quad \Gamma^\theta{}_{\phi\phi} = -\sin\theta\cos\theta$$

$\phi)$ $$\frac{\partial \mathcal{L}}{\partial \dot\phi} = 2r^2\sin^2\theta\,\dot\phi$$

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot\phi}\right) = 2r^2\sin^2\theta\left(\ddot\phi + \frac{2}{r}\dot r\dot\phi + 2\cot\theta\,\dot\theta\dot\phi\right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\Rightarrow \quad \ddot\phi + \frac{2}{r}\dot r\dot\phi + 2\cot\theta\,\dot\theta\dot\phi = 0$$

$$\Rightarrow \quad \Gamma^\phi{}_{r\phi} = \frac{1}{r} \, , \quad \Gamma^\phi{}_{\theta\phi} = \cot\theta$$

The remaining Christoffels either vanish or can be obtained from the previous ones by symmetry. Here $' = \frac{d}{dr}$

b) Recall the definition of the Riemann tensor:

$$R^a{}_{bcd} = \partial_c \Gamma^a{}_{db} - \partial_d \Gamma^a{}_{cb} + \Gamma^a{}_{ce}\Gamma^e{}_{db} - \Gamma^a{}_{de}\Gamma^e{}_{cb}$$

The non-vanishing components of the Riemann are:

$$R^t{}_{rtr} = \partial_t \cancelto{0}{\Gamma^t{}_{rr}} - \partial_r \Gamma^t{}_{tr} + \Gamma^t{}_{te}\Gamma^e{}_{rr} - \Gamma^t{}_{re}\Gamma^e{}_{rt}$$

$$= -\partial_r \Gamma^t{}_{tr} + \Gamma^t{}_{tr}\Gamma^r{}_{rr} - \Gamma^t{}_{rt}\Gamma^t{}_{rt}$$

$$= -A'' + A'B' - (A')^2$$

$$R^t{}_{\theta t\theta} = \partial_t \cancelto{0}{\Gamma^t{}_{\theta\theta}} - \partial_\theta \cancelto{0}{\Gamma^t{}_{t\theta}} + \Gamma^t{}_{te}\Gamma^e{}_{\theta\theta} - \cancelto{0}{\Gamma^t{}_{\theta e}}\Gamma^e{}_{t\theta}$$

$$= \Gamma^t{}_{tr}\Gamma^r{}_{\theta\theta}$$

$$= -re^{-2B}A'$$

$$R^t{}_{\phi t\phi} = \partial_t \cancelto{0}{\Gamma^t{}_{\phi\phi}} - \partial_\phi \cancelto{0}{\Gamma^t{}_{t\phi}} + \Gamma^t{}_{te}\Gamma^e{}_{\phi\phi} - \cancelto{0}{\Gamma^t{}_{\phi e}}\Gamma^e{}_{t\phi}$$

$$= \Gamma^t{}_{tr}\Gamma^r{}_{\phi\phi}$$

$$= -re^{-2B}\sin^2\theta\, A'$$

$$R^r{}_{\theta r\theta} = \partial_r \Gamma^r{}_{\theta\theta} - \partial_\theta \cancelto{0}{\Gamma^r{}_{r\theta}} + \Gamma^r{}_{ra}\Gamma^a{}_{\theta\theta} - \Gamma^r{}_{\theta a}\Gamma^a{}_{r\theta}$$

$$= \partial_r \Gamma^r{}_{\theta\theta} + \Gamma^r{}_{rr}\Gamma^r{}_{\theta\theta} - \Gamma^r{}_{\theta\theta}\Gamma^\theta{}_{r\theta}$$

$$= \partial_r(-re^{-2B}) - re^{-2B}B' + e^{-2B}$$

$$= re^{-2B}B'$$

$$R^r{}_{\phi r\phi} = \partial_r \Gamma^r{}_{\phi\phi} - \partial_\phi \Gamma^r{}_{r\phi} + \Gamma^r{}_{ra}\Gamma^a{}_{\phi\phi} - \Gamma^r{}_{\phi a}\Gamma^a{}_{r\phi}$$

$$= \sin^2\theta \; R^r{}_{\theta r\theta}$$

$$R^\theta{}_{\phi\theta\phi} = \partial_\theta \Gamma^\theta{}_{\phi\phi} - \cancelto{0}{\partial_\phi \Gamma^\theta{}_{\theta\phi}} + \Gamma^\theta{}_{\theta a}\Gamma^a{}_{\phi\phi} - \Gamma^\theta{}_{\phi a}\Gamma^a{}_{\theta\phi}$$

$$= \partial_\theta \Gamma^\theta{}_{\phi\phi} + \Gamma^\theta{}_{\theta r}\Gamma^r{}_{\phi\phi} - \Gamma^\theta{}_{\phi\phi}\Gamma^\phi{}_{\theta\phi}$$

$$= \partial_\theta(-\sin\theta\cos\theta) - e^{-2B}\sin^2\theta + \cos^2\theta$$

$$= \sin^2\theta\,(1 - e^{-2B})$$

c) The components of the Ricci tensor can are computed contracting the Riemann tensor:

$$R_{ab} = R^c{}_{acb}$$

$$R_{tt} = R^r{}_{trt} + R^\theta{}_{t\theta t} + R^\phi{}_{t\phi t}$$

$$= -e^{2(A-B)}\left[A'B' - A'' - (A')^2\right]$$

$$- \frac{1}{r^2} e^{2A}\left[-r e^{-2B} A'\right]$$

$$- \frac{1}{r^2 \sin^2\theta} e^{2A}\left[-r e^{-2B}\sin^2\theta\, A'\right]$$

$$= e^{2(A-B)}\left[A'' + (A')^2 - A'B' + \frac{2}{r}A'\right]$$

$$R_{rr} = R^t{}_{rtr} + R^\theta{}_{r\theta r} + R^\phi{}_{r\phi r}$$

$$= A'B' - A'' - (A')^2$$

$$+ \frac{1}{r^2} e^{2B} \left(r e^{-2B} B' \right)$$

$$+ \frac{1}{r^2 \sin^2\theta} e^{2B} \left(r e^{-2B} \sin^2\theta \, B' \right)$$

$$= -A'' - (A')^2 + A'B' + \frac{2}{r} B'$$

$$R_{\theta\theta} = R^t{}_{\theta t\theta} + R^r{}_{\theta r\theta} + R^\phi{}_{\theta\phi\theta}$$

$$= -r e^{-2B} A' + r e^{-2B} B' + \frac{1}{\sin^2\theta} (1 - e^{-2B}) \sin^2\theta$$

$$= e^{-2B} \left[r(B' - A') - 1 \right] + 1$$

$R_{\phi\phi} = \sin^2\theta \, R_{\theta\theta}$ by spherical symmetry.