Bayesian Credibility has drawbacks.

1. It involves parameters in the prior and likelihood which must be estimated.

2. We may not be able to put the Bayesian estimator in the credibility framework for every case.

Addressing these drawbacks leads to Empirical Bayes Credibility Theory

It is similar to the normal/normal model, but instead of a prior we use collateral data.

There are two versions of EBCT, EBCT1 and EBCT2. EBCT1 assumes the same number of years of data for each collateral risk and gives them equal weight. EBCT2 does not.

Both EBCT1 and EBCT2 assume a r.v. θ, but don't assume any distribution for θ. Instead we estimate $E(X_j|\theta)$ and $Var(X_j|\theta)$ from the collateral data, where X_j is the aggregate claim for risk j.

EBCT 1

Let $X_1, \ldots, X_n$ denote either number of claims or aggregate claims for the risk in question in periods $1, \ldots, n$ which we know. We wan to estimate X_{n+1}.

Assumptions:

- The distribution of each X_i ~~depnd~~ depends on a parameter Θ
- Given Θ, $X_i | \theta$ are i.i.d.

Define $m(\theta) = E(X_i | \theta)$

$s^2(\theta) = Var(X_i | \theta)$

The Basic Estimator of X_{n+1}

$$Z \bar{X} + (1-Z) \, E[m(\theta)]$$

θ is fixed but unknown,

where $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

$$E[m(\Theta)] = E[E(X_j \mid \Theta)] = E[X_j]$$

and $$Z = \frac{n}{n + \frac{E[s^2(\Theta)]}{Var[m(\Theta)]}}$$

- The formula fo Z is similar to the formula for Z in the normal/normal model, for which $E[m(\Theta)] = \mu$, $Var[m(\Theta)] = \sigma_2^2$, $E[s^2(\Theta)] = \sigma_1^2$,
- We don't know $E[m(\Theta)]$, $E[s^2(\Theta)]$ or $Var[m(\Theta)]$ so we will estimate them from collateral risks.

Suppoe $X_{i,j}$ comes from risk i in year j.
Suppose we are interested in risk 1.
We can write $X_{i,j}$ in an array

N risks
n years

$$\begin{array}{cccc}
X_{1,1} & X_{1,2} & \cdots & X_{1,n} \\
X_{2,1} & X_{2,2} & \cdots & X_{2,n} \\
\vdots & & & \\
X_{N,1} & X_{N,2} & \cdots & X_{N,n}
\end{array}$$

collateral data

where N is the number of risks and
n is the number of years of data.

Assumptions about the $X_{i,j}$

1. The distribution of $X_{i,1}, \dots, X_{i,n}$ depends on parameter Θ_i.

2. Given Θ_i, $X_{i,j} \mid \Theta_i$ are iid.

3. For $i \neq j$ the pairs $(\Theta_i, (X_{i1}, \dots, X_{in}))$ and $(\Theta_j, X_{j1}, \dots, X_{jn}))$ are iid

and therefore $\Theta_1, \dots, \Theta_N$ are iid

We don't know the Θ_i.

The credibility estimate of $X_{1,n+1}$

$$Z\bar{X}_1 + (1-Z)E[\mu(\Theta)]$$

where $\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{i,j}$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} \bar{X}_i = \frac{1}{nN}\sum_{i=1}^{N}\sum_{j=1}^{n} X_{i,j}$$

we take

$$Z = \frac{n}{n + \frac{E[s^2(\theta)]}{Var[m(\theta)]}}$$

We use

$$\frac{1}{N}\sum_{i=1}^{N} \frac{1}{n-1}\sum_{j=1}^{n} (X_{i,j} - \bar{X}_i)^2$$

to estimate $E[s^2(\theta)]$

and ~~Var[m(~~

$$\frac{1}{N-1}\sum_{i=1}^{N} (\bar{X}_i - \bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N} \frac{1}{n-1}\sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2$$

to estimate $Var[m(\theta)]$

The first term is the natural estimator for $Var[m(\theta)]$, the second term makes the estimator unbiased.