Solution Quiz 4

Question 1.

Using the following Eviews output determine order p of AR(p) model and order q of MA(q) model you would fit to the data.

Correlogram of Y

Date: 04/10/20 Time: 09:39
Sample: 1 625
Included observations: 625

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.487	-0.487	149.22	0.000
		2	0.289	0.067	201.67	0.000
		3	0.012	0.232	201.77	0.000
		4	-0.016	0.074	201.94	0.000
		5	0.062	0.015	204.35	0.000
		6	-0.013	0.000	204.47	0.000
		7	-0.019	-0.051	204.70	0.000
		8	0.039	-0.001	205.68	0.000
		9	0.007	0.059	205.72	0.000
		10	-0.047	-0.033	207.13	0.000

Write down the model you would fit to the data.

Comment on how would you check the fit of your model to the data.

Brief solution of Question 1

(i) To select the order p for AR(p) model, we use the sample PACF function. We test the hypothesis

$H_0: \rho_k = 0$ against alternative $H_1: \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is the PACF function.

PACF $\hat{\rho}_k$ at lag k is significantly different from 0 at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then PACF at lag k is not significantly different from 0.

Rule: we select for p the largest lag k at which the PACF is significant.

This rule can be used because PACF of the AR(p) model becomes 0 for $k > p$.

(ii) To select the order q of MA(q) model, we use the sample ACF function. We test the hypothesis

$H_0: \rho_k = 0$ against alternative $H_1: \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is ACF function.

Rule: ACF ρ_k is significant at lag k at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

We select for q the largest lag k at which the ACF is significant.

(iii) We have $2/\sqrt{N} = 2/\sqrt{625} = 0.08$. The PACF is significant only at lag 1 and 3. Hence we would fit AR(3) model.

The ACF shows significant correlation at the lags 1 and 2. Hence we would fit MA(2) model.

From the two models AR(3) and MA(2) we select a simpler model MA(2) with smaller number of parameters which should to be fitted to the data.

(iv) According to above, we can fit MA(2) model $Y_t = \phi_0 + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$ where ε_t is white noise.

(v) This MA(2) model fits the data if residuals $\hat{\varepsilon}_t$ are uncorrelated. We could use the correlogram of residuals to test whether residuals are uncorrelated.

Question 2.

Using AIC information criterion values obtained fitting an AR(p) model, select the order p of an AR model you would fit to the data:

p	0	1	2	3	4	5	6
AIC	3	-2.3	-2	-1.1	0.6	1.7	1.8

Write down equation of your AR(p) model.

Solution of Question 2. Using AIC information criterion we select the model which minimizes the AIC value. In this case the minimum value -2.3 corresponds to $p = 1$, so AR(1) model should be fitted to the data.