1) Make use of the following information about three German companies:

	Correlation with				
Company	**Deutsche Lufthansa**	**Volkswagen**	**BMW**	**Risk Premium**	**Standard Deviation**
Deutsche Lufthansa	1	-.23	.58	12.00%	37.0%
Volkswagen	-.23	1	-.18	7%	76.3%
BMW	.58	-.18	1	35%	35.8%

a. Compute the tangency portfolio weights

b. If the annual interest rate is 5%, what is the expected return of the tangency portfolio?

Answer:

Covariance Table:

Company	**Deutsche Lufthansa**	**Volkswagen**	**BMW**	**Risk Premium**
Deutsche Lufthansa	0.1369	-0.06493	0.076827	12.00%
Volkswagen	-0.06493	0.582169	-0.04917	7.00%
BMW	0.076827	-0.04917	0.128164	35.00%

Let K have weight x_1 on DL, x_2 on VW and x_3 w_F on BMW

$$E(R_K) = x_1E(R_{DL}) + x_2E(R_{VW}) + x_3E(R_{BMW})$$

The β representation of the mean variance optimization problem

$$E(R_j) = r + \beta_j(E(R_K) - r)$$

$$\beta_j = \frac{\sigma_{jK}}{\sigma^2{}_K}$$

This means that

$$\frac{E(R_j) - r}{\sigma_{jK}} = \frac{E(R_K) - r}{\sigma^2{}_K}$$

The ratio of risk premium of every stock and its covariance with the tangency portfolio is identical and constant for every stock and let's denote it $\frac{1}{a}$.

$$\frac{E(R_j) - r}{\sigma_{jK}} = constant = \frac{1}{a}$$

$$\sigma_{jK} = [E(R_j) - r)] \times a$$

$$0.1369x_1 - 0.0649x_2 + 0.07682x_3 = 0.12a$$
$$-0.0649x_1 + 0.58217x_2 - 0.04917x_3 = 0.07a$$
$$0.07682x_1 - 0.04917x_2 + 0.12816x_3 = 0.35a$$

Together with the fact that the weights add up to 1, we have a system of 4 equations with 4 unknowns with solutions:

$x_1 =$ -30.90%
$x_2 = 10.96\%$
$x_3 = 119.93\%$

b) The return of **Deutsche Lufthansa:** $R_1 = 12\% + 5\% = 17\%$

The return of **Volkswagen:** $R2 = 7\% + 5\% = 12\%$
The return of **BMW:** $R_3 = 35\% + 5\% = 40\%$
The expected return of the tangency portfolio:

$$-0.309 \times 0.17 + 0.1096 \times 0.12 + 1.1993 \times 0.4 = 0.44$$