

Advanced machine learning

MTH793P 2024

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MATRIX COMPLETION

Introduction

- **Incomplete matrix:** $M = \begin{pmatrix} 1 & ? & 3 & ? & 5 \\ 2 & 4 & 6 & 8 & ? \\ ? & ? & 9 & 12 & 15 \end{pmatrix}$

- **Possible completions:**

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 1

$$\begin{pmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 2

$$\begin{pmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 4 & 6 & 8 & 7 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 3

Introduction

$$M = \begin{pmatrix} 1 & ? & 3 & ? & 5 \\ 2 & 4 & 6 & 8 & ? \\ ? & ? & 9 & 12 & 15 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 1

$$\begin{pmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 2

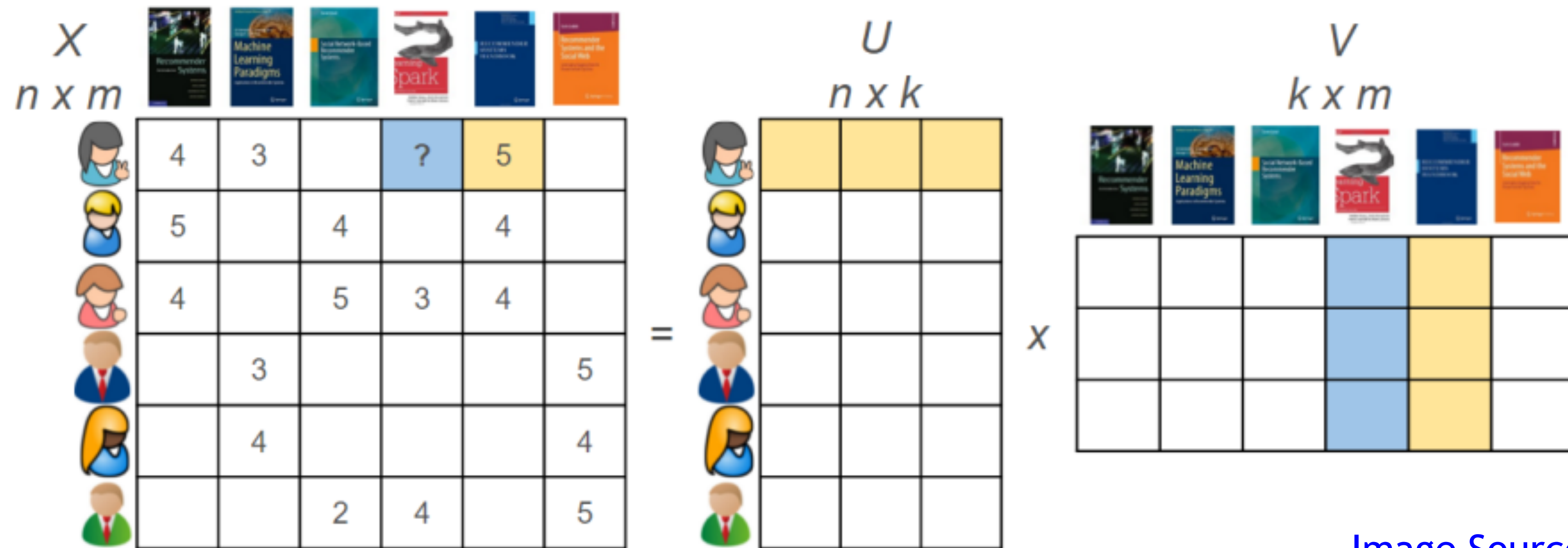
$$\begin{pmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 4 & 6 & 8 & 7 \\ 3 & 6 & 9 & 12 & 15 \end{pmatrix}$$

rank = 3

Goal: Low-rank matrix completion

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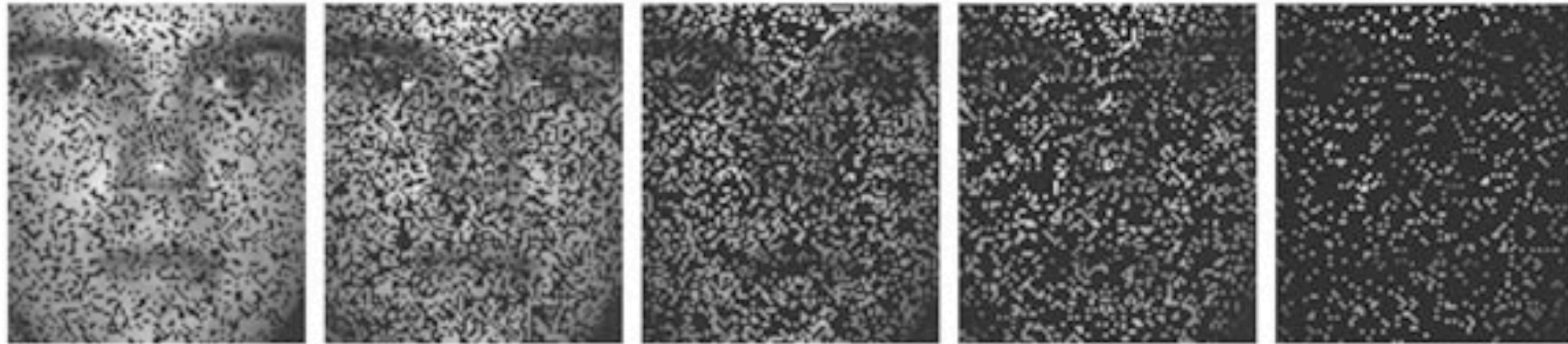
- Recommender systems:



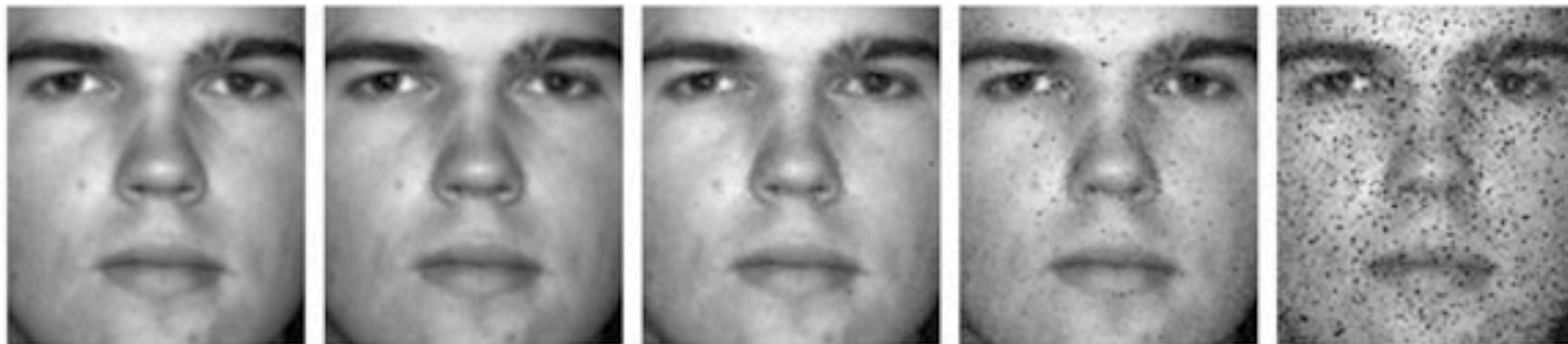
[Image Source](#)

- $n = \text{\#readers}$, $m = \text{\#books}$
- $k = \text{\#traits (clarity, depth, length, graphics, etc.)} = \text{rank}$

Applications



(a) Face images with (30, 50, 70, 80, 90)% percentage of missing entries



(d) Face images reconstructed by convex optimization with $\tau = 4 \times 10^5$