MTH 4104 Assessed Coursework I

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Q1. We see the equation as $[13]_{2024}[X]_{2024} = [4]_{2024}$. We firstly find the multiplicative inverse of $[13]$ in $\mathbb{Z}_{2024}$ by Euclid's algorithm. Since

$$\begin{aligned} 2024 &= 13 \cdot 155 + 9 \\ 13 &= 9 \cdot 1 + 4 \\ 9 &= 4 \cdot 2 + 1, \end{aligned}$$

we have $\gcd(2024, 13) = 1$ and

$$\begin{aligned} 1 &= 9 - 2 \cdot 4 \\ &= 9 - 2 \cdot (13 - 1 \cdot 9) \\ &= (-2) \cdot 13 + 3 \cdot 9 \\ &= (-2) \cdot 13 + 3 \cdot (2024 - 155 \cdot 13) \\ &= 3 \cdot 2024 + (-467) \cdot 13. \end{aligned}$$

Therefore $[-467]$ is the multiplicative inverse of $[13]$. Multiplying $[13][X] = [4]$ by $[-467]$ on both sides, we therefore obtain

$$[X] = [-467][4] = [-1868] = [156].$$

All integers congruent to 156 mod 2024 are the solutions to the congruence equation.

Q2.

$\circ$	(123)	(132)	(213)	(231)	(312)	(321)
(123)	(123)	(132)	(213)	(231)	(312)	(321)
(132)	(132)	(123)	(312)	(321)	(213)	(231)
(213)	(213)	(231)	(123)	(132)	(321)	(312)
(231)	(231)	(213)	(321)	(312)	(123)	(132)
(312)	(312)	(321)	(132)	(123)	(231)	(213)
(321)	(321)	(312)	(231)	(213)	(132)	(123)

Q3. (1) No. For example, $(213) \circ (132) = (231)$ but $(132) \circ (213) = (312)$. (2) $e = (123)$. In lectures, it is explained that the identity map on $\{1, 2, 3\}$ defines the identity element in the group and given the uniqueness of the identity element in a group, (123) has to be the one. Alternatively, one can check from the table that $(abc) \circ (123) = (abc) \circ (123) = (abc)$ for all possible (abc). (3) $s \circ s = e$, $r \circ r \circ r = e$ and $(s \circ r) \circ (s \circ r) = e$. Of course, any variant of $(s \circ r) \circ (s \circ r)$, e.g. $(r \circ s) \circ (r \circ s) = e$ is also admissible.

Marking Scheme. **Q1**. $+1$ for spotting the answer correctly and $+1$ for justification. **Q2**. $+3$ for filling in the table correctly. **Q3**. (1) $+1$ ($+0$ without justification) (2) $+1$ ($+0$ without justification) (3) $+3$ (the hardest one $+2$).