Advanced machine learning

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ROBUST PCA - INTRODUCTION

Motivation

- **Recall:** PCA = find lower dimensional data representation
  = low rank matrix approximation
- **Question:** What happens if we have outliers?
- Example:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ -3 & -6 & -9 & -12 & -15 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & -22 & 10 \\ -3 & 16 & -9 & -12 & -15 \end{pmatrix}$$

rank = 1 (low) rank = 3 (high)

Motivation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ -3 & -6 & -9 & -12 & -15 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & -22 & 10 \\ -3 & 16 & -9 & -12 & -15 \end{pmatrix}$$

rank = 1 (low)

rank = 3 (high)

!!!

$$\begin{pmatrix} 0.2033 & -2.0705 & 0.6100 & 3.1141 & 1.0166 \\ -0.8243 & 8.3939 & -2.4728 & -12.6248 & -4.1213 \\ -1.2101 & 12.3232 & -3.6304 & -18.5347 & -6.0506 \end{pmatrix}$$

rank-1 approximation

Proposed Solution

- Decomposition:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & -22 & 10 \\ -3 & 16 & -9 & -12 & -15 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ -3 & -6 & -9 & -12 & -15 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -30 & 0 \\ 0 & 22 & 0 & 0 & 0 \end{pmatrix}$$

low rank matrix sparse matrix

- Generally - find best decomposition of a matrix X as:

$$X = L_0 + E_0$$

low rank sparse

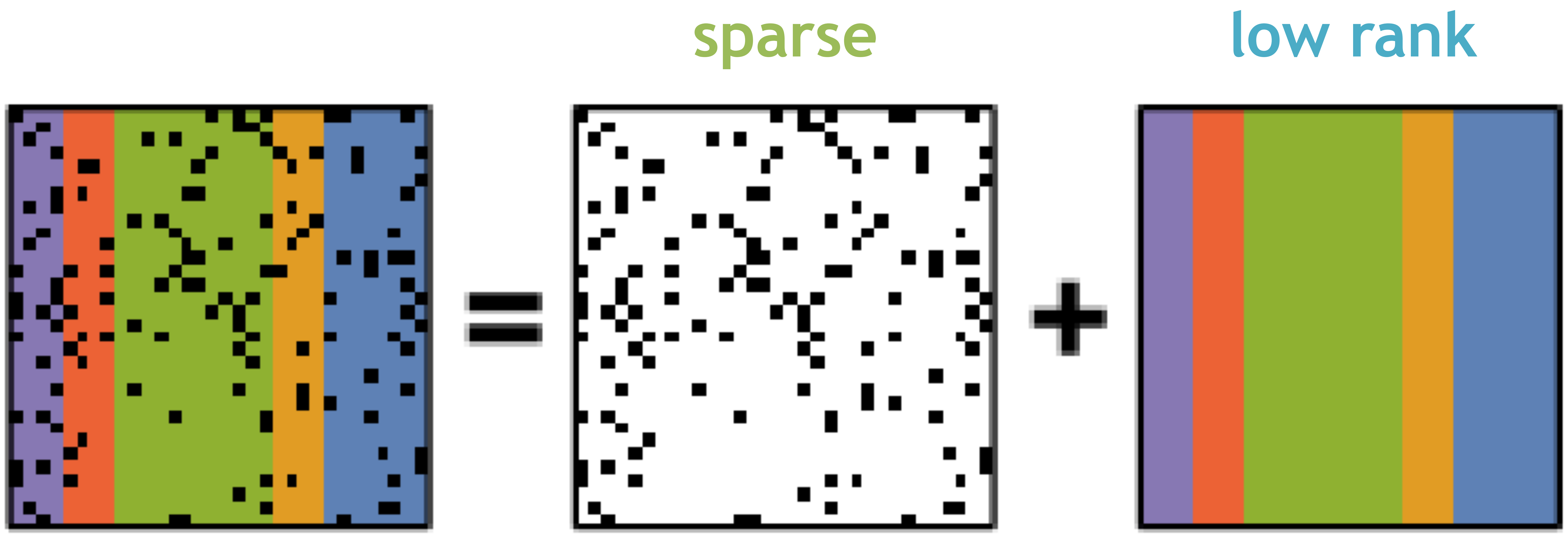

Image source

[Robust Principal Component Analysis? Candès et. al. 2011]

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(a) Original frames (b) Low-rank $\hat{L}$ (c) Sparse $\hat{S}$

[Robust Principal Component Analysis? Candès et. al. 2011]