Lost Monday. Det Let (R,+X) be a ting. If R had an element "1" which statistics to property: $1 \times \alpha = \alpha \times 1 = \alpha \quad \forall \alpha \in R$ Hon wo sky that R his identity. 0 the clentity demont wir, + " in R+2 is often called the additive islantity 1) 1 is often called the multiplication electry.

Example Zn (1) is to isentity Det If (R, +, X) is a ring with Teantity 1 an dement a in R is called a cuit 3bER st. axb = bxa = 1.Det RX ,= the set of wirls in R.

Examples
$$Z^{X} = \{\pm 1\}$$

 $Z(z) := \{a+bz \mid a,b\}$
 $z^{2} = 1$
 $Z(z)^{X} = \{\pm 1,\pm z\}$
 $Z(z)^{X} = \{\pm 1,\pm z\}$

(R, +, X) a ring Proposition 19 with teatity 1 · The isentity 1 is unique If 1 is distinct from the aditive isomity 0, How O is not a unit. e 1 78 a unit 8 its investe w,r,t, X is 1 Thelf

Pf Couk at Prop 14 for inspiration. Suppose 1 & S' are elements in R (1) S,t, $ta = \alpha r = \alpha \quad \forall \alpha \in R$ $S\alpha = \alpha S = \alpha \quad \forall \alpha \in R$ G(AL) r = S. Letting a = \$ in the former, I get VS = SStimilarly, a=r in the Catter, I get F\$ = F Combinios those two, S=r.

(2) Suppose that O is a unit (GOAL: Find contradiction) By definition, there exists a ER $5.4. \qquad 0 \cdot 0 = 0 \cdot a = 1$ OTOH, Prop16 says $\alpha \cdot 0 = 0 \cdot \alpha = 0$ Combining those two , $w_0 get 0 = 1$ This contradicts the assumption 0+1.

(3) Since 1 is the identity, ¥a∈R $1 \times \alpha = \alpha \times 1 = \alpha$ In particular $1 \times 1 = 1$ This equality stays that I is the inverse of 1 wirt. Prop 20 (R, +, X) is a ting with identity 1. · If a is a unit, Hen its inverse is unique.

The inverse is written as
$$a-1$$
.

If a is a a int a .

The inverse a is $a-1$.

The inverse a is a inverse a and a .

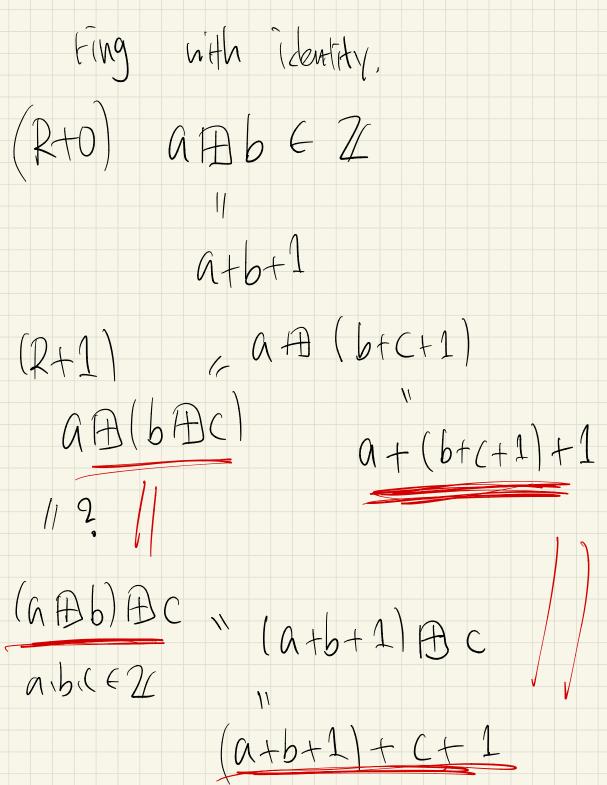
($a-1$)⁻¹ = a .

($a-1$)⁻¹ = a .

So is $ab = a \times b$ 8 to inverse a and a is $ab = a \times b$ 8 to inverse a and $ab = a \times b$ 8 to inverse $ab = a \times b$ 1.

Pt See nutos. They all follow from Prop 14. Thousan 21. If (R, +, X) is a tily with isentity, ten (RX, X) is a group. Is, in particular, R is a community of its,
Hen (RX,X) is an delian group.

 $(2/+,\times)$ Exercise is a tive of integers! Defino new adition a + b + 1 od additor "+" $\alpha Nb =$ a+b+abP P P N N O N X O O N X ¥ abt 21 (Z, B, Ω) is a commutative



(R+3) Given
$$a$$
 in a

I need to find b in a

get. $a + b = -1$.

This is the identity wiret.

Size $a + b + 1 = -1$

Ab $a + b = -4$

Need to check

 $a + b + 1 = -4$

Need to check

 $a + b + 1 = -4$

$$(A \times A) \Rightarrow A = (-1).$$

$$(A \times A) \Rightarrow A \Rightarrow (b+c+bc)$$

$$(A \times B) \Rightarrow C \Rightarrow A \Rightarrow (b+c+bc)$$

$$(A \times B) \Rightarrow C \Rightarrow A \Rightarrow (b+c+bc)$$

$$(A + b+ab) \Rightarrow C \Rightarrow A \Rightarrow (b+c+bc)$$

