

MTH5114 Linear Programming and Game Theory, Spring 2024
Week 6 Coursework Questions Viresh Patel

These exercises should be completed individually and submitted (together with those of weeks 4 and 5) via the course QMPlus page by **9am on Monday, 11 March**.

Make sure you clearly write your **name** and **student ID** number at the top of your submission:

It is generally recommended (here and in the exam) that in your answers, as well as giving the tableaux, you also highlight the appropriate rows and columns and mention the row operations you are carrying out – this way, even if your final answer is incorrect, we can easily award you credit for using the right method.

1. Solve the following linear programme using the 2-phase simplex algorithm. You should give the initial tableau and each further tableau produced during the execution of the algorithm. If the program has an optimal solution, give this solution and state its objective value. If it does not have an optimal solution, say why.

$$\begin{aligned} \text{maximize} \quad & x_1 - 2x_2 + x_3 - 4x_4 \\ \text{subject to} \quad & 2x_1 + x_2 - 2x_3 - x_4 \geq 1, \\ & 5x_1 + x_2 - x_3 - x_4 \leq -1, \\ & 2x_1 + x_2 - x_3 - 3x_4 \geq 2, \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

2. Apply the first phase of the 2-phase simplex algorithm to the following linear programme giving the initial tableau and each further tableau produced. Give the starting tableau for the second phase if there is one.

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & x_2 - x_3 \leq 2, \\ & x_1 + 3x_2 + 2x_3 \geq 3, \\ & 2x_1 + 2x_2 + x_3 = 4, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$