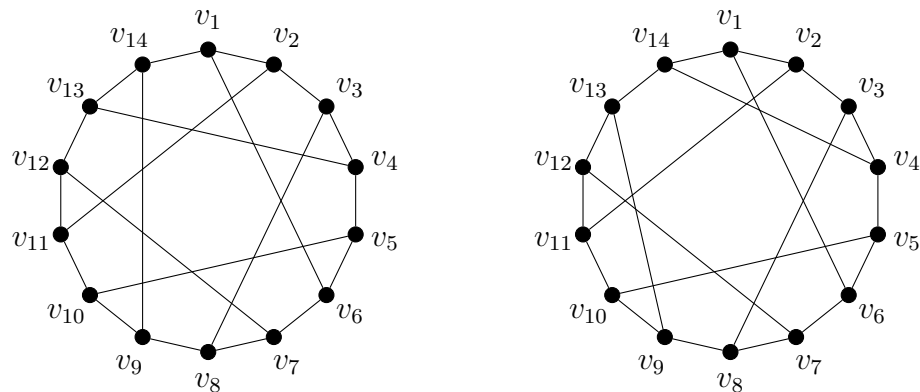
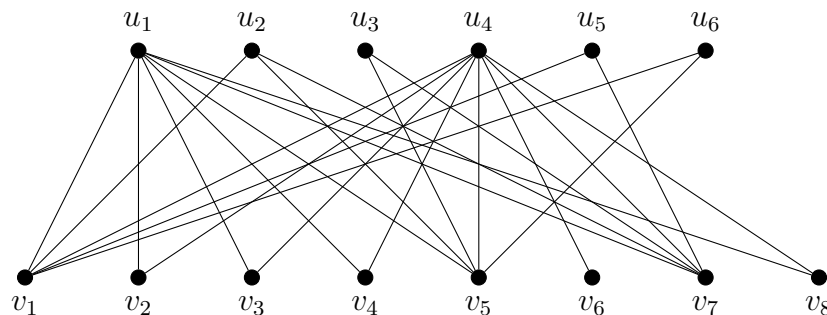


You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

1. For each of the following two graphs, determine if the graph is bipartite. Justify your answer.



2. Consider the following bipartite graph  $G$ .



- (a) Show that  $M = \{u_1v_1, u_2v_5, u_3v_7, u_4v_2\}$  is a matching of  $G$ .  
 (b) Give an  $M$ -augmenting path of  $G$ .  
 (c) Give a maximum matching of  $G$ .
3. A graph  $G$  is called  $k$ -regular, for  $k \in \mathbb{N}$ , if  $d_G(v) = k$  for all  $v \in V(G)$ . Let  $G$  be a  $k$ -regular bipartite graph  $G$  with parts  $L$  and  $R$ .
- (a) Show that  $|L| = |R|$ .  
 (b) Show that the directed network  $(D_G, c_G)$  has an  $s$ - $t$ -flow of size  $|L| = |R|$ .  
 (c) Show that  $G$  has a perfect matching.