MTH6105 – Algorithmic Graph Theory **Spring 2024**

Problem Sheet 9 **F. Fischer**

You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

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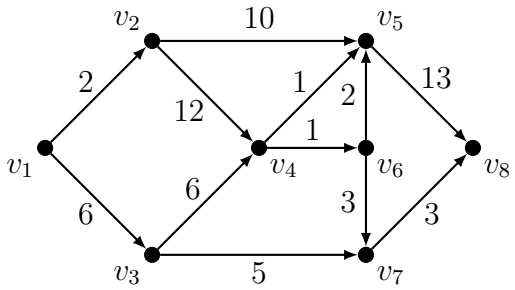
1. Consider the following directed network.

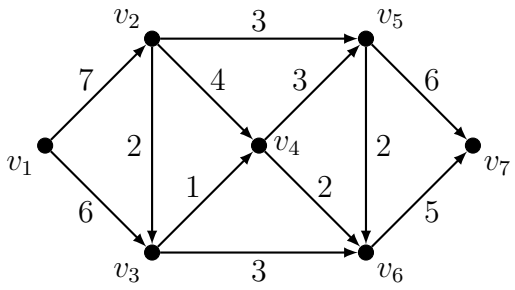


   (a) Apply the Ford-Fulkerson algorithm to the network, drawing the residual network after each iteration.

   (b) Give a maximum v_1-v_8-flow of the network.

   (c) Prove that the v_1-v_8-flow you have found is indeed a maximum v_1-v_8-flow.

2. Consider the following directed network.



   Let g be the v_1-v_7-flow of this network with

$$g(v_1v_2) = 7, \quad g(v_1v_3) = 2, \quad g(v_2v_3) = 2, \quad g(v_2v_4) = 4,$$
$$g(v_2v_5) = 1, \quad g(v_3v_4) = 1, \quad g(v_3v_6) = 3, \quad g(v_4v_5) = 3,$$
$$g(v_4v_6) = 2, \quad g(v_5v_6) = 0, \quad g(v_5v_7) = 4, \quad g(v_6v_7) = 5.$$

   (a) Prove or disprove that g is a maximum v_1-v_7-flow of the network.

   (b) Imagine that $c(v_3v_6)$ is decreased from 3 to 1. Does this affect the size of a maximum flow? Justify your answer.

   (c) Imagine that $c(v_3v_6)$ is increased from 3 to 4. Does this affect the size of a maximum flow? Justify your answer.