Please fill in feedback questionnaire on QMplus!

Recap quiz

An LP max $\underline{c}^T \underline{x}$

sub to $A\underline{x} = \underline{b}$, $\underline{x} \geq \underline{0}$

is called ________ if, for every $k \geq 0$ there exists a ________ $\underline{x}$ such that ____ $\geq k$

Suppose we apply simplex to above LP and final tableau is

	x_1	x_2	s_1	s_2	
s_1	1	-3	1	0	3
s_2	0	-8	2	1	10
	0	11	-3	0	-9

What is the next step?

	x_1	x_2	x_3	s_1	s_2	
s_1	-1	0	0	1	0	6
x_3	$\frac{3}{4}$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}$
s_2	$\frac{1}{2}$	-3	0	0	1	3
$-z$	4	11	0	0	0	-12

	x_1	x_2	x_3	s_1	s_2	
s_1	-1	0	0	1	0	6
x_2	$\frac{3}{2}$	1	2	0	0	3
s_2	5	0	6	0	1	12
$-z$	$-\frac{25}{2}$	0	-22	0	0	-45

Q: We found optimal solution for (B) with objective value 0. (in phase 1).

what would it mean for (A) if instead

(i) (B) had optimal solution whose objective was not zero

(ii) (B) was infeasible

(iii) (B) was unbounded

(A)

maximise $10x_1 + 15x_2 + 8x_3$

sub to

$$8x_1 + 6x_2 + 12x_3 + s_1 = 24$$
$$-4x_1 - 6x_2 - 6x_3 + s_2 = -6$$
$$6x_1 + 4x_2 + 8x_3 = 12$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0.$$

(B)

max $-a_1 - a_2$

sub to

$$8x_1 + 6x_2 + 12x_3 + s_1 = 24$$
$$-4x_1 - 6x_2 - 6x_3 + s_2 - a_1 = -6$$
$$6x_1 + 4x_2 + 8x_3 + a_2 = 12$$
$$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$$

Formal description of 2-phase simplex

① Given LP transform into standard equation form with slack variables just as before

② - If there is constraint with a slack variable s and $b<0$

say $a_1x_1 + a_2x_2 + \cdots + a_nx_n + s = b$

then replace with $a_1x_1 + a_2x_2 + \cdots + a_nx_n + s - a = b$

- If there is constraint with no slack variable

say $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

then replace with $a_1x_1 + a_2x_2 + \cdots + a_nx_n + a' = b$

Here a, a' are artificial variables with sign restriction $a, a' \geq 0$
Each replaced constraint gets a different artificial variable.

③ Form initial tableau as before except

- Variables on the left

  If a constraint has an artificial variable, put it on the left
  If a constraint has no artificial variable, put its slack variable on the left
  (Recall variables listed on the left are the basic variables in our current basic feasible solution)

- Two rows below the line

  We have a row R_w for our phase 1 objective which has a -1 for each artificial variable and zeros everywhere else

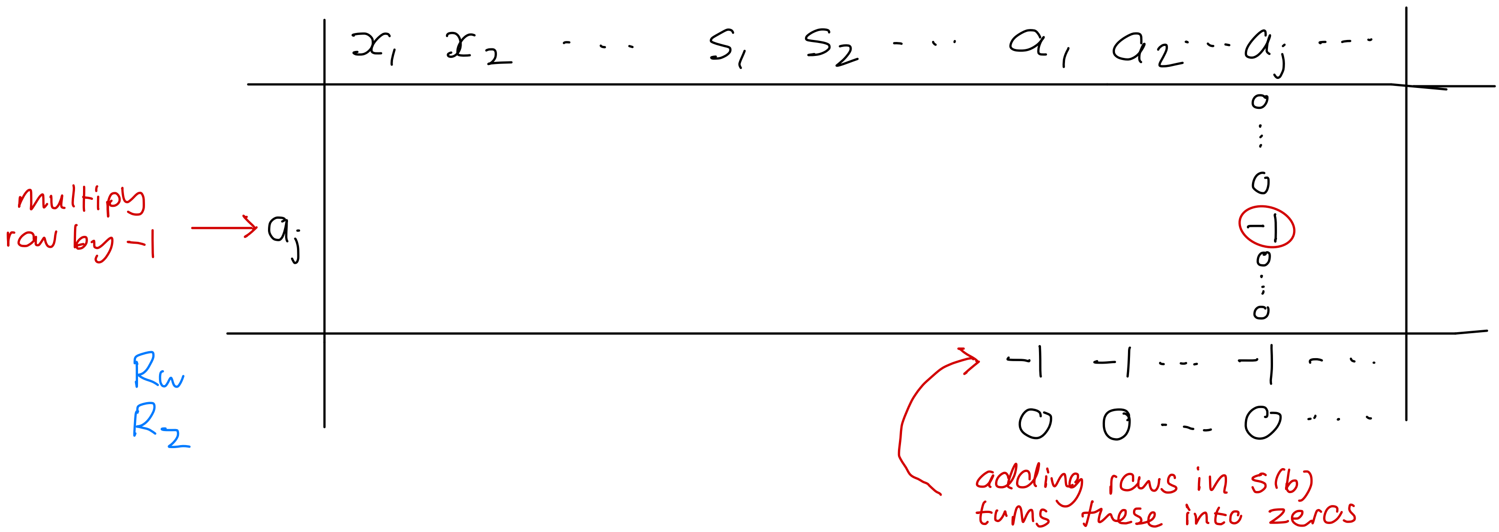
  We have a row R_z for our LP objective just as before.

④ Bring tableau into valid form so that each artificial variable has a single 1 and all other zeros in its column

Do this as follows:

(a) Ignoring R_z and R_w multiply rows by -1 where necessary so that each artificial variable has in its column (above the line) a single 1 (rather than -1)

(b) Every row with an artificial variable on the left is added to R_w (to remove -1's in columns of artificial variables).



⑤ Now apply standard simplex treating R_w as our objective.

(When clearing a column in a pivot, we also make sure we clear the column entry in R_z).

⑥ (a) If far right entry of R_w is 0 then have found our starting basic feasible solution.

Delete R_w and columns of artificial variables.

Apply standard simplex to this tableau with R_z as objective.

(We call this phase 2).

⑥ (a) If far right entry of R_w is 0 then have found our starting basic feasible solution.
Delete R_w and columns of artificial variables.
Apply standard simplex to this tableau with R_z as objective.
(We call this phase 2).

(b) If far right entry of R_w is > 0 then our original LP is infeasible.

<u>Note</u> It could happen in 6(a) that far right entry of R_w is zero, but some artificial variable is basic (i.e. appears on left).

Then we cannot immediately proceed to apply standard simplex.

Small, relatively easy step to deal with this but omitted here (and non-examinable).

Want you to get the main ideas and not be distracted by pathological situations.

Does simplex algorithm always terminate?

- Sometimes a pivot operation does not change far right column, i.e. sometimes BFS stays unchanged and objective does not improve.

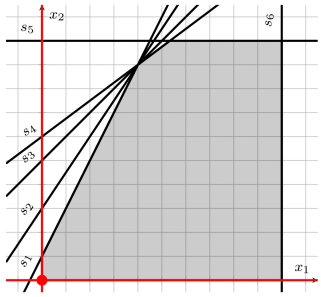
  Example on next page.

- when we apply the rules of simplex it can happen that we end up with exactly the same tableau we saw earlier!

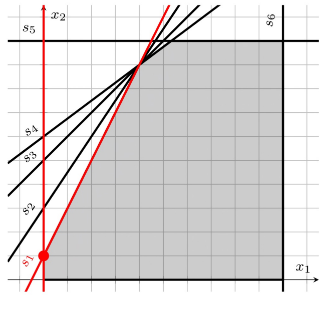
  This is called cycling. (example next page)

- By adjusting the "tie-break" rules, we can avoid this and ensure simplex always terminates. We omit the details here.

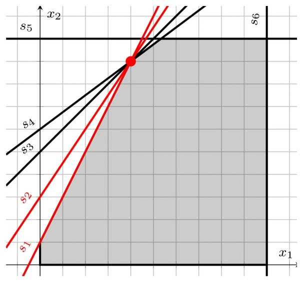
	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
s_1	-2	1	1	0	0	0	0	0	1
s_2	$-\frac{3}{2}$	1	0	1	0	0	0	0	3
s_3	-1	1	0	0	1	0	0	0	5
s_4	$-\frac{3}{4}$	1	0	0	0	1	0	0	6
s_5	0	1	0	0	0	0	1	0	10
s_6	1	0	0	0	0	0	0	1	10
$-z$	0	1	0	0	0	0	0	0	0



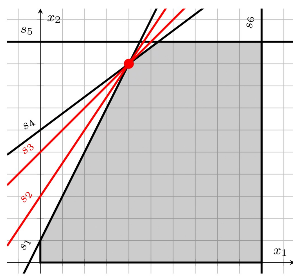
	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
x_2	-2	1	1	0	0	0	0	0	1
s_2	$\frac{1}{2}$	0	-1	1	0	0	0	0	2
s_3	1	0	-1	0	1	0	0	0	4
s_4	$\frac{5}{4}$	0	-1	0	0	1	0	0	5
s_5	2	0	-1	0	0	0	1	0	9
s_6	1	0	0	0	0	0	0	1	10
$-z$	2	0	-1	0	0	0	0	0	-1



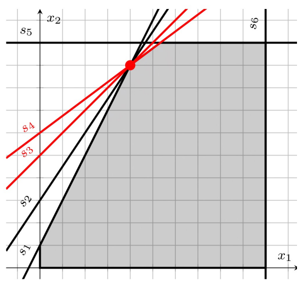
	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
x_2	0	1	-3	4	0	0	0	0	9
x_1	1	0	-2	2	0	0	0	0	4
s_3	0	0	1	-2	1	0	0	0	0
s_4	0	0	$\frac{3}{2}$	$-\frac{5}{2}$	0	1	0	0	0
s_5	0	0	3	-4	0	0	1	0	1
s_6	0	0	2	-2	0	0	0	1	6
$-z$	0	0	3	-4	0	0	0	0	-9



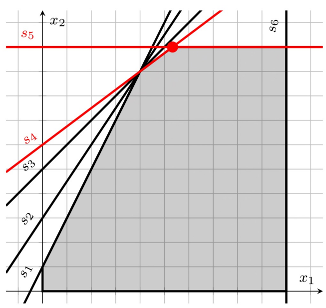
	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
x_2	0	1	0	-2	3	0	0	0	9
x_1	1	0	0	-2	2	0	0	0	4
s_1	0	0	1	-2	1	0	0	0	0
s_4	0	0	0	$\frac{1}{2}$	$-\frac{3}{2}$	1	0	0	0
s_5	0	0	0	2	-3	0	1	0	1
s_6	0	0	0	2	-2	0	0	1	6
$-z$	0	0	0	2	-3	0	0	0	-9



	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
x_2	0	1	0	0	-3	4	0	0	9
x_1	1	0	0	0	-4	4	0	0	4
s_1	0	0	1	0	-5	4	0	0	0
s_2	0	0	0	1	-3	2	0	0	0
s_5	0	0	0	0	3	-4	1	0	1
s_6	0	0	0	0	4	-4	0	1	6
$-z$	0	0	0	0	3	-4	0	0	-9



	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	
x_2	0	1	0	0	0	0	1	0	10
x_1	1	0	0	0	0	$-\frac{4}{3}$	$\frac{4}{3}$	0	$\frac{16}{3}$
s_1	0	0	1	0	0	$-\frac{8}{3}$	$\frac{5}{3}$	0	$\frac{5}{3}$
s_2	0	0	0	1	0	-2	1	0	1
s_3	0	0	0	0	1	$-\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$
s_6	0	0	0	0	0	$\frac{4}{3}$	$-\frac{4}{3}$	1	$\frac{14}{3}$
$-z$	0	0	0	0	0	0	-1	0	-10



	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0.5	−5.5	−2.5	9	1	0	0	0
x_6	0.5	−1.5	−0.5	1	0	1	0	0
x_7	1	0	0	0	0	0	1	1
$-z$	10	−57	−9	−24	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_1	1	−11	−5	18	2	0	0	0
x_6	0	4	2	−8	−1	1	0	0
x_7	0	11	5	−18	−2	0	1	1
$-z$	0	53	41	−204	−20	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_1	1	0	0.5	−4	−0.75	2.75	0	0
x_2	0	1	0.5	−2	−0.25	0.25	0	0
x_7	0	0	−0.5	4	0.75	−2.75	1	1
$-z$	0	0	14.5	−98	−6.75	−13.25	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_3	2	0	1	−8	−1.5	5.5	0	0
x_2	−1	1	0	2	0.5	−2.5	0	0
x_7	1	0	0	0	0	0	1	1
$-z$	−29	0	0	18	15	−93	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_3	−2	4	1	0	0.5	−4.5	0	0
x_4	−0.5	0.5	0	1	0.25	−1.25	0	0
x_7	1	0	0	0	0	0	1	1
$-z$	−20	−9	0	0	10.5	−70.5	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	−4	8	2	0	1	−9	0	0
x_4	0.5	−1.5	−0.5	1	0	1	0	0
x_7	1	0	0	0	0	0	1	1
$-z$	22	−93	−21	0	0	24	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0.5	−5.5	−2.5	9	1	0	0	0
x_6	0.5	−1.5	−0.5	1	0	1	0	0
x_7	1	0	0	0	0	0	1	1
$-z$	10	−57	−9	−24	0	0	0	0