MTH 5125 May 2019. Solutions.

Q1.

(a) P.V. benefits + P.V. expenses = P.V. premiums if gross premium is G.

$$60000 A_{38} + 2000 + 0.03 G \ddot{a}_{38} + 100 A_{38}$$
$$= G \ddot{a}_{38}$$

$$60100 A_{38} + 2000 = 0.97 G \ddot{a}_{38}$$

(b) $\ddot{a}_{38} = \dfrac{1 - A_{38}}{d} = \dfrac{1.04}{0.04}(1 - 0.255) = 19.37$

$$\therefore G = \frac{60100 \times 0.255 + 2000}{0.97 \times 19.37} = \underline{\underline{£922.11}}$$

(c) $L_0 = 60100 v^{K_{38}+1} + 2000 - 0.97 G \ddot{a}_{\overline{K_{38}+1|}}$

(d) we seek smallest G that gives $P(L_0 > 0) < 0.05$ or $P(L_0 \leq 0) \geq 0.95$

if G_n is premium for which $L_0 = 0$ when $K_{38} = n$ we need,

$$P(L_0 \leq 0 \mid G = G_n) = P(K_{38} > n) \geq 0.95$$

now

$P(K_{38} > n) \geq 0.95 \Rightarrow l_{38+n} \geq 0.95 \times l_{38}$

$0.95\, l_{38} = 8949$ $\quad l_{45} = 8980 \quad l_{46} = 8915 \quad \therefore n = 7$

smallest annual premium is

$$G_7 = \frac{60100 v^8 + 2000}{0.97\, \ddot{a}_{\overline{8|}}} \text{ @ } 4\% \qquad v^8 = 0.73069 \quad \ddot{a}_{\overline{8|}} = 7.0027$$

$G_7 = \underline{\underline{£ ~~[illegible]~~ 6760.}}$

Q2

(a) Policy benefit is Last Survivor whilst premiums only payable during joint lifetime

under equivalence principle

P.V. benefits + P.V. expenses = P.V. premiums

if gross premium is G

$$50000 A_{\overline{40:40}} + 1000 + 0.025 G \ddot{a}_{40:40} = G \ddot{a}_{40:40}$$

now

$$A_{\overline{40:40}} = A_{40} + A_{40} - A_{40:40}$$

$$= 0.272 + 0.272 - 0.505 = 0.039$$

$$\ddot{a}_{40:40} = \frac{1 - A_{40:40}}{d} = \frac{1.03}{0.03}(1 - 0.505) = 16.995$$

$$50000 A_{\overline{40:40}} + 1000 = 0.975 G \ddot{a}_{40:40}$$

$\therefore$

$$G = \frac{50000 \times 0.039 + 1000}{0.975 \times 16.995} = \underline{\underline{£178.03}}$$

(b) Prospective gross premium reserve at $t = \overline{10|}$ is

$${}_{10}V = 50000 A_{\overline{40+10:40+10}} - 0.975 \times 178.03 \ddot{a}_{40+10:40+10}$$

$$A_{\overline{50:50}} = 0.383 + 0.383 - 0.592 = 0.174$$

$$\ddot{a}_{50:50} = \frac{1.03}{0.03}(1 - 0.592) = 14.008$$

$${}_{10}V = 50000 \times 0.174 - 0.975 \times 178.03 \times 14.008$$

$$= \underline{\underline{£6268.50}}$$

(c) After the first death the benefit becomes a single life whole life assurance on the remaining life and premiums stop $\therefore$ for both these reasons the reserve jumps up and

$${}_{10}V' = 50000 A_{50}$$

Q3.

(a) $\bar{a}_{xy} = \int_0^\infty v^t \, {}_tp_{xy} \, dt$

if force of mortality is constant ${}_tp_x = e^{-\mu t}$

$\therefore$ here ${}_tp_{xy} = {}_tp_x \, {}_tp_y = e^{-0.03t} e^{-0.04t}$

$$\bar{a}_{xy} = \int_0^\infty \frac{e^{-0.03t} e^{-0.04t}}{(1.02)^t} \, dt$$

$$= \int_0^\infty e^{-(0.03 + 0.04 + \ln(1.02))t} \, dt$$

$$= \frac{1}{0.07 + \ln 1.02} = 11.1355$$

$\therefore$ PV joint life annuity $= 100 \times 11.1355 = \underline{\underline{£1113.55}}$

(b) if amount of reversionary annuity p.a. is R we seek

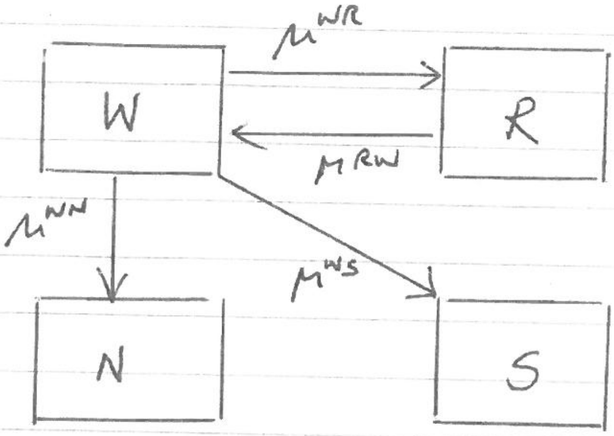
$$R \, \bar{a}_{y|x} = 20000$$

$$\bar{a}_{y|x} = \bar{a}_x - \bar{a}_{xy} = \frac{1}{0.03 + \ln 1.02} - 11.1355$$

$$\therefore R = \frac{20000}{20.079 - 11.136} = \underline{\underline{£2236 \text{ p.a.}}}$$

Q4.

(a)



[other notation fine]

4 states

W - phone working R - phone not working being repaired

N - phone not working needs replacement S - stolen

4 transition intensities

μ^{WR} transition intensity from working to being repaired

μ^{RW} " " " being repaired to working

μ^{WN} " " " working to needs replacement

μ^{WS} " " " working to stolen

(b) We seek ${}_1p_x^{\overline{WW}}$ (t = 1 year)

$${}_1p^{\overline{WW}} = \exp\left\{-\int_0^1 \sum_{i \neq W} \mu_{x+z}^{Wi}\, dz\right\}$$

$$= \exp\left\{-\int_0^1 \left(\mu_{x+z}^{WR} + \mu_{x+z}^{WN} + \mu_{x+z}^{WS}\right) dz\right\}$$

[note - no reason why these transition intensities should be constant over the year]

Q5.

(a) We assume $(a\mu)_x^d = \mu_x^d$ and $(a\mu)_x^c = \mu_x^c$ all x

first calculate (aq) probabilities with

$$(aq)_x = 1 - \exp[-(\mu_x^d + \mu_x^c)]$$

then

$$(aq)_x^d = \frac{\mu_x^d}{\mu_x^d + \mu_x^c}(aq)_x$$ & similarly $(aq)_x^c$'s

x	$(aq)_x$	$(aq)_x^d$	$(aq)_x^c$
40	0.002696	0.002197	0.000499
41	0.002896	0.002397	0.000499
42	0.003295	0.002696	0.000599

then $(ad)_x^d = (al)_x \times (aq)_x^d$

$(ad)_x^c = (al)_x \times (aq)_x^c$

$(al)_{x+1} = (al)_x - (ad)_x^d - (ad)_x^c$

giving the multi-dec table

x	$(al)_x$	$(ad)_x^d$	$(ad)_x^c$
40	100 000	219.7	49.9
41	99 730.4	239.0	49.8
42	99 441.6	268.0	59.6
43	99 114.0		

5 (b) Assume on average any benefit paid half-way through the relevant policy year

$$\text{P.V. death benefit} = \frac{40\,000}{(al)_{40}}\left[(ad)^d_{40} v^{\frac{1}{2}} + (ad)^d_{41} v^{1\frac{1}{2}} + (ad)^d_{42} v^{2\frac{1}{2}}\right]$$

$$\text{P.V. illness benefit} = \frac{70\,000}{(al)_{40}}\left[(ad)^c_{40} v^{\frac{1}{2}} + (ad)^c_{41} v^{1\frac{1}{2}} + (ad)^c_{42} v^{2\frac{1}{2}}\right]$$

$i = 3\%$. $v^{\frac{1}{2}} = 0.985329$ $v^{1\frac{1}{2}} = 0.956630$ $v^{2\frac{1}{2}} = 0.928767$

$$\text{P.V death benefit} = \frac{40\,000}{100\,000} \times 694.0211 = \underline{\underline{£277.61}}$$

$$\text{P.V. illness benefit} = \frac{70\,000}{100\,000} \times 152.1627 = \underline{\underline{£106.51}}$$

Q6

(a)

Time selection - at a given age, mortality rates change over time

e.g. - advances in medical science reducing mortality rates at older ages

Spurious selection - time or temporary initial selection being distorted by other factors

e.g. - change in underwriting practice at annuity provider

Adverse selection - policyholder selecting against insurer

e.g. - groups with greater than average longevity buying annuities

(b) consider 2010 as base year

someone born 1960 was 50 in base year so denote probability of death in next year then as $q_{[50]}$

now in 2019 same life is 59. We can denote their future probability of death in the next year as $q_{[50]+9}$

and with improvement factor of 0.15% p.a.

$$q_{[50]+9} = \frac{q_{[59]}}{(1.0015)^9}$$

where

$q_{[59]}$ is mortality rate from 2010 investigation

$q_{[50]+9}$ is rate to be used in 2019 calculations

of course in practice very unlikely improvement factor will be same for all years and all ages.