The joint life status

- Status that survives so long as all members are alive, and therefore fails upon the first death.
- Notation: (xy) for two lives (x) and (y)
- For two lives: $T_{xy} = \min(T_x, T_y)$

${}_tp_{xy}$ - the probability that both lives (x) and (y) survive after t years.
In the case where T_x and T_y are independent:

$$ {}_tp_{xy} = {}_tp_x \times {}_tp_y $$

$$ {}_tq_{xy} = {}_tq_x + {}_tq_y - {}_tq_x \, {}_tq_y $$

The last survivor status

- Status that survives so long as there is at least one member alive, and therefore fails upon the last death.
- Notation: $(\overline{xy})$
- For two lives: $T_{\overline{xy}} = \max(T_x, T_y)$

$$ {}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy} $$

Interpretation of Survival function

$$ S_{T_{\overline{xy}}}(t) \overset{\text{not}}{=} {}_tp_{\overline{xy}} $$

$$ \Leftrightarrow $$

$$ S_{T_{\overline{xy}}}(t) = {}_tp_x \, {}_tp_y + {}_tp_x \, (1 - {}_tp_y) + {}_tp_y \, (1 - {}_tp_x) $$

- ${}_tp_x \, {}_tp_y$ means that both x and y alive after t years
- ${}_tp_x \, (1 - {}_tp_y)$ means that x is alive and y is dead after t years
- ${}_tp_y \, (1 - {}_tp_x)$ means that y is alive and x is dead after t years

Interpretations of probabilities

${}_tp_{xy}$ is the probability that both lives (x) and (y) will be alive after t years.

${}_tp_{\overline{xy}}$ is the probability that at least one of lives (x) and (y) will be alive after t years.

In contrast:

${}_tq_{xy}$ is the probability that at least one of lives (x) and (y) will be dead within t years.

${}_tq_{\overline{xy}}$ is the probability that both lives (x) and (y) will be dead within t years.

Force of mortality for joint life status
In the case of independence

$$f_{T_{xy}}(t) = {}_tp_x \ {}_tp_y \left(\mu_{x+t} + \mu_{y+t}\right)$$

$$\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t}$$

Force of mortality for life survivor status
Then the pdf of T_{xy} :

$$f_{T_{\overline{xy}}}(t) = {}_tp_{\overline{xy}} \times \mu_{\overline{x+t:y+t}}$$

Life Tables

$$\begin{aligned}
{}_tp_{xy} &= {}_tp_x \ {}_tp_y \\
l_{xy} &= l_x l_y \\
{}_tp_{xy} &= \frac{l_{x+t:y+t}}{l_{x:y}} \\
d_{xy} &= l_{xy} - l_{x+1:y+1} \\
q_{xy} &= \frac{d_{xy}}{l_{xy}}
\end{aligned}$$

Curtate Joint Life

$$\begin{aligned}
P[K_{xy} &= k] = P[k \le T_{xy} \le k+1] \\
&= F_{T_{xy}}(k+1) - F_{T_{xy}}(k) \\
&= (1 - {}_{k+1}p_{xy}) - (1 - {}_kp_{xy}) \\
&= {}_kp_{xy} - {}_{k+1}p_{xy} \\
&= {}_kp_{xy} - {}_kp_{xy} \ p_{x+k:y+k} \\
&= {}_kp_{xy}\,(1 - p_{x+k:y+k}) \\
&= {}_kp_{xy} \ q_{x+k:y+k} \\
&= {}_{k|}q_{xy}
\end{aligned}$$

Curtate Last Survivor lifetime

$$\begin{aligned}
P[K_{\overline{xy}} &= k] = P[k \le T_{\overline{xy}} \le k+1] \\
&= F_{T_{\overline{xy}}}(k+1) - F_{T_{\overline{xy}}}(k) \\
&= F_{T_x}(k+1) + F_{T_y}(k+1) - F_{T_{xy}}(k+1) \\
&\quad - \left(F_{T_x}(k) + F_{T_y}(k) - F_{T_{xy}}(k)\right) \\
&= P[K_x = k] + P[K_y = k] - P[K_{xy} = k] \\
&= {}_{k|}q_x + {}_{k|}q_y - {}_{k|}q_{xy}
\end{aligned}$$