MVPT Practice Set

$E_Q = 0.09$

$E_A = E(R_A) = 0.08$

$E_B = E(R_B) = 0.12$

Q2

Set up the Lagrangian:

$$\mathcal{L}(\lambda, \mu, w_A, w_B) = w_A^2 V_A + w_B^2 V_B + 2 w_A w_B C_{AB} + \lambda(E_Q - w_A R_A - w_B R_B) + \mu(1 - w_A - w_B)$$

F.O.C:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 : \quad 0.09 - 0.08 w_A - 0.12 w_B = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = 0 : \quad 1 - w_A - w_B = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_A} = 0 : \quad 2 w_A V_A + 2 w_B C_{AB} - \lambda \times 0.08 - \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_B} = 0 : \quad 2 w_B V_B + 2 w_A C_{AB} - \lambda \times 0.12 - \mu = 0$$

You don't need λ ; μ => So use the first two equations with 2 unknowns w_A, w_B =>

$w_A = 0.75$; $w_B = 0.25$

Q3:

$E = w_A E_A + w_B E_B = 0.08\, w_A + 0.12(1-w_A) \Rightarrow$

$$w_A = \frac{0.12 - E}{0.04} \qquad w_B = \frac{E - 0.08}{0.04}.$$

Substitute w_A, w_B into:

$$\sigma^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2\rho w_A w_B \sigma_A \sigma_B$$

$$\Rightarrow \quad \sigma^2 = \left(\frac{0.12-E}{0.04}\right)^2 \times 0.0016 + \left(\frac{E-0.08}{0.04}\right)^2 \times 0.0064 + $$

$$+ 2 \times 0.2 \left(\frac{0.12-E}{0.04}\right)\left(\frac{E-0.08}{0.04}\right) \times 0.04 \times 0.08$$

Quadratic equation in E; find solutions: the highest one on the EFFICIENT FRONTIER

Q2 (page 5/8)

$E_A = 0.09$

$E_B = 0.06$

$\sigma_A = 0.2$

$\sigma_B = 0.10$

$\boxed{\rho = -1}$

$$R_F = w_A R_A + (1 - w_A) R_B$$

$$Var(R_F) \equiv \sigma_F^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2$$

$$+ 2 \rho w_A w_B \sigma_A \sigma_B$$

$$\Downarrow$$

$$\sigma_F^2 = w_A^2 \sigma_A + (1 - w_A)^2 \sigma_B^2$$

$$- 2 w_A w_B \sigma_A \sigma_B$$

$$= \left[w_A \sigma_A + (1 - w_A) \sigma_B \right]^2$$

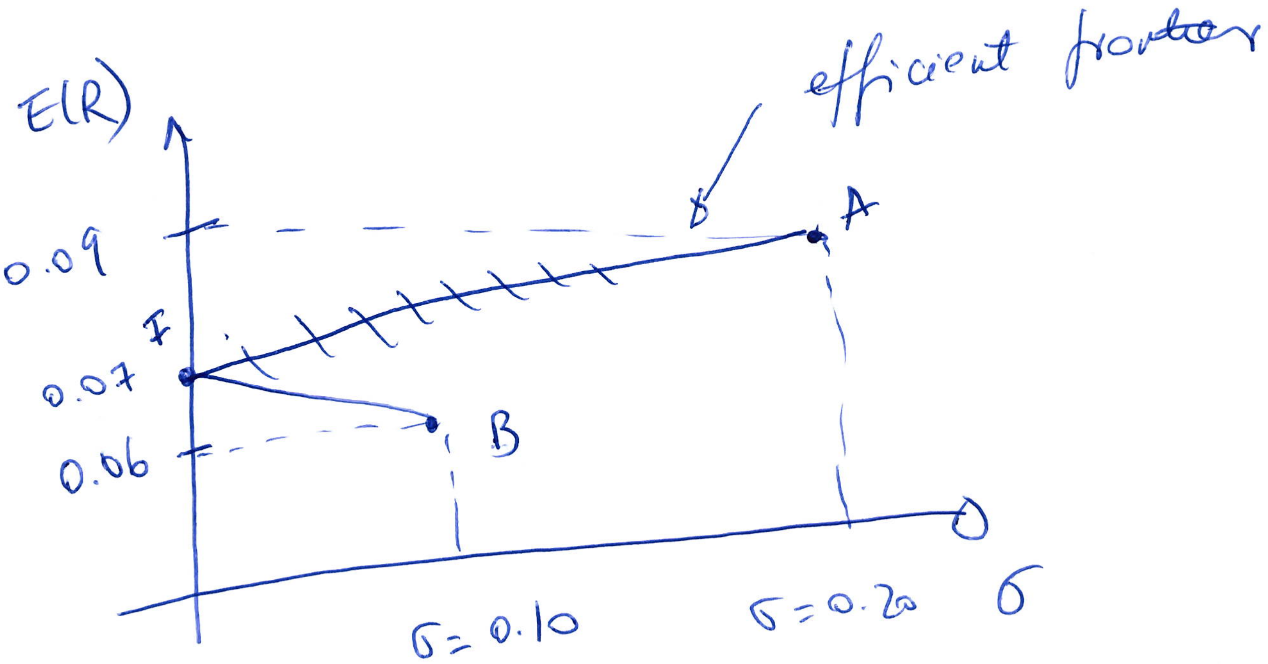
Risk free asset: $\sigma_F = 0 \Rightarrow$

$$w_A = \frac{\sigma_B}{\sigma_A + \sigma_B} \qquad w_B = \frac{\sigma_A}{\sigma_A + \sigma_B}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$w_A = \frac{1}{3} \qquad w_B = \frac{2}{3}$$

F: $\frac{1}{3}$ A and $\frac{2}{3}$ B

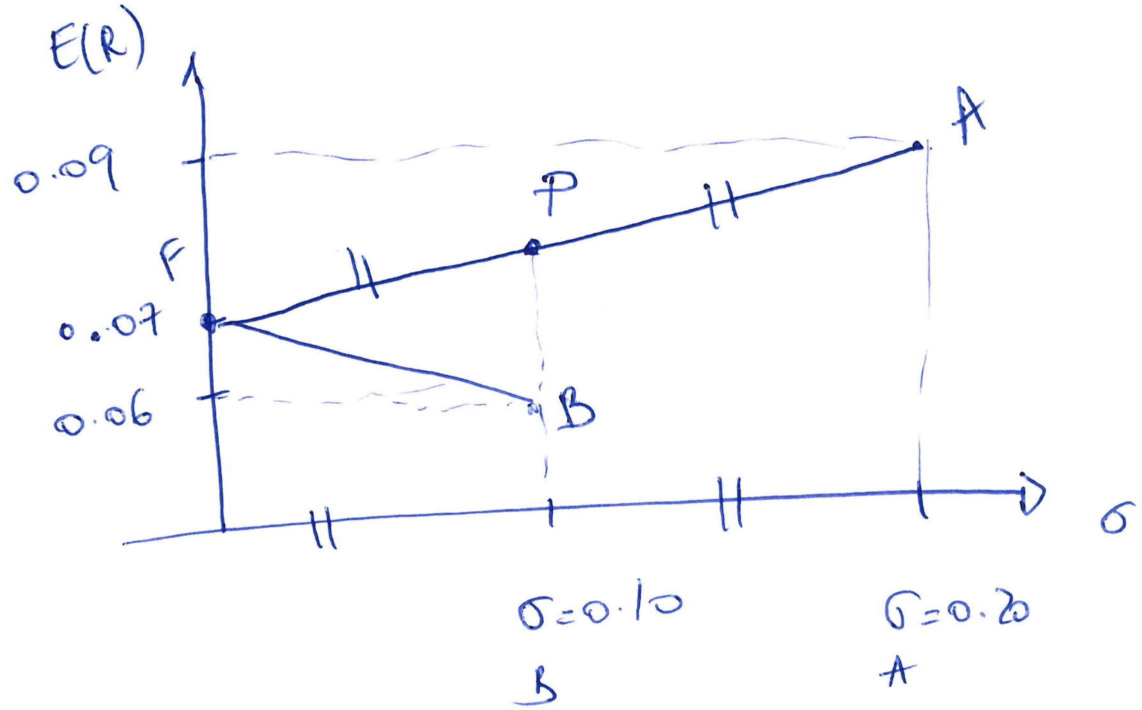


$$E_F = w_A E_A + (1 - w_A) E_B = \frac{1}{3} \times 0.09 + \frac{2}{3} \times 0.06 = 0.07$$

Efficient frontier passes through F and A

$$\Rightarrow \quad \frac{E - 0.09}{\sigma - 0.20} = \frac{0.07 - 0.09}{0 - 0.20}$$

$$\boxed{E = 0.07 + \frac{1}{10}\sigma}$$



$$P = \frac{1}{2} A \text{ and } \frac{1}{2} F$$

$$\equiv$$

$$\frac{1}{2} A \ \boxed{\text{and}} \ \frac{1}{2}\left(\frac{1}{3} A \text{ and } \frac{2}{3} B\right)$$

$$\equiv$$

$$\underbrace{\frac{1}{2} A \ \boxed{\text{and}} \ \frac{1}{6} A}_{} \text{ and } \frac{1}{3} B$$

$$\boxed{\frac{2}{3} A \text{ and } \frac{1}{3} B} \equiv P$$