QUEEN MARY UNIVERSITY OF LONDON

MTH5120   Statistical Modelling I
Exercise Sheet 5

1. A chemist studied the concentration of a solution (Y) over time (x). Fifteen identical solutions were prepared. The solutions were randomly divided into five sets of three, and the five sets were measured, respectively after 1, 3, 5, 7, and 9 hours.

   Without making any plots the chemist entered the data into R, fitted a simple linear regression model and then carried out a goodness of fit test. The following is the Analysis of Variance table she produced but with some figures missing.

   ```
   Analysis of Variance Table

   Response: y
                 Df  Sum Sq Mean Sq F value
   x              1 12.5971
   Residuals     13
     Lack of fit      2.770
     Pure error
   Total         14 15.5218
   ```

   (a) Copy and complete the Analysis of Variance Table without using R.

   (b) Carry out two possible F tests, write down the corresponding null hypotheses and state your conclusions.

2. Write the following models in the form of a general linear model

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

   Hence find the least squares estimators of the parameters and the variance of the estimators.

   (a) The null model
   $$y_i = \beta_0 + \varepsilon_i \quad i = 1, 2, \ldots, n.$$

   (b) The no intercept model

   $$y_i = \beta_1 x_i + \varepsilon_i \quad i = 1, 2, \ldots, n.$$

3. Using the results for $\mathrm{Var}(\widehat{\beta}_0)$, $\mathrm{Var}(\widehat{\beta}_1)$ and $\mathrm{cov}(\widehat{\beta}_0, \widehat{\beta}_1)$ given in lectures find $\mathrm{Var}((\widehat{\beta}_0 + \widehat{\beta}_1 x_0)$