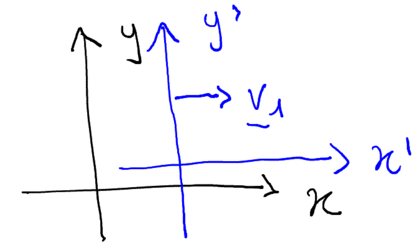
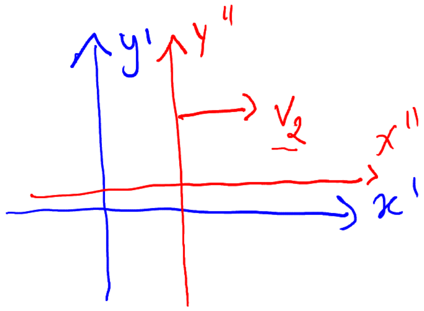
Quiz 1

(1)

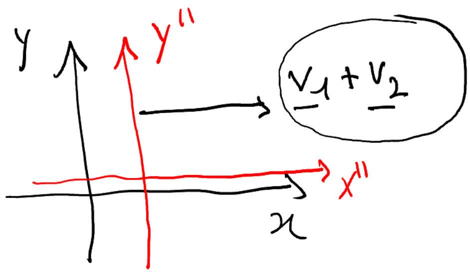


S' with respect to S



S'' with respect to S'

$\Downarrow$ Newtonian theory



S'' with respect to S

(2) $\underline{g} = -\nabla \phi \Rightarrow (g_x, g_y, g_z) = \left(-\frac{\partial \phi}{\partial x}, -\frac{\partial \phi}{\partial y}, -\frac{\partial \phi}{\partial z} \right)$

By using $\phi = gz$ we get $\underline{g} = (0, 0, -g)$

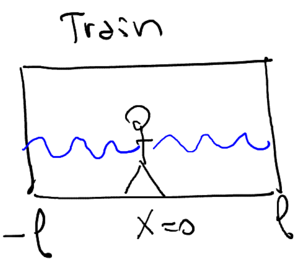
(3) The initial potential energy of the test particle is $U = m\phi = -\frac{Cm}{r_0}$. The initial kinetic energy is $K = \frac{1}{2} m v_0^2$, so we have $K + U = \frac{1}{2} m v_0^2 - \frac{Cm}{r_0}$.

This quantity is conserved and has to be non-negative if the particle escapes to infinity (since $U \to 0$ as

$r \to 0$ and K is always positive]. Thus

$$\frac{1}{2} m v_0^2 - \frac{Cm}{r_0} \geq 0 \qquad v_0 \geq \sqrt{\frac{2C}{r_0}} \xrightarrow[\text{velocity}]{\text{minimal}} v_0 = \sqrt{\frac{2C}{r_0}}$$

(4)



For the observer on the train, the beams cover the same distance at speed c, so they arrive at the same time.

The spacetime positions representing the arrivals are

$$(x^0, x) \begin{array}{l} \nearrow (\ell, \ell) \\ \searrow (\ell, -\ell) \end{array}$$

The frame F' of an observer at the station is boosted by $-v$ with respect to the frame of the passenger. So $(x')^0 = \gamma\left(x^0 + \frac{v}{c} x'\right)$. Thus the spacetime point (ℓ, ℓ) corresponds to a $x'^0 = \gamma \ell \left(1 + \frac{v}{c}\right)$, while $(\ell, -\ell)$ to $x^{0\prime} = \gamma \ell \left(1 - \frac{v}{c}\right)$. Thus the arrival times for the observer at the station are different.

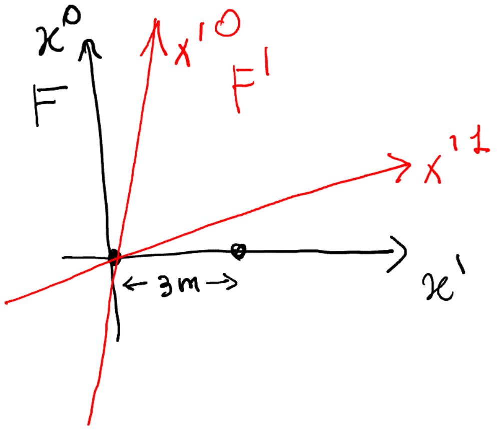
(5) By using $L_{station} = \frac{1}{\gamma} L_{train}$ we have

$$\frac{L}{3} = \sqrt{1 - \frac{v^2}{c^2}}\, L \Rightarrow \frac{1}{9} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{8}{9} \Rightarrow v = c\,\frac{2\sqrt{2}}{3}$$

(6) The answer is $M = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ \sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ as it is

the only transformation for which $M^t \eta M \neq \eta$

(7)



The event $(0,0)$ in F maps to $(0,0)$ in F'. Instead $(0, \ell=3)$ in F maps to

$$x'^0 = \gamma\left(x^0 - \frac{v}{c}x^1\right) \rightarrow \gamma\left(-\frac{v}{c}\ell\right)$$

$$x'^1 = \gamma\left(x^1 - \frac{v}{c}x^0\right) \rightarrow \gamma\ell$$

The time difference between the two events in F' is

$0 - \gamma\left(\frac{v}{c}\ell\right) = c\Delta t'$. Thus $\frac{v/c\,\ell}{\sqrt{1-\frac{v^2}{c^2}}} = c\Delta t' \Rightarrow$

$$\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = c^2\left(\frac{\Delta t'}{\ell}\right)^2 \Rightarrow \left(1+\frac{c^2\Delta t'^2}{\ell^2}\right)\frac{v^2}{c^2} = \frac{c^2\Delta t'^2}{\ell^2} \Rightarrow$$

$$v = c\,\frac{\frac{c\Delta t'}{\ell}}{\left(1+\frac{c^2\Delta t'^2}{\ell^2}\right)^{1/2}} = \frac{c}{\sqrt{2}} \qquad \sqrt{2}\ \frac{1}{\sqrt{2}}$$

where I used $\frac{c\Delta t'}{\ell} = \frac{3\,10^8 \cdot 10^{-8}}{3} = 1$. Thus

$$x'^1 = \gamma\ell = \frac{\ell}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\ell}{\left(1-\frac{1}{2}\right)^{1/2}} = \sqrt{2}\,\ell = \sqrt{2}\,3m$$

(8) $C = (\alpha+2\beta, 0, 2\alpha+\beta, 0)$. Thus $C^2 = 0$ if

$$-(\alpha+2\beta)^2+(2\alpha+\beta)^2=0 \Rightarrow 3\alpha^2-3\beta^2=0$$

So $\alpha = \pm\beta$.

(9) $\sum_a T_{aab}$ is not a tensor as a pair of lower indices is contracted without using the metric.

(10) M is greater than $2m$ since by the 3rd law we have $P_{final}^a = P_1^a + P_2^a$. The spatial components read $p_{final}^i = \gamma(v)\, m\,(v_1^i + v_2^i) = 0$, so the final particle is at rest. Then from the time component

$$p_{final}^0 = Mc = \gamma(v)(m_1+m_2)c = \gamma(v)\,2mc > 2mc$$

since $\gamma(v) > 1$ we have $M > 2m$