Recap quiz (paraphrased defns/theorems)

Consider an LP in standard equation form

$$\begin{aligned} &\text{maximize} \quad \underline{c}^T \underline{x} \\ &\text{subject to} \quad A\underline{x} = \underline{b}, \quad \underline{x} \geq 0. \end{aligned}$$

A basic feasible solution is a feasible solution $\underline{x}$ in which the non-zero entries of $\underline{x}$ correspond to linearly independent columns of A.

Last time we proved two results

1. Every LP (in standard equation form) has an optimal solution that is an extreme point solution (provided it has at least one optimal solution).

2. Given an LP in standard equation form every basic feasible solution is an extreme point solution and vice versa.

   (proof not completed)

1 + 2 imply

Corollary If an LP has an optimal solution, then it also has an optimal solution that is also a basic feasible solution.

A basic feasible solution is a feasible solution $\underline{x}$ in which the non-zero entries of $\underline{x}$ correspond to linearly independent columns of A.

Defn For a basic feasible solution $\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ of an LP in standard equation form

the basic variables are those x_i that are non-zero

the non-basic variables are those x_i that are zero.

Simplex algorithm (weeks 5/6)

General method for finding optimal solutions to LPs

1st half : understand ideas
2nd half : description of algorithm + example

Useful observations about LPs in standard equation form

Example (A) maximise $3x_1 + 2x_2 + x_3$ obj
sub to $x_1 + 2x_2 + 2x_3 = 5$ C_1
$2x_1 - x_2 - x_3 = 4$ C_2
$x_1, x_2, x_3 \geq 0$

Obs 1 Let C and C' be two constraints of LP.
Can replace C with $C + \lambda C'$ for $\lambda \in \mathbb{R}$
and resulting LP has same feasible solutions
same objective values
same optimal solutions

e.g. (B) maximise $3x_1 + 2x_2 + x_3$
sub to $5x_1 = 13$ $C_1 + 2C_2$
$2x_1 - x_2 - x_3 = 4$ C_2
$x_1, x_2, x_3 \geq 0$

(A) and (B) have the same feasible and optimal solutions, so if we solve one then we have solved the other.

(B) maximise $3x_1 + 2x_2 + x_3$

sub to
$$5x_1 = 13 \quad c_1 + 2c_2$$
$$2x_1 - x_2 - x_3 = 4 \quad c_2$$
$$x_1, x_2, x_3 \geq 0$$

<u>Obs 2</u> Can add zero to objective function without changing feasible or optimal solutions

e.g. in (B) c_2 says that $2x_1 - x_2 - x_3 - 4 = 0$

Add this 0 to objective function

(C) maximise $5x_1 + x_2 - 4$

sub to
$$5x_1 = 13$$
$$2x_1 - x_2 - x_3 = 4$$
$$x_1, x_2, x_3 \geq 0.$$

(B) and (C) have same feasible and optimal solutions

(A), (B), (C) all "equivalent"

<u>Rem</u>

- Note that (C) not technically an LP but essentially it is
- Obs 1 and Obs 2 will be used to rewrite LP into a form where it becomes obvious what the optimal solution is.

<u>Example</u> maximise $4x_1 + 3x_2$

sub to $x_1 + x_2 \le 12$

$2x_1 + x_2 \le 16$

$2x_1 + 3x_2 \le 40$ $x_1, x_2, x_3 \ge 0$

Change to standard equation form.

maximise $4x_1 + 3x_2$

sub to

$$
\begin{aligned}
x_1 + x_2 + s_1 \quad\quad &= 12 \quad C_1 \\
2x_1 + x_2 \quad + s_2 \quad &= 16 \quad C_2 \\
2x_1 + 3x_2 \quad\quad + s_3 &= 40 \quad C_3
\end{aligned}
$$

$x_1, x_2, s_1, s_2, s_3 \ge 0$

Idea: Start with a basic feasible solution BFS

In each step, find a better BFS

We know some BFS is optimal

Start BFS1: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ i.e. $\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 12 \\ 16 \\ 40 \end{pmatrix}$

Objective value of BFS1 is $4 \times 0 + 3 \times 0 = 0$.

Can improve by increasing x_1 (or x_2)

How large can we make x_1 (while keeping $x_2 = 0$) if we want to satisfy all constraints and sign restrictions?

Can take $x_1 = 8$ (if $x_1 > 8$ cannot satisfy C_2)

Now BFS2 $\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 4 \\ 0 \\ 24 \end{pmatrix}$

once we decide x_1, x_2, easy to work out s_1, s_2, s_3

(1) maximize $4x_1 + 3x_2$ obj

sub to

$$x_1 + x_2 + s_1 = 12 \quad C_1$$
$$2x_1 + x_2 + s_2 = 16 \quad C_2$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0 \quad 2x_1 + 3x_2 + s_3 = 40 \quad C_3$$

BFS2 $\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 4 \\ 0 \\ 24 \end{pmatrix}$

obj value 32

Rewrite (1) using obs 1 and obs 2 to "eliminate" x_1 to see which other variables we can increase

(2) max $x_2 - 2s_2 + 32$ obj $- 2(C_2 - 16)$

sub to

$$\tfrac{1}{2}x_2 + s_1 - \tfrac{1}{2}s_2 = 4 \quad C_1 - \tfrac{1}{2}C_2$$
$$x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}s_2 = 8 \quad \tfrac{1}{2}C_2$$
$$2x_2 - s_2 + s_3 = 24 \quad C_3 - C_2$$

(1) and (2) are equivalent

Can improve BFS2 by increasing x_2

How much can we increase x_2 (keeping $s_2 = 0$) if we want to satisfy all constraints/ sign restrictions?

C_1: $x_2 \uparrow 8$ if $s_1 \downarrow 0$

C_2: $x_2 \uparrow 16$ if $x_1 \downarrow 0$

C_3: $x_2 \uparrow 12$ if $s_3 \downarrow 0$

can take $x_2 = 8$.

Then BFS3 $\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 0 \\ 0 \\ 8 \end{pmatrix}$

obj value for BFS3 is 40

② max $\qquad x_2 \quad - 2s_2 + 32 \qquad$ obj $- 2(c_2 - 16)$

sub to
$$\begin{aligned} \tfrac{1}{2}x_2 + s_1 - \tfrac{1}{2}s_2 &= 4 & c_1 - \tfrac{1}{2}c_2 \\ x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}s_2 &= 8 & \tfrac{1}{2}c_2 \\ 2x_2 - s_2 + s_3 &= 24 & c_3 - c_2 \end{aligned}$$

BFS 2: $\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 0 \\ 0 \\ 8 \end{pmatrix}$

Obj value = 40

Rewrite ② using obs 1 and 2 to see how we can improve.

③ maximise $-2s_1 - s_2 + 40 \qquad$ obj $- 2(c_1 - 4)$

sub to
$$\begin{aligned} x_2 + 2s_1 + s_2 &= 8 & 2c_1 \\ x_1 - s_1 + s_2 &= 4 & c_2 - c_1 \\ 4s_1 + s_2 + s_3 &= 8 & c_3 - 4c_1 \end{aligned}$$
$$x_1, x_2, s_1, s_2, s_3 \geq 0.$$

③ and ② and ① are equivalent.

max value for objective function is 40 because $s_1, s_2 \geq 0$

so BFS 2: $\begin{pmatrix} 4 \\ 8 \\ 0 \\ 0 \\ 8 \end{pmatrix}$ is an optimal solution.

Summary

- Start with a BFS
- At each step find a BFS with larger objective value by increasing one variable from 0 and decreasing one variable to 0.
- Rewrite LP so it becomes obvious which variable to increase in the next step
- Stop when we see that we cannot increase the objective function by move.

Systematic description of simplex algoritum

Simpler case when $\underline{b} \geq \underline{0}$.

Assume you are given an LP in standard inequality form

$$\text{maximise } \underline{c}^T \underline{x}$$
$$\text{subject to } A\underline{x} \leq \underline{b} \quad \underline{x} \geq \underline{0}.$$

Here A is $m \times n$ matrix, $\underline{c} \in \mathbb{R}^n$, $\underline{b} \in \mathbb{R}^m$, $\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$

(so m constraints, n variables)

① Initialisation:

Put in standard equation form by introducing m slack variables $s_1, \ldots, s_m$.

$$\text{maximise } \underline{c}'^T \underline{x}'$$
$$\text{subjet to } A'\underline{x}' \leq \underline{b}, \quad \underline{x}' \geq \underline{0}$$

$A' = (A \mid I)$ $m \times (n+m)$ matrix

$\underline{c}'^T = (\underline{c} \mid \underline{0}) \in \mathbb{R}^{n+m}$ (add m zeros to $\underline{c}^T$)

$\underline{x}' = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ s_1 \\ \vdots \\ s_m \end{pmatrix} \in \mathbb{R}^{n+m}$

Construct initial tableau (see example)

Basic variables $\rightarrow$

(to begin the basic variables are the slack variables)

$$\begin{array}{c|c|c}
 & x_1 \; x_2 \cdots x_n \; s_1 \; s_2 \cdots s_m & \\
\hline
\begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{matrix} & A' = (A \mid I) & \underline{b} \\
\hline
 & \underbrace{\underline{c}'^T = (\underline{c}^T \mid \underline{0})}_{n+m} & 0
\end{array} \left.\vphantom{\begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{matrix}}\right\} m$$

(2) Repeatedly apply pivot steps as follows.
Consider current tableux

Basic variables appear here

$$\begin{array}{l|ccccccc|c} & x_1 & x_2 & \cdots & x_n & s_1 & s_2 \cdots & s_m & \\ \hline R_1 & & & & & & & & \\ R_2 & & & & A^* & & & & \underline{b}^* \\ \vdots & & & & & & & & \\ R_m & & & & & & & & \\ \hline R_{final} & & & & \underline{c}^{*T} & & & & v \end{array} \quad \Big\} \; m$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{n+m}$$

Label rows $R_1, R_2, \cdots, R_m, R_{final}$ (just so we can refer to them)

(a) Find largest positive entry in $\underline{c}^{*T}$, say c_j^*, and highlight j^{th} column

(b) Look at each entry in highlighted column i.e. the entries A_{rj}^* $r=1,\dots,m$

For each $r=1,\dots,m$ if $\underline{A_{rj}^* \geq 0}$ let $z_r = b_r^*/A_{rj}^*$ and record this number z_r next to b_r^*

Of all z_r, pick smallest, say z_i, and highlight its row, i.e. R_i

Basic variables appear here

$$\begin{array}{l|ccccc|c|l} & x_1 \; x_2 \cdots & & x_j & & \cdots s_m & & \\ \hline R_1 & A_{11}^* & & A_{1j}^* & & A_{1\,n+m}^* & b_1^* & z_1 = b_1^*/A_{1j} \\ R_2 & & & \vdots & & & \vdots & \\ \vdots & & & & & & & \\ R_i & & & A_{ij}^* & & & b_i^* & z_i = b_i^*/A_{ij} \text{ smallest} \\ \vdots & & & \vdots & & & \vdots & \\ R_m & A_{m1}^* & & A_{mj}^* & & A_{m\,n+m}^* & b_m^* & z_m = b_m^*/A_{mj} \\ \hline R_{final} & c_1 \; c_2 & & c_j \text{ largest} & & c_{n+m} & v & \end{array} \quad \Big\} \; m$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{n+m}$$

(c) We "clear" j^{th} column (i.e. highlighted column) using row operations

- Replace i^{th} row R_i (i.e. highlighted row) by $R_i' = R_i/A_{ij}^*$ (so ij^{th} entry is now 1)
- Replace every other row R_r with $R_r' = R_r - A_{rj}^* R_i'$ (<u>including R_{final}</u>) So all entries in highlighted column become zero except ij^{th}.
- Replace highlighted row variable with highlighted column variable (keeping all other variables unchanged).
- Now have our new tableau

(3) Repeat (2) until either

(a) c^{*T} has no positive entries in step 2(a).
In this case the optimal solution is obtained by setting each variable on the far left to the value on the far right and all other variables to zero
The maximum objective value is the negative of the bottom right entry.

(b) There are no positive entries in the highlighted column in step 2(b).
If this happens the LP is unbounded

Important notes

- In each pivot,
  the variable at the top of highlighted column is called the entering variable
  the variable to left of highlighted row is called the leaving variable.

- Tie breaking rules.

When picking largest value in a row / smallest value in a column
If there is a tie-break, pick the one fartherest left / closest to the top.

- How would you summarise this for exam?!
- Roughly what is the reason for steps 2(a) 2(b) 2(c).

Apply simplex algorithm to

$$\text{maximize} \quad 4x_1 + 3x_2$$
$$\text{sub to} \quad x_1 + x_2 \leq 12$$
$$2x_1 + x_2 \leq 16$$
$$2x_1 + 3x_2 \leq 40$$
$$x_1, x_2, x_3 \geq 0$$

① Standard eqn form

$$\text{maximize} \quad 4x_1 + 3x_2$$
$$\text{sub to} \quad x_1 + x_2 + s_1 = 12$$
$$2x_1 + x_2 + s_2 = 16$$
$$2x_1 + 3x_2 + s_3 = 40$$
$$x_1, x_2, s_1, s_2, s_3 \geq 0.$$

Initial tableau

	x_1	x_2	s_1	s_2	s_3	
s_1	1	1	1	0	0	12
s_2	2	1	0	1	0	16
s_3	2	3	0	0	1	40
	4	3	0	0	0	0

Pivot step

2(a), 2(b)

		x_1	x_2	s_1	s_2	s_3		
R_1	s_1	1	1	1	0	0	12	$12/1 = 12$
R_2	s_2	2	1	0	1	0	16	$16/2 = 8$
R_3	s_3	2	3	0	0	1	40	$40/2 = 20$
R_f		4	3	0	0	0	0	

Step 2(c)

$R_1' = R_1 - \frac{1}{2}R_2$

$R_2' = \frac{1}{2}R_2$

$R_3' = R_3 - R_2$

$R_f' = R_f - 2R_2$

	x_1	x_2	s_1	s_2	s_3	
s_1	0	$1/2$	1	$-1/2$	0	4
x_1	1	$1/2$	0	$1/2$	0	8
s_3	0	2	0	-1	1	24
	0	1	0	-2	0	-32

Apply pivot again.

		x_1	x_2	s_1	s_2	s_3		
R_1	s_1	0	$1/2$	1	$-1/2$	0	4	$4/\tfrac{1}{2} = 8$
R_2	x_1	1	$1/2$	0	$1/2$	0	8	$8/\tfrac{1}{2} = 16$
R_3	s_3	0	2	0	-1	1	24	$24/2 = 12$
R_f		0	1	0	-2	0	-32	

		x_1	x_2	s_1	s_2	s_3	
$R_1' = 2R_1$	x_2	0	1	2	-1	0	8
$R_2' = R_2 - R_1$	x_1	1	0	-1	1	0	4
$R_3' = R_3 - 4R_1$	s_3	0	0	-4	1	1	8
$R_f' = R_f - 2R_1$		0	0	-2	-1	0	-40

Last row has no positive entries so 3(a) tells us to stop.

optimal solution is
$$\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 0 \\ 0 \\ 8 \end{pmatrix}$$

optimal value of objective function is 40

Apply simplex to following example

$$\text{maximise} \quad 4x_1 + \tfrac{1}{2}x_2$$

$$\text{sub to} \quad \begin{aligned} x_1 + x_2 &\leq 3 \\ \tfrac{1}{2}x_1 + x_2 &\leq 2 \\ \tfrac{1}{2}x_1 - x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

① Put in standard eqn form

$$\max \quad 4x_1 + \tfrac{1}{2}x_2$$

$$\text{sub to} \quad \begin{aligned} x_1 + x_2 + s_1 \quad\quad\quad &= 3 \\ \tfrac{1}{2}x_1 + x_2 \quad + s_2 \quad &= 2 \\ \tfrac{1}{2}x_1 - x_2 \quad\quad + s_3 &= 1 \\ x_1, x_2, s_1, s_2, s_3 &\geq 0. \end{aligned}$$

$A' =$

Initial tableux

	x_1	x_2	s_1	s_2	s_3	
s_1	1	1	1	0	0	3
s_2	1/2	1	0	1	0	2
s_3	1/2	-1	0	0	1	1
	4	1/2	0	0	0	0

Pivot step 2

2a, 2b

		x_1	x_2	s_1	s_2	s_3	
R_1	s_1	1	1	1	0	0	3
R_2	s_2	$1/2$	1	0	1	0	2
R_3	s_3	$1/2$	-1	0	0	1	1
R_{final}		4	$1/2$	0	0	0	0

$3/1 = 3$

$2/\tfrac{1}{2} = 4$

$\tfrac{1}{2}/1 = \tfrac{1}{2}$

2C

		x_1	x_2	s_1	s_2	s_3	
$R_1' = R_1 - R_3'$	s_1	0	3	1	0	-2	1
$R_2' = R_2 - \frac{1}{2}R_3'$	s_2	0	2	0	1	-1	1
$R_3' = R_3 / \tfrac{1}{2}$	x_1 ~~s_3~~	1	-2	0	0	2	2
$R_{final}' = R_{final} - 4R_3'$		0	$17/2$	0	0	-8	-8

Pivot again with updated table

Apply pivot to updated tableax.

		x_1	x_2	s_1	s_2	s_3		
R_1	s_1	0	3	1	0	-2	1	$1/3$
R_2	s_2	0	2	0	1	-1	1	$1/2$
R_3	x_1	1	-2	0	0	2	2	$-$
R_{final}		0	$17/2$	0	0	-8	-8	

		x_1	x_2	s_1	s_2	s_3	
$R_1' = \frac{1}{3}R_1$	x_2 ~~s_1~~	0	1	$1/3$	0	$-2/3$	$1/3$
$R_2' = R_2 - 2R_1'$	s_2	0	0	$-2/3$	1	$1/3$	$1/3$
$R_3' = R_3 + 2R_1'$	x_1	1	0	$2/3$	0	$2/3$	$8/3$
$R_{final}' = R_{final} - \frac{17}{2}R_1'$		0	0	$-17/6$	0	$-\frac{7}{3}$	$-\frac{65}{6}$

We do not apply another pivot because 3(a) tells us we have found an optimal solution.

Optimal solution

$$\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 8/3 \\ 1/3 \\ 0 \\ 2/3 \\ 0 \end{pmatrix}$$

obj value for this optimal solution is $-\frac{65}{6}$

Example

maximise $2x_1 - x_2 + 8x_3$

subject to

$$2x_3 \leq 1$$
$$2x_1 - 4x_2 + 6x_3 \leq 3$$
$$-x_1 + 3x_2 + 4x_3 \leq 2$$
$$x_1, x_2, x_3 \geq 0$$

standard equation form

maximise $2x_1 - x_2 + 8x_3$

subject to

$$2x_3 + s_1 = 1$$
$$2x_1 - 4x_2 + 6x_3 + s_2 = 3$$
$$-x_1 + 3x_2 + 4x_3 + s_3 = 2$$
$$x_1, x_2, x_3 \geq 0.$$

Initial tableau

	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	0	2	1	0	0	1
s_2	2	-4	6	0	1	0	3
s_3	-1	3	4	0	0	1	2
	2	-1	8	0	0	0	0

		x_1	x_2	x_3	s_1	s_2	s_3		
R_1	s_1	0	0	2	1	0	0	1	$1/2$
R_2	s_2	2	-4	6	0	1	0	3	$3/6 = 1/2$
R_3	s_3	-1	3	4	0	0	1	2	$2/4 = \frac{1}{2}$
R_f		2	-1	8	0	0	0	0	

First pivot

		x_1	x_2	x_3	s_1	s_2	s_3	
$R_1' = \frac{1}{2}R_1$	x_3 $\cancel{s_1}$	0	0	1	$1/2$	0	0	$1/2$
$R_2' = R_2 - 3R_1$	s_2	2	-4	0	-3	1	0	0
$R_3' = R_3 - 2R_1$	s_3	-1	3	0	-2	0	1	0
$R_f' = R_f - 4R_1$		2	-1	0	-4	0	0	-4

		x_1	x_2	x_3	s_1	s_2	s_3		
R_1	x_3	0	0	1	1/2	0	0	1/2	—
R_2	x_1 ~~s_2~~	2	−4	0	−3	1	0	0	0
R_3	s_3	−1	3	0	−2	0	1	0	—
R_f		2	−1	0	−4	0	0	−4	

2^{nd} pivot

		x_1	x_2	x_3	s_1	s_2	s_3	
$R_1' = R_1$	x_3	0	0	1	1/2	0	0	1/2
$R_2' = \frac{1}{2}R_2$	x_1	1	−2	0	−3/2	1/2	0	0
$R_3' = R_3 + \frac{1}{2}R_2$	s_3	0	1	0	−7/2	1/2	1	0
$R_f' = R_f - R_2$		0	3	0	−1	−1	0	−4

		x_1	x_2	x_3	s_1	s_2	s_3		
R_1	x_3	0	0	1	$1/2$	0	0	$1/2$	—
R_2	x_1	1	-2	0	$-3/2$	$1/2$	0	0	—
R_3	x_2 $\cancel{s_3}$	0	1	0	$-7/2$	$1/2$	1	0	0
		0	3	0	-1	-1	0	-4	

3^{rd} pivot

		x_1	x_2	x_3	s_1	s_2	s_3	
$R_1' = R_1$	x_3	0	0	1	$1/2$	0	0	$1/2$
$R_2' = R_2 + 2R_3$	x_1	1	0	0	$-17/2$	$3/2$	0	0
$R_3' = R_3$	x_2	0	1	0	$-7/2$	$1/2$	1	0
$R_f' = R_f - 3R_3$		0	0	0	$19/2$	$-5/2$	-3	-4

		x_1	x_2	x_3	s_1	s_2	s_3		
R_1	s_1 ~~x_3~~	0	0	1	$1/2$	0	0	$1/2$	$\frac{1/2}{1/2} = 1$
R_2	x_1	1	0	0	$-17/2$	$3/2$	0	0	—
R_3	x_2	0	1	0	$-7/2$	$1/2$	1	0	—
		0	0	0	$19/2$	$-5/2$	-3	-4	

$R_1' = 2R_1$

$R_2' = R_2 + 17R_1$

$R_3' = R_3 + 7R_1$

$R_f' = R_f - 19R_1$

	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	0	2	1	0	0	1
x_1	1	0	17	0	$3/2$	0	$17/2$
x_2	0	1	7	0	$1/2$	1	$7/2$
	0	0	-19	0	$-5/2$	-3	$-27/2$

all values ≤ 0
so algorithm stops

Optimal soln is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 17/2 \\ 7/2 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ with obj value $-27/2$

Optimal soln to original LP is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 17/2 \\ 7/2 \\ 0 \end{pmatrix}$.