MTH 4104 Assessed Coursework I

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Please hand in your work, with your name and student ID explicitly written on the front page, by **11am**, **the 4th of March** (the Maths Office).

Q1. Find all integers X satisfying $13X \equiv 4 \bmod 2024$. Show your working.

Q2. Let $(G, *) = (\mathrm{Sym}(\{1,2,3\}), \circ)$ be the group of all bijections on the set $\{1,2,3\}$ with composition $\circ$, as seen in lectures. For brevity, given a triple of distinct integers $\{a, b, c\} = \{1,2,3\}$, we denote by (abc) the bijection that sends 1 to a, 2 to b, and 3 to c. Complete the following Cayley table (calculating the composition of two elements in $\mathrm{Sym}(\{1,2,3\})$):

$\circ$	(123)	(132)	(213)	(231)	(312)	(321)
(123)						
(132)						
(213)		(231)				
(231)						
(312)						
(321)						

For example, the entry provided computes $(213) \circ (132)$, i.e. the bijection (132) followed by (213) (and not the other way around). Indeed, the first bijection (132) sends 1 to 1 which is then sent by the second bijection (213) to 2; similarly, the bijection (132) sends 2 to 3 which is then sent by (213) to 3. Tracking where each integer goes, we conclude that $(213) \circ (132) = (231)$.

Q3. Keep the notation from **Q2**. (1) Is the group in **Q2** abelian? Justify your answer. (2) Find the identity element e. Explain your reasoning. (3) Let $r = (231)$ and $s = (132)$. Find three independent relations that only involve r, s, e and composition $\circ$.

Marking Scheme. **Q1**. $+1$ for spotting the answer correctly and $+1$ for justification. **Q2**. $+3$ for filling in the table correctly. **Q3**. (1) $+1$ ($+0$ without justification) (2) $+1$ ($+0$ without justification) (3) $+3$