Queen Mary
University of London

MTH5114 Linear Programming and Game Theory, Spring 2024

Week 4 Coursework Questions

Viresh Patel

These exercises should be completed individually and submitted (together with those of weeks 5 and 6) via the course QMPlus page by **9am on Monday, 11 March**.

Make sure you clearly write your **name** and **student ID** number at the top of your submission. Note that the credit available for each week's coursework exercises is roughly equal.

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1. Suppose we have a linear program in standard equation form

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^{\mathsf{T}}\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

and suppose $\mathbf{u}$, $\mathbf{v}$. and $\mathbf{w}$ are all optimal solutions to this linear program.

   (a) Prove that $\frac{1}{3}\mathbf{u} + \frac{1}{3}\mathbf{v} + \frac{1}{3}\mathbf{w}$ is a feasible solution.

   (b) Prove that $\frac{1}{3}\mathbf{u} + \frac{1}{3}\mathbf{v} + \frac{1}{3}\mathbf{w}$ is an optimal solution.

   (c) Your proofs for (b) and (c) should work more generally for certain linear combinations of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. State for which linear combinations of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ your proofs still work. (You do not have to justify your answer for part (c)).

   Hint: for this question it is helpful to look at certain parts of the proofs covered in week 4 and to adapt them.