

Actuarial Mathematics II

MTH5125

Problem Set 2 Solutions

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Exercise 7.1

For a whole life insurance issued to (40) , you are given:

- i) The death benefit which is payable at the end of the year of death is \$50,000 in the first 20 years, and \$100,000 thereafter.
- ii) Level annual premiums are payable for 20 years or until earlier death.
- iii) The mortality basis Standard Ultimate Survival Model (Table D).
- iv) The interest basis for policy values is 5% per year.

Calculate the net premium policy value, ${}_{10}V^t$

Exercise 7.1

The net annual premium, P , is calculated as

$$P \ddot{a}_{40:\overline{20}|} = 100\,000 A_{40} - 50\,000 A_{40:\overline{20}|}^1 \Rightarrow P = \$875.38,$$

where $A_{40} = 0.12106$, $A_{40:\overline{20}|}^1 = 0.01463$ and $\ddot{a}_{40:\overline{20}|} = 12.99348$.

The policy value at duration 10 years, ${}_{10}V$, is calculated from

$${}_{10}V = 100\,000 A_{50} - 50\,000 A_{50:\overline{10}|}^1 - P \ddot{a}_{50:\overline{10}|},$$

giving ${}_{10}V = \$11\,149.02$.

Exercise 7.8

You are given the following extract from a select life table with four year select period. A select individual aged 41 purchased a three-year term insurance with a sum insured of \$200,000, with premiums payable annually throughout the term.

$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}	$x+4$
[40]	100 000	99 899	99 724	99 520	99 288	44
[41]	99 802	99 689	99 502	99 283	99 033	45
[42]	99 597	99 471	99 628	99 030	98 752	46

Assume an effective rate of interest of 6% per year, and no expenses.

- Show that the premium for the term insurance is $P = \$323.59$.
- Calculate the mean and standard deviation of the present value of future loss random variable, L_1 , for the term insurance.

Exercise 7.8

- (c) Calculate the sum insured for a three-year endowment insurance for a select life age 41, with the same premium as for the term insurance, $P = \$323.59$.
- (d) Calculate the mean and standard deviation of the present value of future loss random variable, L_1 , for the endowment insurance.
- (e) Comment on the differences between the values for the term insurance and the endowment insurance.

Exercise 7.8 a)

(a) The annual premium, P , is calculated using the equivalence principle from

$$P \ddot{a}_{[41]:\overline{3}|} = 200\,000 A^1_{[41]:\overline{3}|},$$

giving $P = \$323.59$.

Exercise 7.8 b)

- (b) The value of the random variable L_1 depends on whether the policyholder dies in the coming year, dies in the following year or survives for two years. The distribution of L_1 is as follows:

$$L_1 = \begin{cases} 200\,000v - P = \$188\,355.66 & \text{w.p. } q_{[41]+1}, \\ 200\,000v^2 - P(1+v) = \$177\,370.43 & \text{w.p. } {}_1|q_{[41]+1}, \\ -P(1+v) = -\$628.85 & \text{w.p. } {}_2p_{[41]+1}. \end{cases}$$

Hence

$$\begin{aligned} E[L_1] &= 188\,355.66 \times 0.001876 + 177\,370.43 \times 0.002197 \\ &\quad - 628.85 \times 0.995927 \\ &= \$116.68, \end{aligned}$$

Exercise 7.8 b)

and

$$\begin{aligned}V[L_1] &= 188\,355.66^2 \times 0.001876 + 177\,370.43^2 \times 0.002197 \\ &\quad + (-628.85)^2 \times 0.995927 - 116.68^2 \\ &= 11\,663.78^2\end{aligned}$$

so that the standard deviation of L_1 is \$11 663.78.

Exercise 7.8 c)

(c) The sum insured, S , is calculated using the equivalence principle. As

$$P\ddot{a}_{[41]:\overline{3}|} = SA_{[41]:\overline{3}|}$$

we have $S = \$1090.26$.

Exercise 7.8 d)

(d) For the endowment insurance, the distribution of L_1 is:

$$L_1 = \begin{cases} 1090.26 v - P = \$704.96 & \text{w.p. } q_{[41]+1} , \\ 1090.26 v^2 - P(1 + v) = \$341.47 & \text{w.p. } p_{[41]+1} . \end{cases}$$

Note that the loss is the same whether the policyholder dies between ages 42 and 43 or survives to age 43.

Hence, $E[L_1] = \$342.15$ and $SD[L_1] = \$15.73$.

Exercise 7.8 e)

- ▶ $E[L_1]$ is greater for the endowment insurance than for the term insurance;
 - ▶ for endowment insurance a large portion of the premium is needed to fund the maturity benefit.

Exercise 7.8 e)

- ▶ Standard deviations: for the term insurance is \$11 663.78, considerably larger than that for the endowment insurance \$15.73.
 - ▶ future cash flows for the term insurance are much more uncertain than those for the endowment insurance.
 - ▶ for the term insurance L_1 takes very different values, \$177 370.43 and -\$628.85 depending on whether the policyholder dies between ages 42 and 43 or survives to age 43,
 - ▶ for the endowment insurance the value is the same, \$341.47.

Exercise 7.8 e)

For the endowment insurance the insurer knows that the sum insured will be paid at some time within the next two years, the only uncertainty is over the timing; for the term insurance it is not certain that the sum insured will ever be paid.