First coursework due by 9am Mon 19 Feb
Submit on QMplus.

Recap quiz

Given an LP what is an extreme point solution
(i) intuitively/geometrically?
(ii) formally?

What are the 3 main steps in transforming an LP to standard inequality form?

Standard inequality form

$$\begin{aligned} \max\ & \underline{c}^T \underline{x} \quad ② \\ \text{sub to}\ & A\underline{x} \leq \underline{b} \quad ③ \\ & \underline{x} \geq \underline{0} \quad ① \end{aligned}$$

How to transform any linear program to standard inequality form

① For each variable x_i if sign constraint is

$x_i \geq 0$ ✓

$x_i \leq 0$ replace x_i with $\bar{x}_i \geq 0$ where $x_i = -\bar{x}_i$

x_i unrestricted replace x_i with $x_i^+ - x_i^-$ with $x_i^+ \geq 0$ $x_i^- \geq 0$

② If goal is $\min \underline{c}^T \underline{x}$ replace with $\max (-\underline{c})^T \underline{x}$

minimizing a fn is same as maximising negative of that function

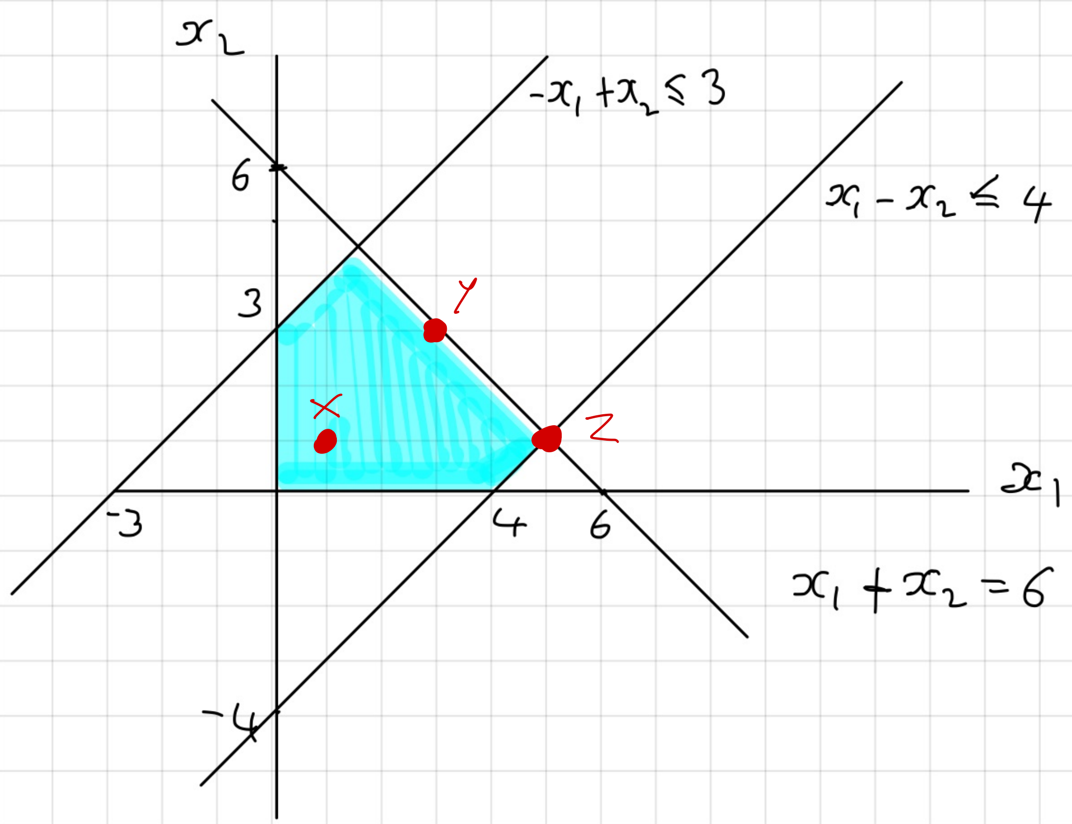
③ For each constraint, if constraint is

$\underline{a}^T \underline{x} \leq b$ ✓

$\underline{a}^T \underline{x} \geq b$ replace with $(-\underline{a})^T \underline{x} \leq -b$

$\underline{a}^T \underline{x} = b$ replace with two constraints

$$\underline{a}^T \underline{x} \leq \underline{b}$$
$$(-\underline{a})^T \underline{x} \leq -b$$



<u>Task</u> : (i) Give standard equation form
(ii) Write down feasable solution of standard equation form that correspond to X, Y, Z

<u>standard inequality form</u>

maximise $2x_1 + 3x_2$

sub to
$$-x_1 + x_2 \leq 3 \quad (1)$$
$$x_1 - x_2 \leq 4 \quad (2)$$
$$x_1 + x_2 \leq 6 \quad (3)$$
$$x_1, x_2 \geq 0$$

X Y Z

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Recap quiz

Given an LP what is an extreme point solution

(i) intuitively/geometrically?

(ii) formally?

Given an LP in standard equation form

$$\begin{aligned} \max\ & \underline{c}^T \underline{x} \\ \text{subject to}\ & A\underline{x} = \underline{b} \\ & \underline{x} \geq \underline{0} \end{aligned}$$

What is an optimal solution

(i) intuitively?

(ii) formally?

Suppose $x \leq 100$ and $y \leq 100$

and average of x and y is exactly 100.

What can we say about x and y?

Example

$$\max \quad x_1 + x_2 + x_3$$
$$\text{sub to} \quad x_1 + \quad\quad + x_3 = 1$$
$$x_2 + 2x_3 = 3$$
$$x_1, x_2, x_3 \geq 0$$

$$\underline{c} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

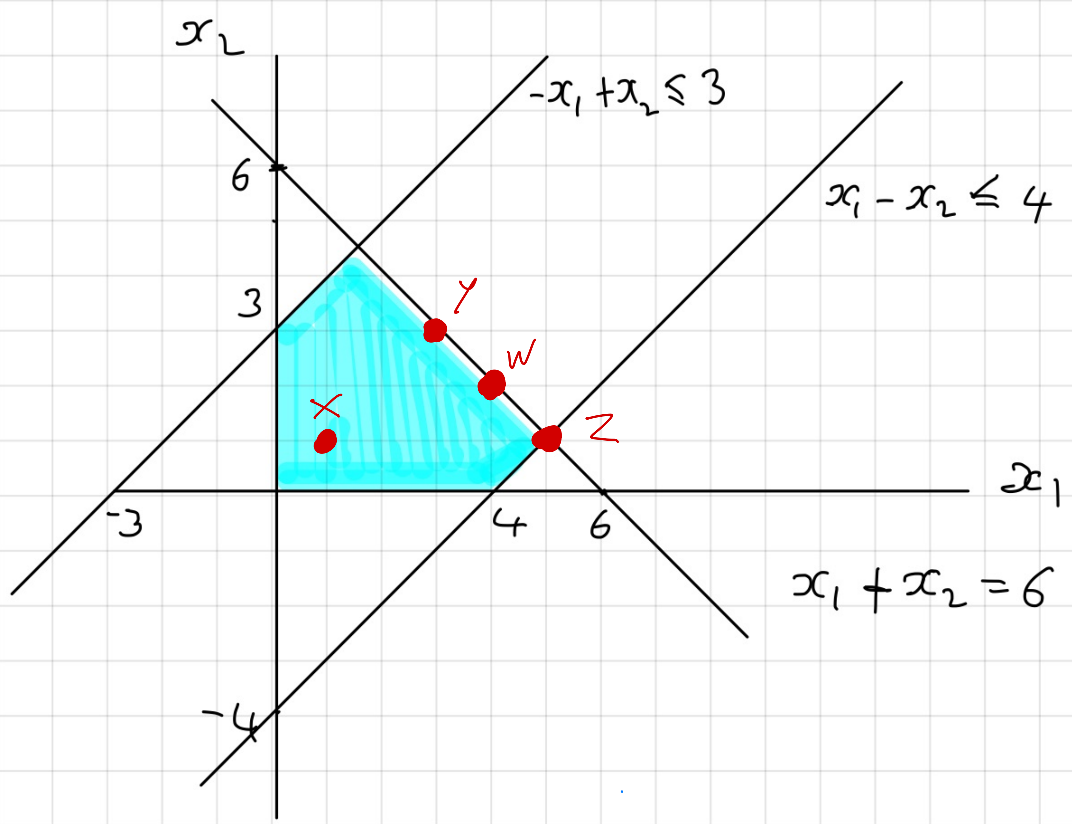
All solutions below are feasible (check). Which are basic feasible?

(a) $\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1/2 \\ 2 \\ 1/2 \end{pmatrix}$

Lin alg recap.

Q If $A = \begin{pmatrix} \underline{c}_1 & \underline{c}_2 & \underline{c}_3 & \underline{c}_4 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 2 \\ 0 \\ 3 \\ 0 \end{pmatrix}$ and $A\underline{v} = \underline{0}$

What can we say about linear dependence/independence of columns $\underline{c}_1, \underline{c}_2, \underline{c}_3, \underline{c}_4$ of A.



standard inequality form

maximise $2x_1 + 3x_2$

sub to
$-x_1 + x_2 \leq 3$ (1)
$x_1 - x_2 \leq 4$ (2)
$x_1 + x_2 \leq 6$ (3)
$x_1, x_2 \geq 0$

standard equation form

maximise $2x_1 + 3x_2$

sub to
$-x_1 + x_2 + s_1 = 3$
$x_1 - x_2 + s_2 = 4$
$x_1 + x_2 + s_3 = 6$
$x_1, x_2, s_1, s_2 \geq 0.$

X Y Z

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

X Y Z

$$\begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 3 \\ 4 \\ 0 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 7 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$