Actuarial Mathematics II
MTH5125

Reserves - Part 2

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Retrospective policy values

The basic formulas we've seen for policy values have been **prospective** in nature, meaning that at time t we're computing the policy value by considering what's expected to happen in the future.

We can also define a **retrospective** policy value by looking from time t back to the time of policy issue. The general form of a retrospective policy value is:

Retrospective policy value at time $t =$ *accumulated value* at time t of *past premiums* - *accumulated value* at time t of *past benefits and expenses*

Retrospective policy values

Define:

$$
\begin{aligned}
L_{0,t} &= PV \text{ at issue of future benefits payable up to } t \\
&\quad - PV \text{ at issue of future Premiums payable up to } t
\end{aligned}
$$

Note that:

$$L_0 = L_{0,t} + 1\,(T_x > t)\, v^t L_t$$

Retrospective policy values

The retrospective net premium policy value is:

$$ {}_tV^R = \frac{-E\left[L_{0,t}\right](1+i)^t}{{}_tp_x} = \frac{-E\left[L_{0,t}\right]}{{}_tE_x} $$

Retrospective policy values

If:

1. the premium is calculated using equivalent principle and
2. the same basis is used for policy values, retrospective policy values and the equivalence priciple then:

$$
\begin{aligned}
E\left[L_0\right] &= E\left[L_{0,t} + 1\left(T_x > t\right) v^t L_t\right] = 0 \\
&\Rightarrow -E\left[L_{0,t}\right] = E\left[1\left(T_x > t\right) v^t L_t\right] \\
&\Rightarrow -E\left[L_{0,t}\right] = {}_t p_x v^t \; {}_t V^P \\
&\Rightarrow {}_t V^R = {}_t V^P
\end{aligned}
$$

Retrospective policy value is equal to prospective policy value.

Retrospective policy values

Example 7.16 An insurer issues a whole life insurance policy to a life aged 40. The death benefit in the first five years of the contract is \$5 000. In subsequent years, the death benefit is \$100 000. The death benefit is payable at the end of the year of death. Premiums are paid annually for a maximum of 20 years. Premiums are level for the first five years, then increase by 50%.

(a) Write down the equation of value for calculating the net premium, using standard actuarial functions.

(b) Write down equations for the net premium policy value at time $t = 4$ using (i) the retrospective policy value approach, and (ii) the prospective policy value approach.

(c) Write down equations for the net premium policy value at time $t = 20$ using (i) the retrospective policy value approach, and (ii) the prospective policy value approach.

Retrospective policy values

a) If we work in units of 1000:

$$P = \frac{5\,A^{\;\;1}_{40:\overline{5|}} + 100\,{}_5E_{40}\,A_{45}}{\ddot{a}_{40:\overline{5|}} + 1.5\,{}_5E_{40}\,\ddot{a}_{45:\overline{15|}}}.$$

from?

Retrospective policy values

c) the retrospective and prospective policy values equations at time $t = 4$ are:

$$ {}_4V^R = \frac{P\,\ddot{a}_{40:\overline{4}|} - 5A^{\;1}_{40:\overline{4}|}}{{}_4E_{40}} $$

and

$$ {}_4V^P = 5\,A^{\;1}_{44:\overline{1}|} + 100\,{}_1E_{44}\,A_{45} - P\left(\ddot{a}_{44:\overline{1}|} + 1.5\,{}_1E_{44}\,\ddot{a}_{45:\overline{15}|}\right). $$

Retrospective policy values

For $t = 20$

$$ {}_{20}V^R = \frac{P(\ddot{a}_{40:\overline{5}|} + 1.5\,{}_5E_{40}\,\ddot{a}_{45:\overline{15}|}) - 5\,A^{\;1}_{40:\overline{5}|} - 100\,{}_5E_{40}\,A_{45:\overline{15}|}}{{}_{20}E_{40}} $$

and

$$ {}_{20}V^P = 100\,A_{60}. $$

Retrospective policy values

From these equations we see that the retrospective policy value offers an effcient methos at the start of the contract, when the premium and benefit are charged ahead, and the prospective approach is more effcient at later durations, when the changes are in the past.