Practice Set 4

1. An investor is considering entering a business venture which involves a 50-50 chance of an income of £900 or £400 depending on the state of the economy. The initial investment involves a payment of I_0 =£650. Investor's initial wealth is $w_0 = £1{,}000$.

   a) Using the definition of a "fair gamble", explain whether this business venture is a fair gamble for the investor.

   b) Assume that the investor's utility function of wealth is $v(w) = \sqrt{w}$, with $w > 0$. Explain, giving reasons, whether the investor would invest I_0 or any other amount in this venture.

   c) Suppose the investor has instead a utility function described by $v(y) = ay$ where $a > 0$. What is the amount the investor is prepared to invest to take part in the venture?

   d) If the investor's utility function is $v(w) = w^2$, is he willing to invest the required initial investment of £650?

   e) Explain the reason for the different investment values (under which the investor is willing to invest) in the situations described in points b), d), and e).

2. An investor has the choice between two assets, A and B.
   The annual return on asset A is a continuous random variable uniformly distributed between 3% and 7%. Note that the probability density function for a continuous uniform distribution on the interval $[a, b]$ is $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$ and $f(x) = 0$ otherwise.

   The annual return on investment B will only take discrete values with probabilities given in the following table:

Probability	0.3	0.3	0.2	0.2
Return	4%	−4%	10%	−8%

   For each investment calculate the following statistics:
   i) Expected annual return.
   ii) Variance of annual return.
   iii) Semi-variance of annual return.
   iv) Shortfall probability with a required level of annual return of 6%.
   v) Explain which investment you would choose based on your previous answers.

3. There are 100 possible states for the market over the next year with each state being equally likely. Two assets, *A* and *B* have returns over the next year of R_A and R_B respectively. These returns depend on the state of the market and are given below as percentages:

Market condition	R_A	R_B
1	-5.0	-10.0
2	-3.5	-7.5
3	-3.0	-6.5
4	-2.8	-5.0
5	-2.5	-2.5
6	-2.0	-2.0
...	...	...
98	6.5	6.5
99	9.0	9.0
100	10.0	10.0

For $X = A,B$: calculate:
(i) $SP(R_X;\ -2.5)$
(ii) $VaR(R_X; q = 5\%)$ and $VaR(R_X; q = 1\%)$