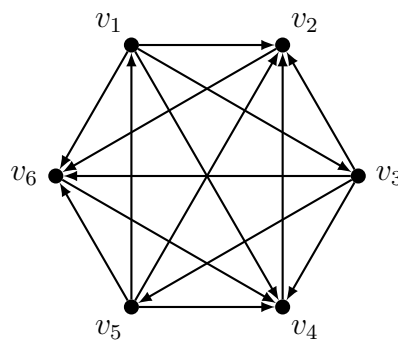


You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

1. Consider a graph  $G$ , and let  $e \in E(G)$ .
  - (a) Show that  $v_0e_1v_1e_2v_2 \dots e_mv_mv_0$  is a shortest cycle containing  $e$  in  $G$  if and only if  $v_0e_1v_1e_2v_2 \dots e_m$  is a shortest  $v_0-v_m$ -path in the graph  $G'$  with  $V(G') = V(G)$  and  $E(G') = E(G) \setminus \{e\}$ .
  - (b) Give an algorithm for finding a shortest cycle in  $G$  that contains  $e$ .
  - (c) Give an efficient algorithm for finding a shortest cycle in  $G$ . Show that the algorithm is indeed efficient.
2. Find the strongly connected components of the following digraph.



3. Consider the following spreadsheet, in which some cells contain a formula that depends on the values of other cells.

	a	b	c	d	e	f	g
1	e1+g5	a1-c5	110	a1+c1	180	f5-e1	c1+c2
2	a1+b1	a2+c4	240	a2+c2	120	f5-e2	e3+e5
3	a2+b2	a3-c3	100	a3+c1	200	f5-e3	f1+f2
4	a3+b3	a4+c2	220	a4+c2	100	f5-e4	f3+f4
5	a4+b4	a5-c1	130	a5+c5	120	g3+g4	g1+g2

- (a) Which of the values in columns c and e can be changed without changing the value of cell b2?
- (b) Is it possible to compute the values of all cells in the spreadsheet? Justify your answer.

To answer these questions, you may want to consider the digraph  $D$  where  $V(D)$  is the set of cells in the spreadsheet and  $E(D)$  contains an arc from cell  $u$  to cell  $v$  if cell  $u$  contains a formula that depends on the value of cell  $v$ .