MTH6105 – Algorithmic Graph Theory **Spring 2024**

Problem Sheet 6 **F. Fischer**

You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

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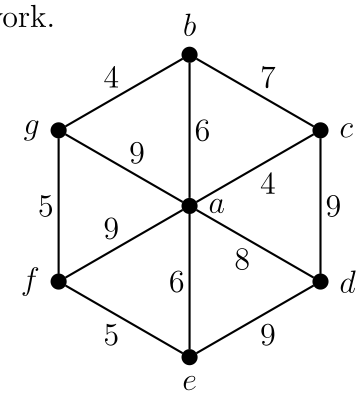
1. Consider a network (G, w), and let T be a minimum spanning tree of (G, w). Show that (G, w) has a unique minimum spanning tree if and only if the following condition is satisfied: for every edge $e \in E(G) \setminus E(T)$ with endpoints $u, v \in V(G)$ and every edge $d \in E(T)$ contained in the unique u–v-path in T, $w(e) > w(d)$.

2. Consider the following network.



   (a) Use Prim's algorithm starting from vertex b to find a minimum spanning tree of the network.

   (b) Give another minimum spanning tree of the network.

3. Consider a directed network (D, w) such that $w(e) \geq 0$ for all $e \in A(D)$. Let $v \in V(D)$.

   (a) Give an algorithm with running time $O(|V(D)| \cdot |A(D)|)$ that finds a shortest directed v–u-path in (D, w) for every $u \in V(D)$ for which such a path exists. Provide a brief justification for the claimed running time.

   (b) Give an algorithm with running time $O(|V(D)| \cdot |A(D)|)$ that finds a shortest directed u–v-path in (D, w) for every $u \in V(D)$ for which such a path exists.

   (c) Find shortest directed v_8–u-paths and shortest directed u–v_1-paths in the following directed network for all $u \in \{v_1, v_2, \ldots, v_8\}$.