

MTH5114      Linear Programming and Games, Spring 2024  
Week 3 Seminar Questions      Viresh Patel

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**Warm-Up Questions (from 2019 Midterm):** Sketch each of the following linear programs in 2 variables, then find its optimal solution. If an optimal solution does not exist, say why.

$$\begin{aligned} &\text{maximize} && 2x_1 + x_2 \\ &\text{subject to} && 2x_1 + 3x_2 \leq 12, \\ & && 4x_1 + 2x_2 \leq 12, \\ & && x_1, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} &\text{maximize} && 2x_1 - x_2 \\ &\text{subject to} && x_1 - x_2 \leq 1, \\ & && 2x_1 - x_2 \geq 1, \\ & && 2x_1 + 2x_2 \geq 4, \\ & && x_1, x_2 \geq 0 \end{aligned}$$

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**Discussion Questions:**

1. Suppose that  $p, q \in \mathbb{R}$  are both constants and consider the following linear program:

$$\begin{aligned} &\text{maximize} && px_1 + qx_2 \\ &\text{subject to} && x_1 + 3x_2 \leq 6, \\ & && 3x_1 + x_2 \leq 6, \\ & && x_1 + x_2 \geq 1, \\ & && x_1, x_2 \geq 0 \end{aligned}$$

- (a) Sketch the feasible region for these constraints.
- (b) Find a value for the constants  $p$  and  $q$  so that  $x_1 = 3/2, x_2 = 3/2$  is the unique optimal solution of the resulting program.
- (c) Find a value for the constants  $p$  and  $q$  so that  $x_1 = 0, x_2 = 1$  is the unique optimal solution of the resulting program.
- (d) Find a value for the constants  $p$  and  $q$  for which *both*  $x_1 = 3/2, x_2 = 3/2$  and  $x_1 = 2, x_2 = 0$  are optimal solutions of the resulting program.

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- (e) Find a value for the constants  $p$  and  $q$  for which *both*  $x_1 = 3/2$ ,  $x_2 = 3/2$  and  $x_1 = 0, x_2 = 2$  are optimal solutions of the resulting program.
- (f) Determine all  $p$  and  $q$  such that  $x_1 = 3/2$ ,  $x_2 = 3/2$  is an optimal solution to the resulting program?
2. (a) A fruit farmer has 3 hectares of apple trees and 1.5 hectares of orange trees and 4 days of work available to complete his harvest. Picking each hectare of apples takes 1 day of work and produces 0.5 truckloads of apples. Picking each hectare of oranges takes 1.5 days of work and produces 0.25 truckloads of oranges. Suppose that each truckload of apples sells for 600 pounds, and each truckload of oranges sells for 800 pounds. Write a linear program to find the most money that the farmer can make.
- (b) Sketch your linear program and find the optimal solution. Note you may need to eliminate unnecessary variables to reduce the total number of variables to 2. Additionally, it may be helpful to change the “units” for the objective (for example, to 100’s of pounds) in order to make it easier to sketch.
- (c) For next year, should the farmer plant more apples, more oranges, or find a way to increase the number of days available (by hiring additional labour, for example)?