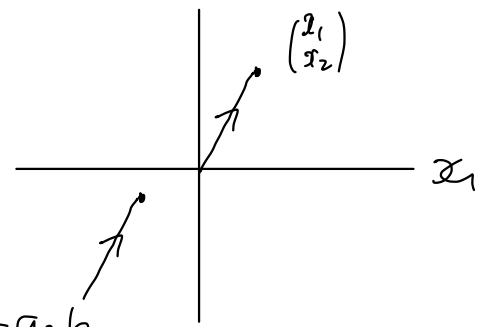Facts about geometry of $\mathbb{R}^2$

$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$ can be thought of as a point in $\mathbb{R}^2$ or as a vector

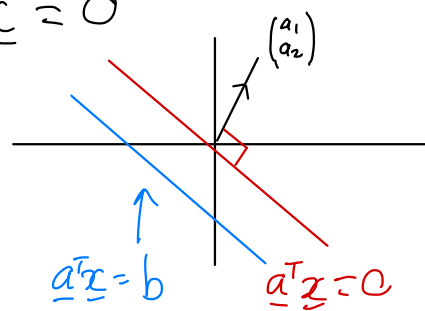If $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\underline{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$\underline{a}^T \underline{b} = a_1 b_1 + a_2 b_2$

If $\underline{a}^T \underline{b} = 0$ means $\underline{a}$ and $\underline{b}$ are perpendicular.

Fix $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$. Which $\underline{x} \in \mathbb{R}^2$ satisfy $\underline{a}^T \underline{x} = 0$

ans: all $\underline{x}$ on the line perpendicular to $\underline{a}$ that goes through $\underline{0}$.

$$a_1 x_1 + a_2 x_2 = 0$$

Which $\underline{x} \in \mathbb{R}^2$ satisfy $\underline{a}^T \underline{x} = b$ $b \in \mathbb{R}$

ans: all $\underline{x}$ on the line perpendicular to $\underline{a}$ but through some other point e.g. b/a_2.

$$a_1 x_1 + a_2 x_2 = b \quad \text{goes through} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ b/a_2 \end{pmatrix}$$

What happens to the line $\underline{a}^T \underline{x} = b$ as b increases.

ans: it stays perpendicular to $\underline{a}$ but moves in the direction of $\underline{a}$.

Lecture 3(a) Geometry of linear programming

Aim: to solve LPs with two variables by drawing

Recap quiz

$3x_1 - 2x_2 = 5$ is line in $\mathbb{R}^2$.

Find a vector perpendicular to this line

Geometrically what happens to the line if we replace 5 with 6?

Given an LP, what is a

(i) feasible solution?

(ii) optimal solution?