MTH5126 Statistics for Insurance

Formula given in the final exam

You can utilize the following formula directly without the need for proof, unless specified otherwise.

- $M_X(t) = E[e^{tX}] = \sum_{k=0}^{\infty} E[X^k] \frac{t^k}{k!}$
  $E[X^k] = \frac{d^k}{dt^k} M_X(t)|_{t=0}$ (should know how to prove)
- Relationship between Gamma and Chi-squared distributions: If $X \sim Gamma(\alpha, \lambda)$ and 2α is an integer, then $2\lambda X \sim \chi^2_{2\alpha}$
- The PDF of the reinsurer's claims is $g(w) = \frac{f(w+M)}{1-F(M)},\ w > 0$ (should know how to prove)
- The law of total expectation: $E(S) = E[E[S|N]]$
- The law of total variance: $var(S) = E[var[S|N]] + var[E[S|N]]$
- $E(S) = E(N)E(X)$ (should know how to prove)
- $var(S) = E(N)var(X) + var(N)[E(X)]^2$ (should know how to prove)
- $M_S(t) = M_N(lnM_X(t))$ (should know how to prove)
- For the case where there are 3 variables, Archimedean copulas take the form: $C[u, v, w] = \psi^{[-1]}(\psi(u) + \psi(v) + \psi(w))$.
- $U(t) = U + ct - S(t)$.
- $\lambda M_X(R) = \lambda + cR$
- $M_X(R) = 1 + (1 + \theta)m_1 R$ (should know how to prove)
- $c = (1 + \theta)\lambda m_1$
- Lundberg's inequality: $\Psi(U) \leqslant e^{-RU}$
- $\Psi(U) \approx e^{RU}$

- Upper bound for R: $R < \frac{2(c-\lambda m_1)}{\lambda m_2}$ or $R < \frac{2\theta m_1}{m_2}$;
  Lower bound for R ($0 \leqslant X \leqslant M$): $R > \frac{1}{M} ln(\frac{c}{\lambda m_1})$ or $R > \frac{1}{M} ln(1+\theta)$
  (should know how to prove)

- The formula for $\Psi(U)$ when individual claim amounts are exponentially distributed with mean 1 and when the premium loading factor is θ is given by the following result:
  When $F(x) = 1 - e^{-x}$: $\Psi(U) = \frac{1}{1+\theta} e^{-\frac{\theta U}{1+\theta}}$

In addition, (1) the mean, variance, MGF, CDF and PDF of common distributions, (2) Extreme Value distributions, and (3) copula functions will be provided in the final exam, except the mean and variance of Normal and Poisson distributions.