

QUEEN MARY UNIVERSITY OF LONDON

MTH5120
Exercise Sheet 2

Statistical Modelling I

1.

A fire insurance company wants to relate the amount of fire damage in major residential fires to the distance between the residence and the nearest fire station. The study was conducted in a large suburb of a major city; a sample of fifteen recent fires in the suburb was selected. The amount of damage, y (£000), and the distance, x (km), between the fire and the nearest fire station are given in the following table.

x	3.4	1.8	4.6	2.3	3.1	5.5	0.7	3.0
y	26.2	17.8	31.3	23.1	27.5	36.0	14.1	22.3
x	2.6	4.3	2.1	1.1	6.1	4.8	3.8	
y	19.6	31.3	24.0	17.3	43.2	36.4	26.1	

- (a) Fit a simple linear regression model to these data. Check if the residual plots give any reason to doubt the usual assumptions of the model.
- (b) Write a short report giving your conclusions. You do not need to upload the output from R. Include a possible interpretation of the intercept and slope parameters.
2. (a) Derive the Least Squares Estimator $\hat{\beta}$ of the parameter β in the no-intercept model

$$Y_i = \beta x_i + \varepsilon_i,$$

where $\varepsilon_i \stackrel{iid}{\sim} N(0, \sigma^2)$.

- (b) Obtain the distribution of $\hat{\beta}$ including the mean and the variance of the estimator.
3. Suppose Y_i is an i.i.d sequences of random variables, such as $Y_i \sim \mathcal{N}(\theta, 1)$ for $i = 1, \dots, n$. Let us consider the following estimators:

$$\hat{\theta}_1 = \frac{1}{n-1} \sum_{i=1}^n y_i, \quad \hat{\theta}_2 = \frac{1}{2}(y_1 + y_n)$$

- (a) Determine the mean of the estimators;
- (b) Determine the variance of the estimators;
- (c) Are the estimators biased?