

1. In the last Budget the Chancellor had announced the launch of a new three-year "Investment Guaranteed Growth" bond which will pay 2.2% annual return on an investment up to £3,000. There is a penalty for cashing in early, which equals 90 days' interest on the amount cashed in. The bond is available from today to anyone aged 16 or over.

At the same time the Atom Bank offers a two-year bond which pays a 2.1 % annual return on any amount invested with a penalty of only 30 days' interest on the amount cashed in. You can only buy the Atom Bank bonds through an app. As these bonds have different characteristics (in terms of return, maturity, penalty for cashing in early and availability over the internet or not) you need to analyse very carefully what is your preference for them. After a lot of introspection, you realise that your preference for these two bonds is described by the utility function $U(X, Y) = (X + 200)(Y + 100)$ where X is the amount invested in the government bond and Y the amount invested in the Atom Bank bond. Your budget for investing in these two bonds is £3,000. Both the bonds are selling at par, i.e. you can assume that for £1 face value you pay £1, hence $p_X = p_Y = £1$

- a. List the assumptions you would need to make about your preferences so that you can apply utility theory to find your optimum investment in each bond.
- b. Explain what you understand by your indifference curve for these investments.
- c. Write an equation for your indifference curve that goes through the point $(X, Y) = (200, 800)$. Sketch this indifference curve.
- d. What is your budget constraint and your marginal rate of substitution?
- e. Write your optimization problem and find the optimum amounts invested in the two bonds.
- f. If these bonds had been offered in a different country by both a government and a private bank with histories of defaulting on debt, would you still use the same method to find your optimal choice for investment? If yes, explain briefly why. If not, explain briefly what method would you use instead? Assume that now you know the expected returns of these bonds and their risks expressed as their standard deviation.

2. A consumer has utility from consuming goods x_1 and x_2 $f(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$.
- (a) If the price of x_1 is p_1 and the price of x_2 is p_2 while her income is m find this consumer's demand function (the optimal choice for x_1 and x_2).
- (b) If $p_1 = £2$ is $p_2 = £1$ and the consumer has an income of £100 what are the demands for x_1 and x_2 of this consumer.
3. An investor, who prefers more to less, has preferences can be modelled by the utility function such that
 $U'(w) = 7.5 - 2w$ ($w > 0$)
- a) Over what range can this utility function be satisfactorily applied?
- b) Show how the investor's absolute amount and relative amount of holdings in risky assets change as his wealth increases.
4. A homeowner has initial wealth of £100,000 and utility function of wealth
 $U(w) = \ln(w)$. In the next year there is a chance that her house will suffer damage as follows:

| Value of damage | Probability |
|-----------------|-------------|
| £0 | 0.89 |
| £5,000 | 0.1 |
| £50,000 | 0.01 |

An insurer offers a policy that will fully protect the householder against losses arising from such damage in the next year. The insurer calculates that the premium charged for the policy should be 20% higher than the expected costs of claims.

- a) Calculate the premium that the insurer will charge.
- b) Calculate the maximum price that the householder is prepared to pay and hence determine whether the policy will be purchased.