Mathematical Tools for Asset Management

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Practice 1 Solutions

1. Assume that there are only three possible states of the economy (recession state, normal state and an expansion state). From past experience and your personal beliefs, you expect the economy will be in a recession state 25% of the time, in the normal state 50% of the time and in the expansion state 25% of the time. There are two stocks, A and B with the following returns in different states of the economy.

	Probability	Ret A	Ret B
Recession	0.25	-4%	0%
Normal	0.5	4%	1%
Expansion	0.25	5%	10%

a) What is the expected return of stock A? What is the expected return of stock B?

b) What is the variance of return on stock A? What is the variance of return on stock B?

c) What is the standard deviation for stock A? What is the standard deviation for stock B?

d) What is the covariance between returns of Stock A and returns of Stock B? What is their correlation coefficient?

e) What are the expected return and variance of a portfolio of 50% stock A and 50% stock B?

f) What are the expected return and variance of a portfolio of 70% stock A and 30% stock B?

g) What are the expected return and variance of a portfolio of 30% stock A and 70% stock B?

a) What is the expected return of stock A? What is the expected return of stock B?

	Probability	Ret A	Ret B
Recession	0.25	-4%	0%
Normal	0.5	4%	1%
Expansion	0.25	5%	10%

What is the expected return of stock A? What is the expected return of stock B?

Expected Return of Stock A

$$E(R_A) = 0.25 \times (-4\%) + 0.5 \times (4\%) + 0.25 \times (5\%) = 2.25\%$$

or

$$E(R_A) = 0.25 \times (-0.04) + +0.5 \times 0.04 + 0.25 \times 0.05 = 0.0225$$

Expected Return of Stock B

$$E(R_B) = 0.25 \times (0\%) + 0.5 \times (1\%) + 0.25 \times (10\%) = 3.00\%$$

$$E(R_B) = 0.25 \times 0 + 0.5 \times 0.01 + 0.025 \times 0.1 = 0.03$$

What is the variance of return on stock A? What is the variance of return on stock B?

	Probability	Ret A	Ret B
Recession	0.25	-4%	0%
Normal	0.5	4%	1%
Expansion	0.25	5%	10%

What is the variance of return on stock A? What is the variance of return on stock B?

Variance of Return on Stock A:

$$var(R_A) = 0.25(-0.04 - 0.0225)^2 + 0.5(0.04 - 0.0225)^2 + 0.25(0.05 - 0.0225)^2$$
$$= 0.00131875$$

Variance of Return on Stock B

$$var(R_B) = 0.25(0 - 0.03)^2 + 0.5(0.01 - 0.03)^2 + 0.25(0.1 - 0.03)^2 = 0.00165$$

What is the standard deviation for stock A? What is the standard deviation for stock B?

Standard Deviation for Stock A is

$$StdDev(R_A) = \sqrt{0.001319} = 0.0363$$ or 3.63%

Standard Deviation for Stock B

$$StdDev(R_B) = \sqrt{0.00165} = 0.0406$$ or 4.06%

What is the covariance between returns of Stock A and returns of Stock B? What is their correlation coefficient?

	Probability	Ret A	Ret B
Recession	0.25	-4%	0%
Normal	0.5	4%	1%
Expansion	0.25	5%	10%

$$Co\,var(R_A R_B) = 0.25 \times (-0.04 - 0.0225) \times (0 - 0.03) + 0.5 \times (0.04 - 0.0225) \times (0.01 - 0.03)$$
$$+ 0.25 \times (0.05 - 0.0225) \times (0.1 - 0.03) = 0.000775$$

$$corr(R_A, R_B) = \frac{0.000775}{0.0363 \times 0.0406} = 0.5254$$

Summing up:

	Return A	Return B
Expected Return	2.25%	3.00%
Variance	0.001319	0.001650
St. Dev	3.6315%	4.0620%
Covariance	0.000775	
Correlation	0.525386	

h) What are the expected return and variance of a portfolio of 50% stock A and 50% stock B?

i) What are the expected return and variance of a portfolio of 70% stock A and 30% stock B?

j) What are the expected return and variance of a portfolio of 30% stock A and 70% stock B?

The expected return and the expected variance of a portfolio of N assets with returns R_i and weights w_i are:

$$E[R_P] = \sum_{i=1}^{N} w_i E[R_i]$$

$$\text{var}[R_P] = \sum_{i=1}^{N} w_i^2 \text{var}[R_i] + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \text{cov}[R_i, R_j]$$

Portfolio	50% - 50%	70% - 30%	30% - 70%
Expected Return	2.63%	2.48%	2.78%
Variance	0.001130	0.001120	0.001253
St. Dev	3.3611%	3.3469%	3.5393%

2. An American investor holds a portfolio comprising of two stocks: Microsoft and Bethlehem Steel. In particular his portfolio is made of 100 Microsoft shares and 650 Bethlehem Steel. The market price of Microsoft shares is $130 and of Bethlehem Steel is $10. Their respective expected returns are 0.10 and 0.16 while their variances are 0.25 and 0.49.

a) What is the portfolio weight of Bethlehem Steel? What is the portfolio weight of Microsoft?

b) What is the market price of this portfolio?

What is the portfolio weight of Bethlehem Steel? What is the portfolio weight of Microsoft?

$$w_{BS} = \frac{650 \times 10}{100 \times 130 + 650 \times 10} = \frac{6500}{19500} = 0.33$$

$$w_M = \frac{100 \times 130}{100 \times 130 + 650 \times 10} = 0.67$$

What is the market price of this portfolio?

$$P_{BS+M} = 100 \times \$130 + 650 \times \$10 = \$19500$$