MTH6105 – Algorithmic Graph Theory

Felix Fischer

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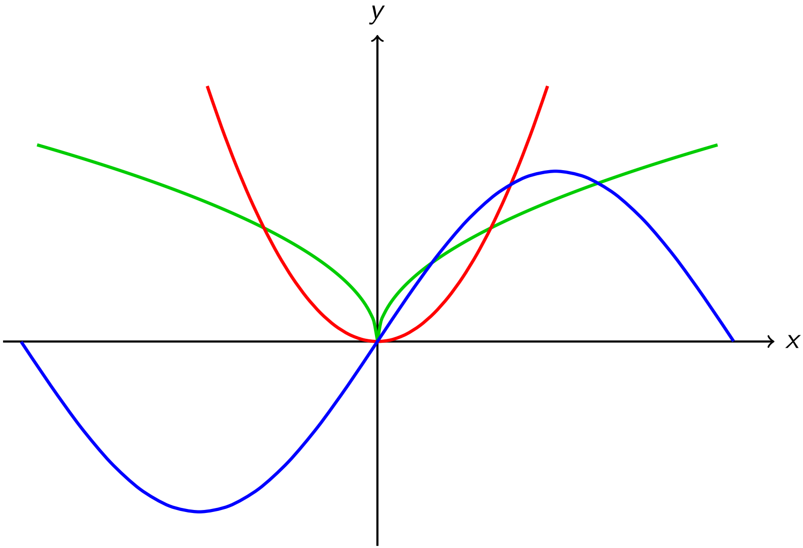
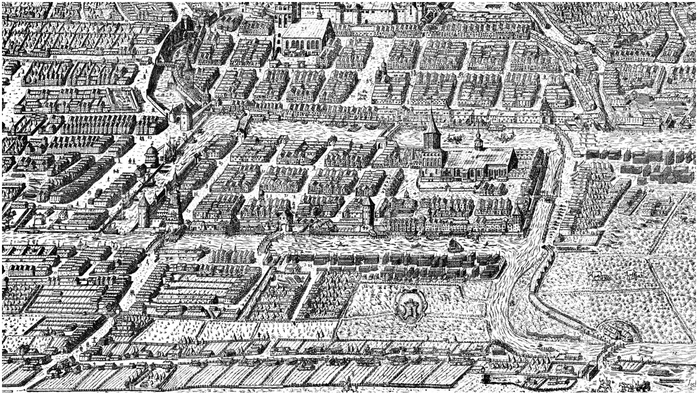
Not These Graphs



The Bridges of Königsberg



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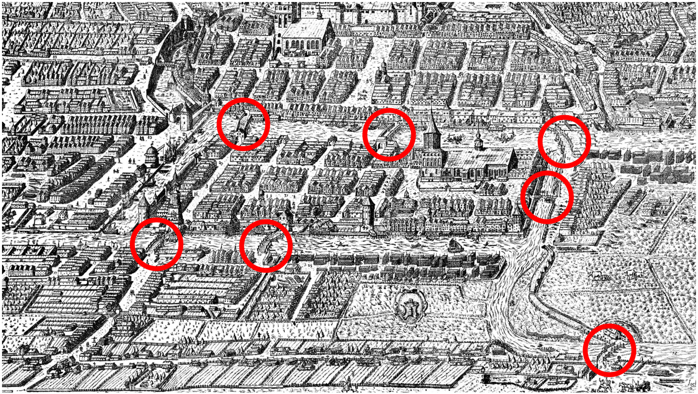

- Seven bridges connected banks of Pregel river and two islands
- Residents wandered and wondered: can we go on a walk and cross every bridge exactly once?

Bridges and Mathematics

- In 1736 the mayor of Danzig, now Gdansk, consulted eminent Swiss mathematician Leonhard Euler
- Euler was bored, then intrigued, and in 1741 published a solution

  > This type of solution bears little relationship to mathematics, and I do not understand why you would expect a mathematician to produce it, rather than anyone else. – Euler to Leonhard Gottlieb Ehler, mayor of Danzig

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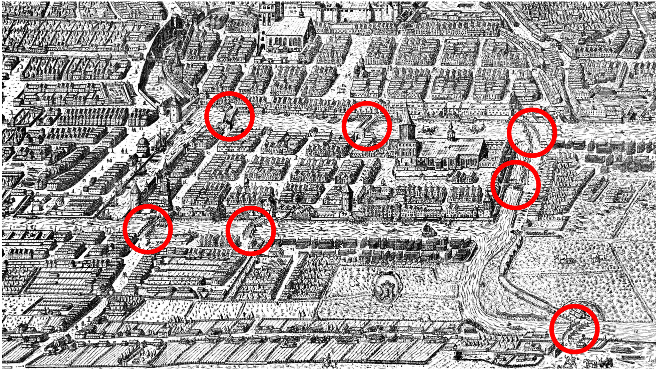
- Arguably the first result in graph theory and topology
- Also points out importance of computational efficiency

A Mathematical Model

- Enumeration of all possible paths not desirable
- What we do while on a land mass is irrelevant
- Bridges between same pair of land masses are equivalent



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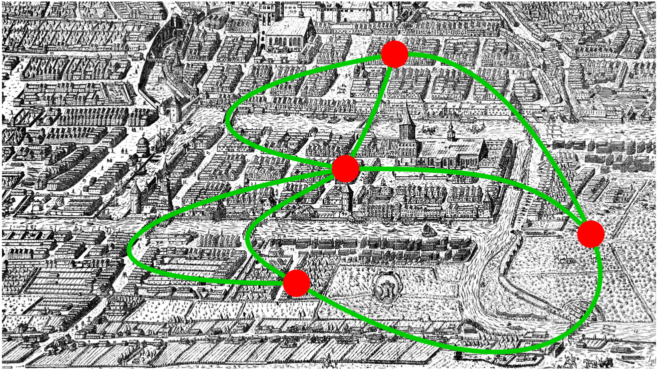

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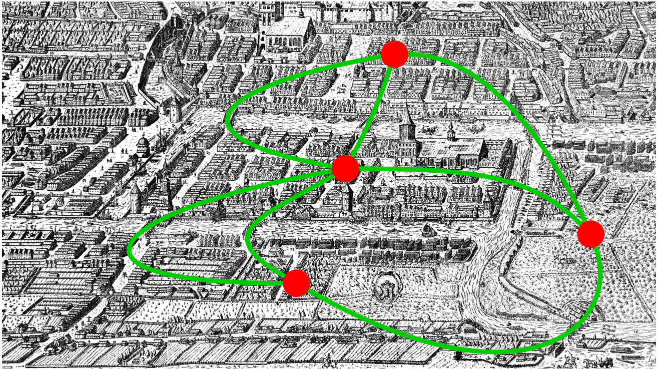

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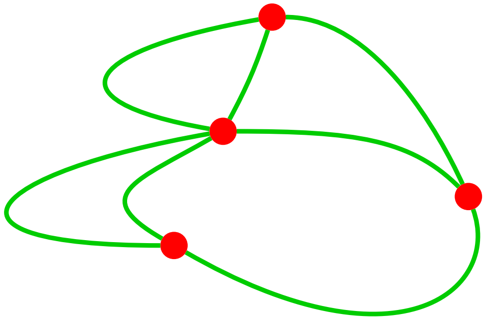

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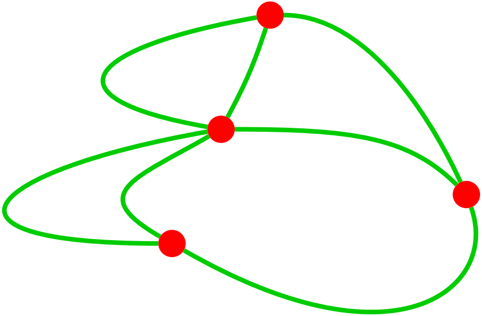

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- Is there an "Euler walk," a walk visiting every edge exactly once?

Graphs, Algorithms, and Combinatorial Optimisation

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  - example: locations on a map and roads between locations

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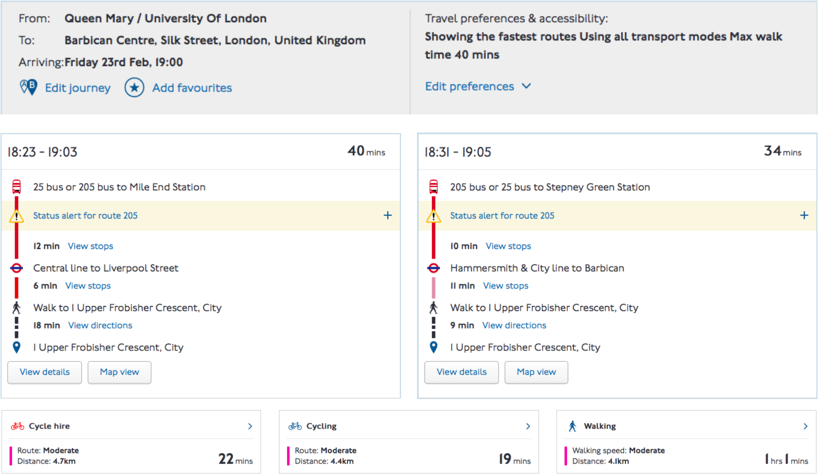
- Compare to MTH5114 Linear Programming and Games
  - among solutions of a set of linear inequalities . . .
  - . . . find one that maximises or minimises a linear objective

Another Problem



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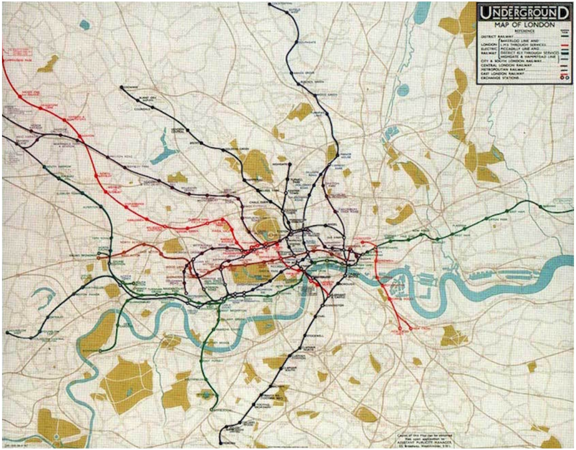

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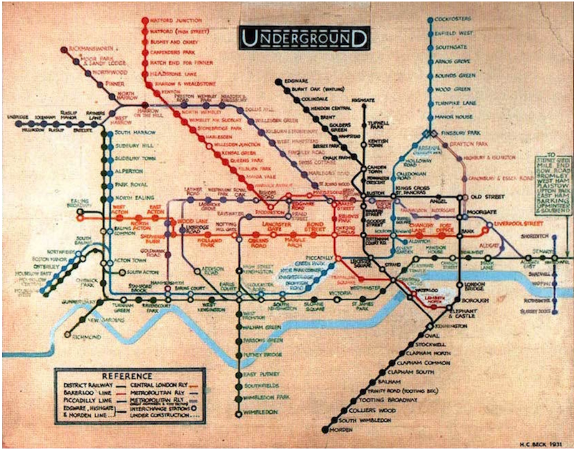

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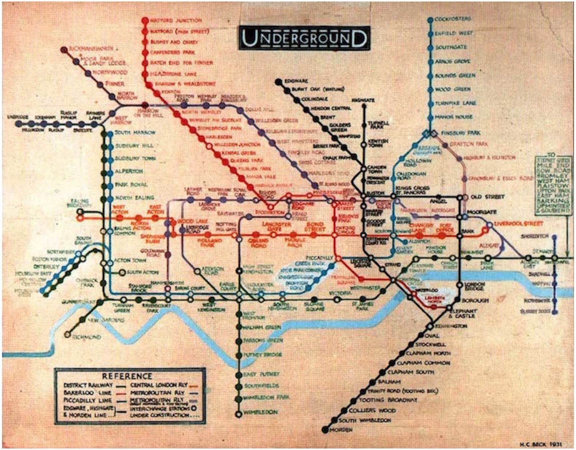

- Use a more detailed version, annotated with travel time, cost, . . .
- Find a walk with lowest travel time, cost, number of changes, . . .

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- Applications: network design, scheduling, project selection, . . .

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Please Interact

- Attend in person and ask questions
- Online attendance and recordings are just a backup

- Provide feedback on what works and what doesn't work

- Learning Support Hour: Thursday 14-15, Maths Social Hub (as part of the Learning Cafe)
- When I am in my office, MB-G23
- At other times or online, by request
- Email: `felix.fischer@qmul.ac.uk`