TUTORIAL Week 1

(A) The value of G

(B) escape velocity

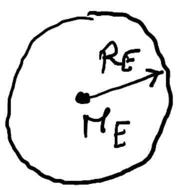
↳ A preview on the Schwarzschild radius

(A) By using the divergence theorem, you can prove that a spherically symmetric mass distribution, i.e. a $\rho(\underline{x})$ which actually depends only on the radial coordinate $r = |\underline{x}|$, produces the same gravitational field as a particle with $M = \int \rho(|\underline{x}|)\, d^3\underline{x}$ in the region where $\rho(|\underline{x}|) = 0$.

Then the gravitational field on the earth surface $\underline{g}$ should be

$$|\underline{g}_E| = \frac{G M_E}{R_E^2}$$

where



Experimentally we have

$$|\underline{g}_E| \simeq 9.8 \ \frac{m}{s^2} \quad \text{and} \quad R_E \simeq 6.4 \ 10^6 \ m$$

Thus we deduce $G M_E = |\underline{g}_E| R_E^2$. Newton guessed that the average mass density of the earth is ~ 5

times the one of water $\rho_{water} \sim 1 \frac{Kg}{(0.1\ m)^3} = 10^3 \frac{Kg}{m^3}$

If this is the case we have

$$G = \frac{|g_E|\ R_E^2}{5 \cdot \rho_{water} \frac{4}{3} \pi R_E^3} \simeq \frac{9.8 \frac{m}{s^2}}{5\ 10^3 \frac{Kg}{m^3} (4.2) (6.4\ 10^6\ m)} \sim$$

$$\sim 7\ 10^{-11} \frac{m^3}{Kg\ s^2}$$

First experimental measurements $\sim$ 100 years after Newton guess. Modern value

$$G \simeq 6.67\ 10^{-11} \frac{m^3}{Kg\ s^2}$$

Comment: it yields tiny forces for "normal" objects!

(B) An object at the earth surface has a velocity $\underline{v}$ pointing upwards $\underline{v} = |\underline{v}| \frac{\underline{R}_E}{R_E}$. For what values of $|\underline{v}|$ does it escape the earth's (we will ignore the effect of the atmosphere).

Start by calculating the initial energy

$$\frac{1}{2} m \underline{v}^2 - G \frac{m M_E}{R_E} = E_{TOT}$$

We know that it is conserved, so this object can travel infinitely far away from the earth only if $E_{TOT} \geq 0$ (because when the distance from the earth becomes very large the contribution from the potential energy becomes negligible so the total energy cannot be negative). From $E_{TOT} = 0$ we get

$$v_{esc}^2 = \frac{2GM_E}{R_E}$$

Notice that the escape velocity depends only on the "planet" and not on the test-body escaping.

One may wonder whether there are objects for which v_{esc} = velocity of light (= c)... would they be completely black as light cannot escape?

<u>Question</u>: How small should the earth be to have an escape velocity equal to c according to the formula above?