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PREFACE

This new edition of the Formulae and Tables represents a considerable overhaul of its predecessor “green book” first published in 1980.

The contents have been updated to reflect more fully the evolving syllabus requirements of the profession, and also in the case of the Tables to reflect more contemporary experience and methods. Correspondingly, there has been some modest removal of material which has either become redundant with syllabus changes or obviated by the availability of pocket calculators.

As in the predecessor book, it is important to note that these tables have been produced for the sole use of examination candidates. The profession does not express any opinion whatsoever as to the circumstances in which any of the tables may be suitable for other uses.

FORMULAE

This section is intended to help candidates with formulae that may be hard to remember. Derivations of these formulae may still be required under the relevant syllabuses.

Note. In these tables, log denotes logarithms to base *e*.

1 MATHEMATICAL METHODS

1.1 SERIES

Exponential function

$$\exp(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Natural log function

$$\log(1+x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \quad (-1 < x \leq 1)$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + b^n$$

where n is a positive integer

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \cdots$$

$$(-1 < x < 1)$$

1.2 CALCULUS

Taylor series (one variable)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots$$

Taylor series (two variables)

$$f(x+h, y+k) = f(x,y) + h\, f'_x(x,y) + k\, f'_y(x,y)$$
$$+ \frac{1}{2!}\Big(h^2 f''_{xx}(x,y) + 2hk\, f''_{xy}(x,y) + k^2 f''_{yy}(x,y) \Big) + \cdots$$

Integration by parts

$$\int_a^b u \frac{dv}{dx} dx = \left[uv \right]_a^b - \int_a^b v \frac{du}{dx} dx$$

Double integrals (changing the order of integration)

$$\int_a^b \left(\int_a^x f(x,y)\, dy \right) dx = \int_a^b \left(\int_y^b f(x,y) dx \right) dy \quad \text{or}$$

$$\int_a^b dx \int_a^x dy\, f(x,y) = \int_a^b dy \int_y^b dx\, f(x,y)$$

The domain of integration here is the set of values (x, y) for which $a \le y \le x \le b$.

Differentiating an integral

$$\frac{d}{dy} \int_{a(y)}^{b(y)} f(x,y) dx = b'(y) f[b(y), y] - a'(y) f[a(y), y]$$
$$+ \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y) dx$$

1.3 SOLVING EQUATIONS

Newton-Raphson method

If x is a sufficiently good approximation to a root of the equation $f(x) = 0$ then (provided convergence occurs) a better approximation is

$$x^* = x - \frac{f(x)}{f'(x)}.$$

Integrating factors

The integrating factor for solving the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is:

$$\exp\left(\int P(x)dx\right)$$

Second-order difference equations

The general solution of the difference equation $ax_{n+2} + bx_{n+1} + cx_n = 0$ is:

if $b^2 - 4ac > 0$: $x_n = A\lambda_1^n + B\lambda_2^n$
(distinct real roots, $\lambda_1 \neq \lambda_2$)

if $b^2 - 4ac = 0$: $x_n = (A + Bn)\lambda^n$
(equal real roots, $\lambda_1 = \lambda_2 = \lambda$)

if $b^2 - 4ac < 0$: $x_n = r^n(A\cos n\theta + B\sin n\theta)$
(complex roots, $\lambda_1 = \overline{\lambda}_2 = re^{i\theta}$)

where λ_1 and λ_2 are the roots of the quadratic equation $a\lambda^2 + b\lambda + c = 0$.

1.4 GAMMA FUNCTION

Definition

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt , \quad x > 0$$

Properties

$$\Gamma(x) = (x-1)\Gamma(x-1)$$

$$\Gamma(n) = (n-1)!, \quad n = 1,2,3,\ldots$$

$$\Gamma(½) = \sqrt{\pi}$$

1.5 BAYES' FORMULA

Let $A_1, A_2, ..., A_n$ be a collection of mutually exclusive and exhaustive events with $P(A_i) \neq 0$, $i = 1, 2, \ldots, n$.

For any event B such that $P(B) \neq 0$:

$$P(A_i | B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)}, \, i = 1, 2, ..., n.$$

2 STATISTICAL DISTRIBUTIONS

Notation

PF = Probability function, $p(x)$
PDF = Probability density function, $f(x)$
DF = Distribution function, $F(x)$
PGF = Probability generating function, $G(s)$
MGF = Moment generating function, $M(t)$

Note. Where formulae have been omitted below, this indicates that (a) there is no simple formula or (b) the function does not have a finite value or (c) the function equals zero.

2.1 DISCRETE DISTRIBUTIONS

Binomial distribution

Parameters: n, p ($n =$ positive integer, $0 < p < 1$ with $q = 1 - p$)

PF: $p(x) = \binom{n}{x} p^x q^{n-x}$, $x = 0,1,2,\ldots,n$

DF: The distribution function is tabulated in the statistical tables section.

PGF: $G(s) = (q + ps)^n$

MGF: $M(t) = (q + pe^t)^n$

Moments: $E(X) = np$, $\text{var}(X) = npq$

Coefficient of skewness: $\dfrac{q - p}{\sqrt{npq}}$

Bernoulli distribution

The Bernoulli distribution is the same as the binomial distribution with parameter $n = 1$.

Poisson distribution

Parameter: μ ($\mu > 0$)

PF: $p(x) = \dfrac{e^{-\mu}\mu^x}{x!}$, $x = 0,1,2,...$

DF: The distribution function is tabulated in the statistical tables section.

PGF: $G(s) = e^{\mu(s-1)}$

MGF: $M(t) = e^{\mu(e^t - 1)}$

Moments: $E(X) = \mu$, $\text{var}(X) = \mu$

Coefficient of skewness: $\dfrac{1}{\sqrt{\mu}}$

Negative binomial distribution – Type 1

Parameters: k, p ($k =$positive integer, $0 < p < 1$ with $q = 1 - p$)

PF: $p(x) = \binom{x-1}{k-1} p^k q^{x-k}$, $x = k, k+1, k+2, \ldots$

PGF: $G(s) = \left(\dfrac{ps}{1 - qs} \right)^k$

MGF: $M(t) = \left(\dfrac{pe^t}{1 - qe^t} \right)^k$

Moments: $E(X) = \dfrac{k}{p}$, $\operatorname{var}(X) = \dfrac{kq}{p^2}$

Coefficient
of skewness: $\dfrac{2 - p}{\sqrt{kq}}$

Negative binomial distribution – Type 2

Parameters: k, p ($k > 0$, $0 < p < 1$ with $q = 1 - p$)

PF: $p(x) = \dfrac{\Gamma(k+x)}{\Gamma(x+1)\Gamma(k)} p^k q^x$, $x = 0,1,2,\ldots$

PGF: $G(s) = \left(\dfrac{p}{1-qs}\right)^k$

MGF: $M(t) = \left(\dfrac{p}{1-qe^t}\right)^k$

Moments: $E(X) = \dfrac{kq}{p}$, $\text{var}(X) = \dfrac{kq}{p^2}$

Coefficient of skewness: $\dfrac{2-p}{\sqrt{kq}}$

Geometric distribution

The geometric distribution is the same as the negative binomial distribution with parameter $k = 1$.

Uniform distribution (discrete)

Parameters: a, b, h $(a < b,\ h > 0,\ b - a$ is a multiple of $h)$

PF: $p(x) = \dfrac{h}{b-a+h},\ x = a, a+h, a+2h, \ldots, b-h, b$

PGF: $G(s) = \dfrac{h}{b-a+h}\left(\dfrac{s^{b+h} - s^{a}}{s^{h} - 1}\right)$

MGF: $M(t) = \dfrac{h}{b-a+h}\left(\dfrac{e^{(b+h)t} - e^{at}}{e^{ht} - 1}\right)$

Moments: $E(X) = \dfrac{1}{2}(a+b)$, $\operatorname{var}(X) = \dfrac{1}{12}(b-a)(b-a+2h)$

2.2 CONTINUOUS DISTRIBUTIONS

Standard normal distribution – $N(0,1)$

Parameters: none

PDF: $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2},\ -\infty < x < \infty$

DF: The distribution function is tabulated in the statistical tables section.

MGF: $M(t) = e^{\frac{1}{2}t^2}$

Moments: $E(X) = 0$, $\operatorname{var}(X) = 1$

$$E(X^r) = \frac{1}{2^{r/2}} \frac{\Gamma(1+r)}{\Gamma\left(1 + \frac{r}{2}\right)},\ r = 2, 4, 6, \ldots$$

Normal (Gaussian) distribution – $N(\mu,\sigma^2)$

Parameters: μ, σ^2 ($\sigma > 0$)

PDF: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \ -\infty < x < \infty$

MGF: $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Moments: $E(X) = \mu$, $\operatorname{var}(X) = \sigma^2$

Exponential distribution

Parameter: λ ($\lambda > 0$)

PDF: $f(x) = \lambda e^{-\lambda x}, \ x > 0$

DF: $F(x) = 1 - e^{-\lambda x}$

MGF: $M(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}, \ t < \lambda$

Moments: $E(X) = \frac{1}{\lambda}$, $\operatorname{var}(X) = \frac{1}{\lambda^2}$

$E(X^r) = \frac{\Gamma(1+r)}{\lambda^r}, \ r = 1,2,3,\ldots$

Coefficient
of skewness: 2

Gamma distribution

Parameters: $\alpha, \lambda \ (\alpha > 0, \lambda > 0)$

PDF: $f(x) = \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \ x > 0$

DF: When 2α is an integer, probabilities for the gamma distribution can be found using the relationship:

$$2\lambda X \sim \chi^2_{2\alpha}$$

MGF: $M(t) = \left(1 - \dfrac{t}{\lambda}\right)^{-\alpha}, \ t < \lambda$

Moments: $E(X) = \dfrac{\alpha}{\lambda}, \ \ \text{var}(X) = \dfrac{\alpha}{\lambda^2}$

$$E(X^r) = \frac{\Gamma(\alpha + r)}{\Gamma(\alpha)\lambda^r}, \ r = 1, 2, 3, \ldots$$

Coefficient
of skewness: $\dfrac{2}{\sqrt{\alpha}}$

Chi-square distribution – χ^2_υ

The chi-square distribution with υ degrees of freedom is the same as the gamma distribution with parameters $\alpha = \dfrac{\upsilon}{2}$ and $\lambda = \dfrac{1}{2}$.

The distribution function for the chi-square distribution is tabulated in the statistical tables section.

Uniform distribution (continuous) – *U*(*a*, *b*)

Parameters: $a, b \ (a < b)$

PDF: $f(x) = \frac{1}{b-a}, \ a < x < b$

DF: $F(x) = \frac{x-a}{b-a}$

MGF: $M(t) = \frac{1}{(b-a)} \frac{1}{t} (e^{bt} - e^{at})$

Moments: $E(X) = \frac{1}{2}(a+b), \ \ \mathrm{var}(X) = \frac{1}{12}(b-a)^2$

$$E(X^r) = \frac{1}{(b-a)} \frac{1}{r+1} (b^{r+1} - a^{r+1}), \ r = 1,2,3,\ldots$$

Beta distribution

Parameters: $\alpha, \beta \ (\alpha > 0, \ \beta > 0)$

PDF: $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \ 0 < x < 1$

Moments: $E(X) = \frac{\alpha}{\alpha+\beta}, \ \ \mathrm{var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$$E(X^r) = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\alpha+\beta+r)}, \ r = 1,2,3,\ldots$$

Coefficient of skewness: $\frac{2(\beta-\alpha)}{(\alpha+\beta+2)} \sqrt{\frac{\alpha+\beta+1}{\alpha\beta}}$

Lognormal distribution

Parameters: μ, σ^2 ($\sigma > 0$)

PDF: $$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{x}\exp\left\{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right\},\ x > 0$$

Moments: $$E(X) = e^{\mu + \frac{1}{2}\sigma^2},\quad \operatorname{var}(X) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$$

$$E(X^r) = e^{r\mu + \frac{1}{2}r^2\sigma^2},\ r = 1,2,3,\ldots$$

Coefficient
of skewness: $\left(e^{\sigma^2} + 2\right)\sqrt{e^{\sigma^2} - 1}$

Pareto distribution (two parameter version)

Parameters: α, λ ($\alpha > 0$, $\lambda > 0$)

PDF: $$f(x) = \frac{\alpha\lambda^\alpha}{(\lambda + x)^{\alpha+1}},\ x > 0$$

DF: $$F(x) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^\alpha$$

Moments: $$E(X) = \frac{\lambda}{\alpha - 1}\ (\alpha > 1),\ \operatorname{var}(X) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)}\ (\alpha > 2)$$

$$E(X^r) = \frac{\Gamma(\alpha - r)\Gamma(1 + r)}{\Gamma(\alpha)}\lambda^r,\ r = 1,2,3,\ldots,\ r < \alpha$$

Coefficient
of skewness: $\frac{2(\alpha+1)}{(\alpha-3)}\sqrt{\frac{\alpha-2}{\alpha}}\ (\alpha > 3)$

Pareto distribution (three parameter version)

Parameters: α, λ, k ($\alpha > 0$, $\lambda > 0$, $k > 0$)

PDF: $f(x) = \dfrac{\Gamma(\alpha + k)\lambda^{\alpha} x^{k-1}}{\Gamma(\alpha)\Gamma(k)(\lambda + x)^{\alpha+k}}$, $x > 0$

Moments: $E(X) = \dfrac{k\lambda}{\alpha - 1}$ ($\alpha > 1$), $\text{var}(X) = \dfrac{k(k + \alpha - 1)\lambda^2}{(\alpha - 1)^2(\alpha - 2)}$ ($\alpha > 2$)

$$E(X^r) = \frac{\Gamma(\alpha - r)\Gamma(k + r)}{\Gamma(\alpha)\Gamma(k)}\lambda^r, \; r = 1,2,3,\ldots, \; r < \alpha$$

Weibull distribution

Parameters: c, γ ($c > 0$, $\gamma > 0$)

PDF: $f(x) = c\gamma x^{\gamma-1} e^{-cx^{\gamma}}$, $x > 0$

DF: $F(x) = 1 - e^{-cx^{\gamma}}$

Moments: $E(X^r) = \Gamma\left(1 + \dfrac{r}{\gamma}\right)\dfrac{1}{c^{r/\gamma}}$, $r = 1,2,3,\ldots$

Burr distribution

Parameters: α, λ, γ ($\alpha > 0$, $\lambda > 0$, $\gamma > 0$)

PDF: $f(x) = \dfrac{\alpha\gamma\lambda^{\alpha} x^{\gamma-1}}{(\lambda + x^{\gamma})^{\alpha+1}}$, $x > 0$

DF: $F(x) = 1 - \left(\dfrac{\lambda}{\lambda + x^{\gamma}}\right)^{\alpha}$

Moments: $E(X^r) = \Gamma\left(\alpha - \dfrac{r}{\gamma}\right)\Gamma\left(1 + \dfrac{r}{\gamma}\right)\dfrac{\lambda^{r/\gamma}}{\Gamma(\alpha)}$, $r = 1,2,3,\ldots, r < \alpha\gamma$

2.3 COMPOUND DISTRIBUTIONS

Conditional expectation and variance

$$E(Y) = E[E(Y \mid X)]$$

$$\text{var}(Y) = \text{var}[E(Y \mid X)] + E[\text{var}(Y \mid X)]$$

Moments of a compound distribution

If $X_1, X_2, \ldots$ are IID random variables with MGF $M_X(t)$ and N is an independent nonnegative integer-valued random variable, then $S = X_1 + \cdots + X_N$ (with $S = 0$ when $N = 0$) has the following properties:

Mean: $E(S) = E(N)E(X)$

Variance: $\text{var}(S) = E(N)\,\text{var}(X) + \text{var}(N)[E(X)]^2$

MGF: $M_S(t) = M_N[\log M_X(t)]$

Compound Poisson distribution

Mean: λm_1

Variance: λm_2

Third central moment: λm_3

where $\lambda = E(N)$ and $m_r = E(X^r)$

Recursive formulae for integer-valued distributions

$(a,b,0)$ *class of distributions*

Let $g_r = P(S = r)$, $r = 0,1,2,\ldots$ and $f_j = P(X = j)$, $j = 1,2,3,\ldots$.

If $p_r = P(N = r)$, where $p_r = \left(a + \frac{b}{r}\right) p_{r-1}$, $r = 1,2,3,\ldots$, then

$$g_0 = p_0 \quad \text{and} \quad g_r = \sum_{j=1}^{r} \left(a + \frac{bj}{r}\right) f_j \, g_{r-j}, \; r = 1,2,3,\ldots$$

Compound Poisson distribution

If N has a Poisson distribution with mean λ, then $a = 0$ and $b = \lambda$, and

$$g_0 = e^{-\lambda} \quad \text{and} \quad g_r = \frac{\lambda}{r} \sum_{j=1}^{r} j \, f_j \, g_{r-j}, \; r = 1,2,3,\ldots$$

2.4 TRUNCATED MOMENTS

Normal distribution

If $f(x)$ is the PDF of the $N(\mu, \sigma^2)$ distribution, then

$$\int_L^U x\, f(x)dx = \mu[\Phi(U') - \Phi(L')] - \sigma[\phi(U') - \phi(L')]$$

where $L' = \dfrac{L - \mu}{\sigma}$ and $U' = \dfrac{U - \mu}{\sigma}$.

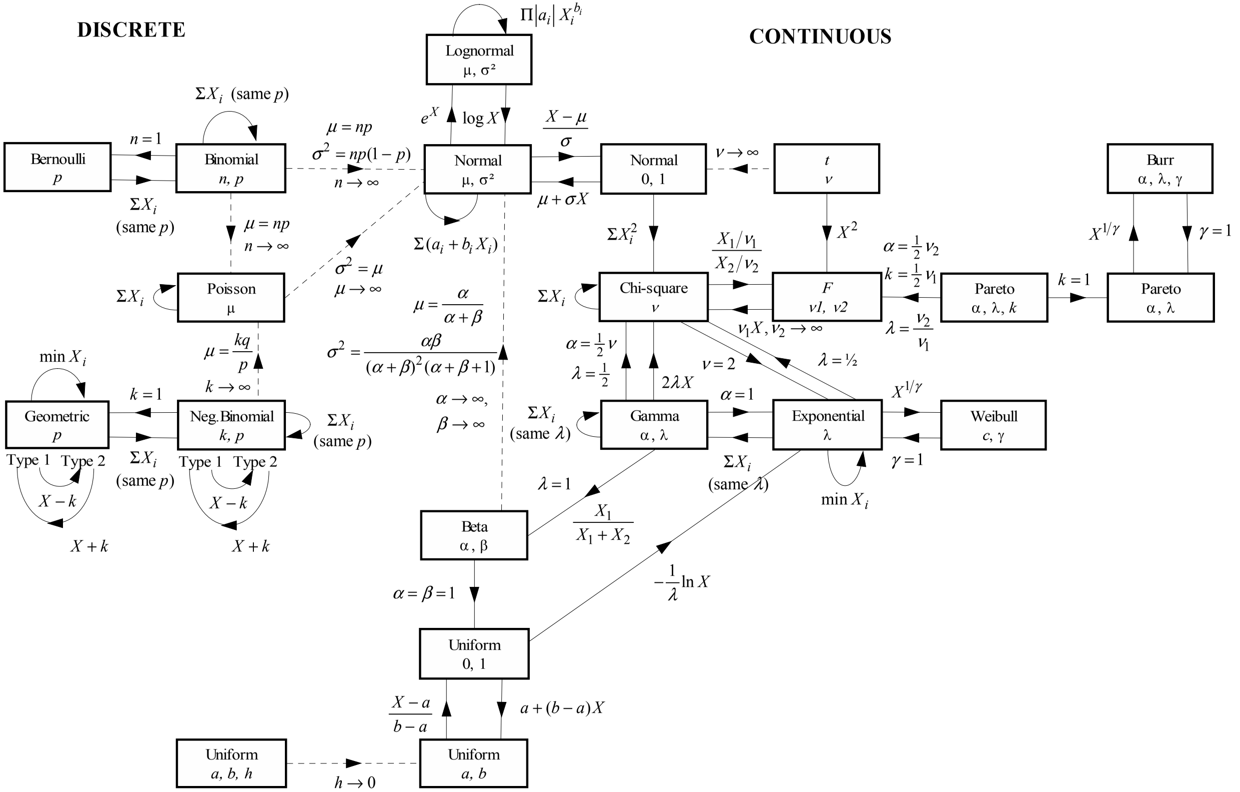
Lognormal distribution

If $f(x)$ is the PDF of the lognormal distribution with parameters μ and σ^2, then

$$\int_L^U x^k\, f(x)dx = e^{k\mu + ½k^2\sigma^2}[\Phi(U_k) - \Phi(L_k)]$$

where $L_k = \dfrac{\log L - \mu}{\sigma} - k\sigma$ and $U_k = \dfrac{\log U - \mu}{\sigma} - k\sigma$.

RELATIONSHIPS BETWEEN STATISTICAL DISTRIBUTIONS



EXPLANATION OF THE DISTRIBUTION DIAGRAM

The distribution diagram shows the main interrelationships between the distributions in the statistics section. The relationships shown are of four types:

Special cases

For example, the arrow marked “$n = 1$” connecting the binomial distribution to the Bernoulli distribution means:

> In the special case where $n = 1$, the binomial distribution is equivalent to a Bernoulli distribution.

Transformations

For example, the arrow marked “e^X” connecting the normal distribution to the lognormal distribution means:

> If X has a normal distribution, the function e^X will have a lognormal distribution.

Note that the parameters of the transformed distributions may differ from those of the basic distributions shown.

Sums, products and minimum values

For example, the arrow marked “ΣX_i (same p)” connecting the binomial distribution to itself means:

> The sum of a fixed number of independent random variables, each having a binomial distribution with the same value for the parameter p, also has a binomial distribution.

Similarly, “ΠX_i” and “$\min X_i$” denote the product and the minimum of a fixed set of independent random variables. Where a sum or product includes “a_i” or “b_i”, these denote arbitrary constants.

Limiting cases (indicated by dotted lines)

For example, the arrow marked “$\mu = np$, $n \to \infty$” connecting the binomial distribution to the Poisson distribution means:

> For large values of n, the binomial distribution with parameters n and p will approximate to the Poisson distribution with parameter μ, where $\mu = np$.

3 STATISTICAL METHODS

3.1 SAMPLE MEAN AND VARIANCE

The random sample $(x_1, x_2, \ldots, x_n)$ has the following sample moments:

Sample mean: $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Sample variance: $$s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right\}$$

3.2 PARAMETRIC INFERENCE (NORMAL MODEL)

One sample

For a single sample of size n under the normal model $X \sim N(\mu, \sigma^2)$:

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \quad \text{and} \quad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

Two samples

For two independent samples of sizes m and n under the normal models $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$:

$$\frac{S_X^2 / \sigma_X^2}{S_Y^2 / \sigma_Y^2} \sim F_{m-1,n-1}$$

Under the additional assumption that $\sigma_X^2 = \sigma_Y^2$:

$$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S_p\sqrt{\frac{1}{m}+\frac{1}{n}}} \sim t_{m+n-2}$$

where $S_p^2 = \frac{1}{m+n-2}\left\{(m-1)S_X^2+(n-1)S_Y^2\right\}$ is the pooled sample variance.

3.3 MAXIMUM LIKELIHOOD ESTIMATORS

Asymptotic distribution

If $\hat{\theta}$ is the maximum likelihood estimator of a parameter θ based on a sample $\underline{X}$, then $\hat{\theta}$ is asymptotically normally distributed with mean θ and variance equal to the Cramér-Rao lower bound

$$CRLB(\theta) = -1 \Big/ E\left[\frac{\partial^2}{\partial\theta^2}\log L(\theta,\underline{X})\right]$$

Likelihood ratio test

$$-2(\ell_p - \ell_{p+q}) = -2\log\left(\frac{\max\limits_{H_0} L}{\max\limits_{H_0\cup H_1} L}\right) \sim \chi_q^2 \text{ approximately (under } H_0)$$

where $\ell_p = \max\limits_{H_0} \log L$ is the maximum log-likelihood for the model under H_0 (in which there are p free parameters)

and $\ell_{p+q} = \max\limits_{H_0\cup H_1} \log L$ is the maximum log-likelihood for the model under $H_0 \cup H_1$ (in which there are $p+q$ free parameters).

3.4 LINEAR REGRESSION MODEL WITH NORMAL ERRORS

Model

$$Y_i \sim N(\alpha + \beta x_i, \sigma^2), \ \ i = 1, 2, \ldots, n$$

Intermediate calculations

$$s_{xx} = \sum_{i=1}^{n} (x_i - \overline{x})^2 = \sum_{i=1}^{n} x_i^2 - n\overline{x}^2$$

$$s_{yy} = \sum_{i=1}^{n} (y_i - \overline{y})^2 = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$$

$$s_{xy} = \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) = \sum_{i=1}^{n} x_i y_i - n\,\overline{x}\,\overline{y}$$

Parameter estimates

$$\hat{\alpha} = \overline{y} - \hat{\beta}\overline{x}\,, \ \ \hat{\beta} = \frac{s_{xy}}{s_{xx}}$$

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \frac{1}{n-2}\left(s_{yy} - \frac{s_{xy}^2}{s_{xx}} \right)$$

Distribution of $\hat{\beta}$

$$\frac{\hat{\beta} - \beta}{\sqrt{\hat{\sigma}^2 / s_{xx}}} \sim t_{n-2}$$

Variance of predicted mean response

$$\text{var}(\hat{\alpha}+\hat{\beta}x_0)=\left\{\frac{1}{n}+\frac{(x_0-\overline{x})^2}{s_{xx}}\right\}\sigma^2$$

An additional σ^2 must be added to obtain the variance of the predicted <u>individual</u> response.

Testing the correlation coefficient

$$r=\frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}}$$

If $\rho=0$, then $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}\sim t_{n-2}$.

Fisher Z transformation

$$z_r \sim N\left(z_\rho,\frac{1}{n-3}\right) \text{ approximately}$$

where $z_r=\tanh^{-1}r=\tfrac{1}{2}\log\left(\frac{1+r}{1-r}\right)$ and $z_\rho=\tanh^{-1}\rho=\tfrac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right)$.

Sum of squares relationship

$$\sum_{i=1}^{n}(y_i-\overline{y})^2=\sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(\hat{y}_i-\overline{y})^2$$

3.5 ANALYSIS OF VARIANCE

Single factor normal model

$$Y_{ij} \sim N(\mu + \tau_i, \sigma^2), \ \ i = 1,2,\ldots,k, \ \ j = 1,2,\ldots,n_i$$

where $n = \sum_{i=1}^{k} n_i$, with $\sum_{i=1}^{k} n_i \tau_i = 0$

Intermediate calculations (sums of squares)

Total: $$SS_T = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{\bullet\bullet})^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{\bullet\bullet}^2}{n}$$

Between treatments: $$SS_B = \sum_{i=1}^{k} n_i(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2 = \sum_{i=1}^{k}\frac{y_{i\bullet}^2}{n_i} - \frac{y_{\bullet\bullet}^2}{n}$$

Residual: $$SS_R = SS_T - SS_B$$

Variance estimate

$$\hat{\sigma}^2 = \frac{SS_R}{n-k}$$

Statistical test

Under the appropriate null hypothesis:

$$\left.\frac{SS_B}{k-1}\right/\frac{SS_R}{n-k} \sim F_{k-1,n-k}$$

3.6 GENERALISED LINEAR MODELS

Exponential family

For a random variable Y from the exponential family, with natural parameter θ and scale parameter ϕ:

Probability (density) function: $f_Y(y;\theta,\phi) = \exp\left[\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right]$

Mean: $E(Y) = b'(\theta)$

Variance: $\text{var}(Y) = a(\phi)b''(\theta)$

Canonical link functions

Binomial: $g(\mu) = \log\frac{\mu}{1-\mu}$

Poisson: $g(\mu) = \log\mu$

Normal: $g(\mu) = \mu$

Gamma: $g(\mu) = \frac{1}{\mu}$

3.7 BAYESIAN METHODS

Relationship between posterior and prior distributions

$$\textit{Posterior} \propto \textit{Prior} \times \textit{Likelihood}$$

The posterior distribution $f(\theta \mid \underline{x})$ for the parameter θ is related to the prior distribution $f(\theta)$ via the likelihood function $f(\underline{x} \mid \theta)$:

$$f(\theta \mid \underline{x}) \propto f(\theta) \times f(\underline{x} \mid \theta)$$

Normal / normal model

If $\underline{x}$ is a random sample of size n from a $N(\mu, \sigma^2)$ distribution, where σ^2 is known, and the prior distribution for the parameter μ is $N(\mu_0, \sigma_0^2)$, then the posterior distribution for μ is:

$$\mu \mid \underline{x} \sim N(\mu_*, \sigma_*^2)$$

where $\mu_* = \left(\frac{n\overline{x}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right) \Bigg/ \left(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2} \right)$ and $\sigma_*^2 = 1 \Bigg/ \left(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2} \right)$

3.8 EMPIRICAL BAYES CREDIBILITY – MODEL 1

Data requirements

$$\{X_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\}$$

X_{ij} represents the aggregate claims in the j th year from the i th risk.

Intermediate calculations

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}\ , \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} \bar{X}_i$$

Parameter estimation

Quantity	*Estimator*
$E[m(\theta)]$	$\bar{X}$
$E[s^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2\right\}$
$\text{var}[m(\theta)]$	$\frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2\right\}$

Credibility factor

$$Z = \frac{n}{n + \dfrac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$$

3.9 EMPIRICAL BAYES CREDIBILITY – MODEL 2

Data requirements

$$\{Y_{ij}, i = 1,2,\ldots,N, j = 1,2,\ldots,n\},\ \{P_{ij}, i = 1,2,\ldots,N, j = 1,2,\ldots,n\}$$

Y_{ij} represents the aggregate claims in the j th year from the i th risk;
P_{ij} is the corresponding risk volume.

Intermediate calculations

$$\overline{P}_i = \sum_{j=1}^{n} P_{ij}\,,\quad \overline{P} = \sum_{i=1}^{N} \overline{P}_i\,,\ P^* = \frac{1}{Nn-1}\sum_{i=1}^{N} \overline{P}_i\left(1-\frac{\overline{P}_i}{\overline{P}}\right)$$

$$X_{ij} = \frac{Y_{ij}}{P_{ij}},\quad \overline{X}_i = \sum_{j=1}^{n} \frac{P_{ij}X_{ij}}{\overline{P}_i},\quad \overline{X} = \sum_{i=1}^{N}\sum_{j=1}^{n} \frac{P_{ij}X_{ij}}{\overline{P}}$$

Parameter estimation

Quantity	*Estimator*
$E[m(\theta)]$	$\overline{X}$
$E[s^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n} P_{ij}(X_{ij}-\overline{X}_i)^2\right\}$
$\mathrm{var}[m(\theta)]$	$\frac{1}{P^*}\left(\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n} P_{ij}(X_{ij}-\overline{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n} P_{ij}(X_{ij}-\overline{X}_i)^2\right\}\right)$

Credibility factor

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\mathrm{var}[m(\theta)]}}$$

4 COMPOUND INTEREST

Increasing/decreasing annuity functions

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}, \quad (Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

Accumulation factor for variable interest rates

$$A(t_1, t_2) = \exp\left(\int_{t_1}^{t_2} \delta(t)\,dt \right)$$

5 SURVIVAL MODELS

5.1 MORTALITY “LAWS”

Survival probabilities

$$_t p_x = \exp\left(-\int_0^t \mu_{x+s} ds\right)$$

Gompertz’ Law

$$\mu_x = Bc^x, \quad _t p_x = g^{c^x(c^t - 1)} \quad \text{where} \quad g = e^{-B/\log c}$$

Makeham’s Law

$$\mu_x = A + Bc^x, \quad _t p_x = s^t g^{c^x(c^t - 1)} \quad \text{where} \quad s = e^{-A}$$

Gompertz-Makeham formula

The Gompertz-Makeham graduation formula, denoted by $\text{GM}(r,s)$, states that

$$\mu_x = poly_1(t) + \exp[poly_2(t)]$$

where t is a linear function of x and $poly_1(t)$ and $poly_2(t)$ are polynomials of degree r and s respectively.

5.2 EMPIRICAL ESTIMATION

Greenwood's formula for the variance of the Kaplan-Meier estimator

$$\text{var}[\tilde{F}(t)] = \left[1 - \hat{F}(t)\right]^2 \sum_{t_j \le t} \frac{d_j}{n_j(n_j - d_j)}$$

Variance of the Nelson-Aalen estimate of the integrated hazard

$$\text{var}[\tilde{\Lambda}_t] = \sum_{t_j \le t} \frac{d_j(n_j - d_j)}{n_j^3}$$

5.3 MORTALITY ASSUMPTIONS

Balducci assumption

$${}_{1-t}q_{x+t} = (1-t)q_x \quad (x \text{ is an integer, } 0 \le t \le 1)$$

5.4 GENERAL MARKOV MODEL

Kolmogorov forward differential equation

$$\frac{\partial}{\partial t}\, {}_t p_x^{gh} = \sum_{j \ne h} \left({}_t p_x^{gj} \mu_{x+t}^{jh} - {}_t p_x^{gh} \mu_{x+t}^{hj} \right)$$

5.5 GRADUATION TESTS

Grouping of signs test

If there are n_1 positive signs and n_2 negative signs and G denotes the observed number of positive runs, then:

$$P(G=t)=\frac{\binom{n_1-1}{t-1}\binom{n_2+1}{t}}{\binom{n_1+n_2}{n_1}} \text{ and, approximately,}$$

$$G \sim N\left(\frac{n_1(n_2+1)}{n_1+n_2}, \frac{(n_1 n_2)^2}{(n_1+n_2)^3}\right)$$

Critical values for the grouping of signs test are tabulated in the statistical tables section for small values of n_1 and n_2. For larger values of n_1 and n_2 the normal approximation can be used.

Serial correlation test

$$r_j \approx \frac{\frac{1}{m-j}\sum_{i=1}^{m-j}(z_i-\bar{z})(z_{i+j}-\bar{z})}{\frac{1}{m}\sum_{i=1}^{m}(z_i-\bar{z})^2} \quad \text{where } \bar{z}=\frac{1}{m}\sum_{i=1}^{m} z_i$$

$r_j \times \sqrt{m} \sim N(0,1)$ approximately.

Variance adjustment factor

$$r_x = \frac{\sum_i i^2 \pi_i}{\sum_i i\pi_i}$$

where π_i is the proportion of lives at age x who have exactly i policies.

5.6 MULTIPLE DECREMENT TABLES

For a multiple decrement table with three decrements α, β and γ, each uniform over the year of age $(x, x+1)$ in its single decrement table, then

$$(aq)_x^{\alpha} = q_x^{\alpha}\left[1 - \tfrac{1}{2}(q_x^{\beta} + q_x^{\gamma}) + \tfrac{1}{3}q_x^{\beta}q_x^{\gamma}\right]$$

5.7 POPULATION PROJECTION MODELS

Logistic model

$$\frac{1}{P(t)}\frac{dP(t)}{dt} = \rho - kP(t) \text{ has general solution } P(t) = \frac{\rho}{C\rho e^{-\rho t} + k}$$

where C is a constant.

6 ANNUITIES AND ASSURANCES

6.1 APPROXIMATIONS FOR NON ANNUAL ANNUITIES

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}$$

$$\ddot{a}_{x:\overline{n|}}^{(m)} \approx \ddot{a}_{x:\overline{n|}} - \frac{m-1}{2m}\left(1 - \frac{D_{x+n}}{D_x}\right)$$

6.2 MOMENTS OF ANNUITIES AND ASSURANCES

Let K_x and T_x denote the curtate and complete future lifetimes (respectively) of a life aged exactly x.

Whole life assurances

$$E[v^{K_x+1}] = A_x, \quad \text{var}[v^{K_x+1}] = {}^2A_x - (A_x)^2$$

$$E[v^{T_x}] = \bar{A}_x, \quad \text{var}[v^{T_x}] = {}^2\bar{A}_x - (\bar{A}_x)^2$$

Similar relationships hold for endowment assurances (with status $\cdots_{x:\overline{n|}}$), pure endowments (with status $_{x:\overset{\,1}{\overline{n|}}}$), term assurances (with status $_{\overset{1}{x}:\overline{n|}}$) and deferred whole life assurances (with status $_{m|}\cdots_x$).

Whole life annuities

$$E[\ddot{a}_{\overline{K_x+1|}}] = \ddot{a}_x, \quad \text{var}[\ddot{a}_{\overline{K_x+1|}}] = \frac{{}^2A_x - (A_x)^2}{d^2}$$

$$E[\bar{a}_{\overline{T_x|}}] = \bar{a}_x, \quad \text{var}[\bar{a}_{\overline{T_x|}}] = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$$

Similar relationships hold for temporary annuities (with status $\cdots_{x:\overline{n|}}$).

6.3 PREMIUMS AND RESERVES

Premium conversion relationship between annuities and assurances

$$A_x = 1 - d\ddot{a}_x, \quad \bar{A}_x = 1 - \delta \bar{a}_x$$

Similar relationships hold for endowment assurance policies (with status $\cdots_{x:\overline{n|}}$).

Net premium reserve

$${}_tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}, \quad {}_t\bar{V}_x = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

Similar formulae hold for endowment assurance policies (with statuses $\cdots_{x:\overline{n|}}$ and $\cdots_{x+t:\overline{n-t|}}$).

6.4 THIELE'S DIFFERENTIAL EQUATION

Whole life assurance

$$\frac{\partial}{\partial t}\, {}_t\bar{V}_x = \delta\, {}_t\bar{V}_x + \bar{P}_x - (1 - {}_t\bar{V}_x)\mu_{x+t}$$

Similar formulae hold for other types of policies.

Multiple state model

$$\frac{\partial}{\partial t}\, {}_tV_x^j = \delta\, {}_tV_x^j + b_{x+t}^j - \sum_{k \neq j} \mu_{x+t}^{jk} \left(b_{x+t}^{jk} + {}_tV_x^k - {}_tV_x^j \right)$$

7 STOCHASTIC PROCESSES

7.1 MARKOV “JUMP” PROCESSES

Kolmogorov differential equations

Forward equation: $\frac{\partial}{\partial t} p_{ij}(s,t) = \sum_{k \in S} p_{ik}(s,t)\sigma_{kj}(t)$

Backward equation: $\frac{\partial}{\partial s} p_{ij}(s,t) = -\sum_{k \in S} \sigma_{ik}(s) p_{kj}(s,t)$

where $\sigma_{ij}(t)$ is the transition rate from state i to state j ($j \neq i$) at time t, and $\sigma_{ii} = -\sum_{j \neq i} \sigma_{ij}$.

Expected time to reach a subsequent state k

$$m_i = \frac{1}{\lambda_i} + \sum_{j \neq i, j \neq k} \frac{\sigma_{ij}}{\lambda_i} m_j \text{, where } \lambda_i = \sum_{j \neq i} \sigma_{ij}$$

7.2 BROWNIAN MOTION AND RELATED PROCESSES

Martingales for standard Brownian motion

If $\{B_t, t \geq 0\}$ is a standard Brownian motion, then the following processes are martingales:

$$B_t,\ B_t^2 - t \text{ and } \exp(\lambda B_t - \tfrac{1}{2}\lambda^2 t)$$

Distribution of the maximum value

$$P\left[\max_{0 \leq s \leq t} (B_s + \mu s) > y\right] = \Phi\left(\frac{-y + \mu t}{\sqrt{t}}\right) + e^{2\mu y}\Phi\left(\frac{-y - \mu t}{\sqrt{t}}\right), \quad y > 0$$

Hitting times

If $\tau_y = \min_{s \geq 0}\{s : B_s + \mu s = y\}$ where $\mu > 0$ and $y < 0$, then

$$E[e^{-\lambda\tau_y}] = e^{y(\mu + \sqrt{\mu^2 + 2\lambda})}, \ \ \lambda > 0$$

Ornstein-Uhlenbeck process

$$dX_t = -\gamma X_t dt + \sigma dB_t, \ \ \gamma > 0$$

7.3 MONTE CARLO METHODS

Box-Muller formulae

If U_1 and U_2 are independent random variables from the $U(0,1)$ distribution then

$$Z_1 = \sqrt{-2\log U_1}\cos(2\pi U_2) \ \text{ and } \ Z_2 = \sqrt{-2\log U_1}\sin(2\pi U_2)$$

are independent standard normal variables.

Polar method

If V_1 and V_2 are independent random variables from the $U(-1,1)$ distribution and $S = V_1^2 + V_2^2$ then, conditional on $0 < S \leq 1$,

$$Z_1 = V_1\sqrt{\frac{-2\log S}{S}} \ \text{ and } \ Z_2 = V_2\sqrt{\frac{-2\log S}{S}}$$

are independent standard normal variables.

Pseudorandom values from the $U(0,1)$ distribution and the $N(0,1)$ distribution are included in the statistical tables section.

8 TIME SERIES

8.1 TIME SERIES – TIME DOMAIN

Sample autocovariance and autocorrelation function

Autocovariance: $\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(x_t - \hat{\mu})(x_{t-k} - \hat{\mu})$, where $\hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} x_t$

Autocorrelation: $\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$

Autocorrelation function for ARMA(1,1)

For the process $X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$:

$$\rho_k = \frac{(1+\alpha\beta)(\alpha+\beta)}{(1+\beta^2+2\alpha\beta)}\alpha^{k-1}, \quad k = 1,2,3,\ldots$$

Partial autocorrelation function

$$\phi_1 = \rho_1, \quad \phi_2 = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$$

$$\phi_k = \frac{\det P_k^*}{\det P_k}, \; k = 2,3,\ldots,$$

where $P_k = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{pmatrix}$

and P_k^* equals P_k, but with the last column replaced with $(\rho_1, \rho_2, \rho_3, ..., \rho_k)^T$.

Partial autocorrelation function for MA(1)

For the process $X_t = \mu + e_t + \beta e_{t-1}$:

$$\phi_k = (-1)^{k+1} \frac{(1-\beta^2)\beta^k}{1-\beta^{2(k+1)}}, \quad k = 1,2,3,\ldots$$

8.2 TIME SERIES – FREQUENCY DOMAIN

Spectral density function

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-ik\omega} \gamma_k, \quad -\pi < \omega < \pi$$

Inversion formula

$$\gamma_k = \int_{-\pi}^{\pi} e^{ik\omega} f(\omega) d\omega$$

Spectral density function for ARMA(*p*,*q*)

The spectral density function of the process $\phi(B)(X_t - \mu) = \theta(B)e_t$, where $\text{var}(e_t) = \sigma^2$, is

$$f(\omega) = \frac{\sigma^2}{2\pi} \frac{\theta(e^{i\omega})\theta(e^{-i\omega})}{\phi(e^{i\omega})\phi(e^{-i\omega})}$$

Linear filters

For the linear filter $Y_t = \sum_{k=-\infty}^{\infty} a_k X_{t-k}$:

$$f_Y(\omega) = |A(\omega)|^2 f_X(\omega),$$

where $A(\omega) = \sum_{k=-\infty}^{\infty} e^{-ik\omega} a_k$ is the transfer function for the filter.

8.3 TIME SERIES – BOX-JENKINS METHODOLOGY

Ljung and Box “portmanteau” test of the residuals for an ARMA(*p*,*q*) model

$$n(n+2)\sum_{k=1}^{m}\frac{r_k^2}{n-k}\sim\chi^2_{m-(p+q)}$$

where r_k ($k=1,2,\ldots,m$) is the estimated value of the k th autocorrelation coefficient of the residuals and n is the number of data values used in the $ARMA(p,q)$ series.

Turning point test

In a sequence of n independent random variables the number of turning points T is such that:

$$E(T)=\tfrac{2}{3}(n-2) \quad \text{and} \quad \text{var}(T)=\frac{16n-29}{90}$$

9 ECONOMIC MODELS

9.1 UTILITY THEORY

Utility functions

Exponential: $U(w) = -e^{-aw}, \quad a > 0$

Logarithmic: $U(w) = \log w$

Power: $U(w) = \gamma^{-1}(w^{\gamma} - 1), \ \gamma \leq 1, \ \gamma \neq 0$

Quadratic: $U(w) = w + dw^2, d < 0$

Measures of risk aversion

Absolute risk aversion: $A(w) = -\dfrac{U''(w)}{U'(w)}$

Relative risk aversion: $R(w) = w\,A(w)$

9.2 CAPITAL ASSET PRICING MODEL (CAPM)

Security market line

$$E_i - r = \beta_i(E_M - r) \quad \text{where} \quad \beta_i = \frac{\text{cov}(R_i, R_M)}{\text{var}(R_M)}$$

Capital market line (for efficient portfolios)

$$E_P - r = (E_M - r)\frac{\sigma_P}{\sigma_M}$$

9.3 INTEREST RATE MODELS

Spot rates and forward rates for zero-coupon bonds

Let $P(\tau)$ be the price at time 0 of a zero-coupon bond that pays 1 unit at time τ.

Let $s(\tau)$ be the spot rate for the period $(0,\tau)$.

Let $f(\tau)$ be the instantaneous forward rate at time 0 for time τ.

Spot rate

$$P(\tau)=e^{-\tau s(\tau)} \text{ or } s(\tau)=-\frac{1}{\tau}\log P(\tau)$$

Instantaneous forward rate

$$P(\tau)=\exp\left(-\int_0^\tau f(s)ds\right) \text{ or } f(\tau)=-\frac{d}{d\tau}\log P(\tau)$$

Vasicek model

Instantaneous forward rate

$$f(\tau)=e^{-\alpha\tau}R+(1-e^{-\alpha\tau})L+\frac{\beta}{\alpha}e^{-\alpha\tau}(1-e^{-\alpha\tau})$$

Price of a zero-coupon bond

$$P(\tau)=\exp\left[-D(\tau)R-(\tau-D(\tau))L-\frac{\beta}{2}D(\tau)^2\right]$$

where $D(\tau)=\dfrac{1-e^{-\alpha\tau}}{\alpha}$

10 FINANCIAL DERIVATIVES

Note. In this section, q denotes the (continuously-payable) dividend rate.

10.1 PRICE OF A FORWARD OR FUTURES CONTRACT

For an asset with fixed income of present value I:

$$F = (S_0 - I)e^{rT}$$

For an asset with dividends:

$$F = S_0 e^{(r-q)T}$$

10.2 BINOMIAL PRICING ("TREE") MODEL

Risk-neutral probabilities

Up-step probability $= \dfrac{e^{r\Delta t} - d}{u - d}$,

where $u \approx e^{\sigma\sqrt{\Delta t} + q\Delta t}$

and $d \approx e^{-\sigma\sqrt{\Delta t} + q\Delta t}$.

10.3 STOCHASTIC DIFFERENTIAL EQUATIONS

Generalised Wiener process

$$dx = adt + bdz$$

where a and b are constant and dz is the increment for a Wiener process (standard Brownian motion).

Ito process

$$dx = a(x,t)dt + b(x,t)dz$$

Ito's lemma for a function *G*(*x*, *t*)

$$dG = \left(a\frac{\partial G}{\partial x} + \frac{1}{2}b^2\frac{\partial^2 G}{\partial x^2} + \frac{\partial G}{\partial t} \right)dt + b\frac{\partial G}{\partial x}dz$$

Models for the short rate r_t

Ho-Lee: $dr = \theta(t)dt + \sigma dz$

Hull-White: $dr = [\theta(t) - ar]dt + \sigma dz$

Vasicek: $dr = a(b - r)dt + \sigma dz$

Cox-Ingersoll-Ross: $dr = a(b - r)dt + \sigma\sqrt{r}dz$

10.4 BLACK-SCHOLES FORMULAE FOR EUROPEAN OPTIONS

Geometric Brownian motion model for a stock price S_t

$$dS_t = S_t(\mu dt + \sigma dz)$$

Black-Scholes partial differential equation

$$\frac{\partial f}{\partial t} + (r - q)S_t\frac{\partial f}{\partial S_t} + \tfrac{1}{2}\sigma^2 S_t^2\frac{\partial^2 f}{\partial S_t^2} = rf$$

Garman-Kohlhagen formulae for the price of call and put options

Call: $c_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$

Put: $p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t e^{-q(T-t)}\Phi(-d_1)$

where $d_1 = \dfrac{\log(S_t/K) + (r - q + ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$

and $d_2 = \dfrac{\log(S_t/K) + (r - q - ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$

10.5 PUT-CALL PARITY RELATIONSHIP

$$c_t + Ke^{-r(T-t)} = p_t + S_t e^{-q(T-t)}$$

COMPOUND INTEREST TABLES

Compound Interest

½%

i	0.005 000
$i^{(2)}$	0.004 994
$i^{(4)}$	0.004 991
$i^{(12)}$	0.004 989
δ	0.004 988
$(1+i)^{1/2}$	1.002 497
$(1+i)^{1/4}$	1.001 248
$(1+i)^{1/12}$	1.000 416
v	0.995 025
$v^{1/2}$	0.997 509
$v^{1/4}$	0.998 754
$v^{1/12}$	0.999 584
d	0.004 975
$d^{(2)}$	0.004 981
$d^{(4)}$	0.004 984
$d^{(12)}$	0.004 987
$i/i^{(2)}$	1.001 248
$i/i^{(4)}$	1.001 873
$i/i^{(12)}$	1.002 290
i/δ	1.002 498
$i/d^{(2)}$	1.003 748
$i/d^{(4)}$	1.003 123
$i/d^{(12)}$	1.002 706

n	$(1+i)^n$	v^n	$s_{\overline{n}\|}$	$a_{\overline{n}\|}$	$(Ia)_{\overline{n}\|}$	$(Da)_{\overline{n}\|}$	n
1	1.005 00	0.995 02	1.000 0	0.995 0	0.995 0	0.995 0	1
2	1.010 03	0.990 07	2.005 0	1.985 1	2.975 2	2.980 1	2
3	1.015 08	0.985 15	3.015 0	2.970 2	5.930 6	5.950 4	3
4	1.020 15	0.980 25	4.030 1	3.950 5	9.851 6	9.900 9	4
5	1.025 25	0.975 37	5.050 3	4.925 9	14.728 5	14.826 7	5
6	1.030 38	0.970 52	6.075 5	5.896 4	20.551 6	20.723 1	6
7	1.035 53	0.965 69	7.105 9	6.862 1	27.311 4	27.585 2	7
8	1.040 71	0.960 89	8.141 4	7.823 0	34.998 5	35.408 2	8
9	1.045 91	0.956 10	9.182 1	8.779 1	43.603 4	44.187 2	9
10	1.051 14	0.951 35	10.228 0	9.730 4	53.116 9	53.917 6	10
11	1.056 40	0.946 61	11.279 2	10.677 0	63.529 7	64.594 7	11
12	1.061 68	0.941 91	12.335 6	11.618 9	74.832 5	76.213 6	12
13	1.066 99	0.937 22	13.397 2	12.556 2	87.016 4	88.769 7	13
14	1.072 32	0.932 56	14.464 2	13.488 7	100.072 2	102.258 4	14
15	1.077 68	0.927 92	15.536 5	14.416 6	113.990 9	116.675 1	15
16	1.083 07	0.923 30	16.614 2	15.339 9	128.763 7	132.015 0	16
17	1.088 49	0.918 71	17.697 3	16.258 6	144.381 7	148.273 6	17
18	1.093 93	0.914 14	18.785 8	17.172 8	160.836 2	165.446 4	18
19	1.099 40	0.909 59	19.879 7	18.082 4	178.118 4	183.528 8	19
20	1.104 90	0.905 06	20.979 1	18.987 4	196.219 6	202.516 2	20
21	1.110 42	0.900 56	22.084 0	19.888 0	215.131 4	222.404 1	21
22	1.115 97	0.896 08	23.194 4	20.784 1	234.845 1	243.188 2	22
23	1.121 55	0.891 62	24.310 4	21.675 7	255.352 4	264.863 9	23
24	1.127 16	0.887 19	25.432 0	22.562 9	276.644 9	287.426 8	24
25	1.132 80	0.882 77	26.559 1	23.445 6	298.714 2	310.872 4	25
26	1.138 46	0.878 38	27.691 9	24.324 0	321.552 1	335.196 4	26
27	1.144 15	0.874 01	28.830 4	25.198 0	345.150 3	360.394 4	27
28	1.149 87	0.869 66	29.974 5	26.067 7	369.500 9	386.462 1	28
29	1.155 62	0.865 33	31.124 4	26.933 0	394.595 6	413.395 2	29
30	1.161 40	0.861 03	32.280 0	27.794 1	420.426 5	441.189 2	30
31	1.167 21	0.856 75	33.441 4	28.650 8	446.985 6	469.840 0	31
32	1.173 04	0.852 48	34.608 6	29.503 3	474.265 1	499.343 3	32
33	1.178 91	0.848 24	35.781 7	30.351 5	502.257 1	529.694 8	33
34	1.184 80	0.844 02	36.960 6	31.195 5	530.953 8	560.890 4	34
35	1.190 73	0.839 82	38.145 4	32.035 4	560.347 6	592.925 7	35
36	1.196 68	0.835 64	39.336 1	32.871 0	590.430 8	625.796 8	36
37	1.202 66	0.831 49	40.532 8	33.702 5	621.195 9	659.499 3	37
38	1.208 68	0.827 35	41.735 4	34.529 9	652.635 2	694.029 1	38
39	1.214 72	0.823 23	42.944 1	35.353 1	684.741 4	729.382 2	39
40	1.220 79	0.819 14	44.158 8	36.172 2	717.506 9	765.554 4	40
41	1.226 90	0.815 06	45.379 6	36.987 3	750.924 5	802.541 7	41
42	1.233 03	0.811 01	46.606 5	37.798 3	784.986 9	840.340 0	42
43	1.239 20	0.806 97	47.839 6	38.605 3	819.686 7	878.945 3	43
44	1.245 39	0.802 96	49.078 8	39.408 2	855.016 9	918.353 5	44
45	1.251 62	0.798 96	50.324 2	40.207 2	890.970 3	958.560 7	45
46	1.257 88	0.794 99	51.575 8	41.002 2	927.539 8	999.562 9	46
47	1.264 17	0.791 03	52.833 7	41.793 2	964.718 4	1 041.356 1	47
48	1.270 49	0.787 10	54.097 8	42.580 3	1 002.499 1	1 083.936 4	48
49	1.276 84	0.783 18	55.368 3	43.363 5	1 040.875 1	1 127.299 9	49
50	1.283 23	0.779 29	56.645 2	44.142 8	1 079.839 4	1 171.442 7	50
60	1.348 85	0.741 37	69.770 0	51.725 6	1 500.371 4	1 654.887 8	60
70	1.417 83	0.705 30	83.566 1	58.939 4	1 972.582 2	2 212.116 5	70
80	1.490 34	0.670 99	98.067 7	65.802 3	2 490.447 8	2 839.538 9	80
90	1.566 55	0.638 34	113.310 9	72.331 3	3 048.408 2	3 533.740 1	90
100	1.646 67	0.607 29	129.333 7	78.542 6	3 641.336 1	4 291.471 0	100

Compound Interest

1%

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.010 00	0.990 10	1.000 0	0.990 1	0.990 1	0.990 1	1
2	1.020 10	0.980 30	2.010 0	1.970 4	2.950 7	2.960 5	2
3	1.030 30	0.970 59	3.030 1	2.941 0	5.862 5	5.901 5	3
4	1.040 60	0.960 98	4.060 4	3.902 0	9.706 4	9.803 4	4
5	1.051 01	0.951 47	5.101 0	4.853 4	14.463 7	14.656 9	5
6	1.061 52	0.942 05	6.152 0	5.795 5	20.116 0	20.452 4	6
7	1.072 14	0.932 72	7.213 5	6.728 2	26.645 0	27.180 5	7
8	1.082 86	0.923 48	8.285 7	7.651 7	34.032 9	34.832 2	8
9	1.093 69	0.914 34	9.368 5	8.566 0	42.261 9	43.398 2	9
10	1.104 62	0.905 29	10.462 2	9.471 3	51.314 8	52.869 5	10
11	1.115 67	0.896 32	11.566 8	10.367 6	61.174 4	63.237 2	11
12	1.126 83	0.887 45	12.682 5	11.255 1	71.823 8	74.492 3	12
13	1.138 09	0.878 66	13.809 3	12.133 7	83.246 4	86.626 0	13
14	1.149 47	0.869 96	14.947 4	13.003 7	95.425 8	99.629 7	14
15	1.160 97	0.861 35	16.096 9	13.865 1	108.346 1	113.494 7	15
16	1.172 58	0.852 82	17.257 9	14.717 9	121.991 2	128.212 6	16
17	1.184 30	0.844 38	18.430 4	15.562 3	136.345 6	143.774 9	17
18	1.196 15	0.836 02	19.614 7	16.398 3	151.394 0	160.173 1	18
19	1.208 11	0.827 74	20.810 9	17.226 0	167.121 0	177.399 2	19
20	1.220 19	0.819 54	22.019 0	18.045 6	183.511 9	195.444 7	20
21	1.232 39	0.811 43	23.239 2	18.857 0	200.551 9	214.301 7	21
22	1.244 72	0.803 40	24.471 6	19.660 4	218.226 7	233.962 1	22
23	1.257 16	0.795 44	25.716 3	20.455 8	236.521 8	254.417 9	23
24	1.269 73	0.787 57	26.973 5	21.243 4	255.423 4	275.661 3	24
25	1.282 43	0.779 77	28.243 2	22.023 2	274.917 6	297.684 4	25
26	1.295 26	0.772 05	29.525 6	22.795 2	294.990 9	320.479 6	26
27	1.308 21	0.764 40	30.820 9	23.559 6	315.629 8	344.039 2	27
28	1.321 29	0.756 84	32.129 1	24.316 4	336.821 2	368.355 7	28
29	1.334 50	0.749 34	33.450 4	25.065 8	358.552 1	393.421 5	29
30	1.347 85	0.741 92	34.784 9	25.807 7	380.809 8	419.229 2	30
31	1.361 33	0.734 58	36.132 7	26.542 3	403.581 7	445.771 5	31
32	1.374 94	0.727 30	37.494 1	27.269 6	426.855 4	473.041 1	32
33	1.388 69	0.720 10	38.869 0	27.989 7	450.618 8	501.030 7	33
34	1.402 58	0.712 97	40.257 7	28.702 7	474.859 9	529.733 4	34
35	1.416 60	0.705 91	41.660 3	29.408 6	499.566 9	559.142 0	35
36	1.430 77	0.698 92	43.076 9	30.107 5	524.728 2	589.249 5	36
37	1.445 08	0.692 00	44.507 6	30.799 5	550.332 4	620.049 0	37
38	1.459 53	0.685 15	45.952 7	31.484 7	576.368 2	651.533 7	38
39	1.474 12	0.678 37	47.412 3	32.163 0	602.824 6	683.696 7	39
40	1.488 86	0.671 65	48.886 4	32.834 7	629.690 7	716.531 4	40
41	1.503 75	0.665 00	50.375 2	33.499 7	656.955 9	750.031 1	41
42	1.518 79	0.658 42	51.879 0	34.158 1	684.609 5	784.189 2	42
43	1.533 98	0.651 90	53.397 8	34.810 0	712.641 2	818.999 2	43
44	1.549 32	0.645 45	54.931 8	35.455 5	741.040 8	854.454 6	44
45	1.564 81	0.639 05	56.481 1	36.094 5	769.798 2	890.549 2	45
46	1.580 46	0.632 73	58.045 9	36.727 2	798.903 7	927.276 4	46
47	1.596 26	0.626 46	59.626 3	37.353 7	828.347 5	964.630 1	47
48	1.612 23	0.620 26	61.222 6	37.974 0	858.120 0	1 002.604 1	48
49	1.628 35	0.614 12	62.834 8	38.588 1	888.211 8	1 041.192 1	49
50	1.644 63	0.608 04	64.463 2	39.196 1	918.613 7	1 080.388 2	50
60	1.816 70	0.550 45	81.669 7	44.955 0	1 237.761 2	1 504.496 2	60
70	2.006 76	0.498 31	100.676 3	50.168 5	1 578.816 0	1 983.148 6	70
80	2.216 72	0.451 12	121.671 5	54.888 2	1 934.765 3	2 511.179 4	80
90	2.448 63	0.408 39	144.863 3	59.160 9	2 299.728 4	3 083.911 9	90
100	2.704 81	0.369 71	170.481 4	63.028 9	2 668.804 6	3 697.112 1	100

i	0.010 000
$i^{(2)}$	0.009 975
$i^{(4)}$	0.009 963
$i^{(12)}$	0.009 954
δ	0.009 950
$(1+i)^{1/2}$	1.004 988
$(1+i)^{1/4}$	1.002 491
$(1+i)^{1/12}$	1.000 830
v	0.990 099
$v^{1/2}$	0.995 037
$v^{1/4}$	0.997 516
$v^{1/12}$	0.999 171
d	0.009 901
$d^{(2)}$	0.009 926
$d^{(4)}$	0.009 938
$d^{(12)}$	0.009 946
$i/i^{(2)}$	1.002 494
$i/i^{(4)}$	1.003 742
$i/i^{(12)}$	1.004 575
i/δ	1.004 992
$i/d^{(2)}$	1.007 494
$i/d^{(4)}$	1.006 242
$i/d^{(12)}$	1.005 408

Compound Interest

1½%

i	0.015 000
$i^{(2)}$	0.014 944
$i^{(4)}$	0.014 916
$i^{(12)}$	0.014 898
δ	0.014 889
$(1+i)^{1/2}$	1.007 472
$(1+i)^{1/4}$	1.003 729
$(1+i)^{1/12}$	1.001 241
v	0.985 222
$v^{1/2}$	0.992 583
$v^{1/4}$	0.996 285
$v^{1/12}$	0.998 760
d	0.014 778
$d^{(2)}$	0.014 833
$d^{(4)}$	0.014 861
$d^{(12)}$	0.014 879
$i/i^{(2)}$	1.003 736
$i/i^{(4)}$	1.005 608
$i/i^{(12)}$	1.006 857
i/δ	1.007 481
$i/d^{(2)}$	1.011 236
$i/d^{(4)}$	1.009 358
$i/d^{(12)}$	1.008 107

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.015 00	0.985 22	1.000 0	0.985 2	0.985 2	0.985 2	1
2	1.030 23	0.970 66	2.015 0	1.955 9	2.926 5	2.941 1	2
3	1.045 68	0.956 32	3.045 2	2.912 2	5.795 5	5.853 3	3
4	1.061 36	0.942 18	4.090 9	3.854 4	9.564 2	9.707 7	4
5	1.077 28	0.928 26	5.152 3	4.782 6	14.205 5	14.490 3	5
6	1.093 44	0.914 54	6.229 6	5.697 2	19.692 8	20.187 5	6
7	1.109 84	0.901 03	7.323 0	6.598 2	26.000 0	26.785 7	7
8	1.126 49	0.887 71	8.432 8	7.485 9	33.101 7	34.271 7	8
9	1.143 39	0.874 59	9.559 3	8.360 5	40.973 0	42.632 2	9
10	1.160 54	0.861 67	10.702 7	9.222 2	49.589 7	51.854 4	10
11	1.177 95	0.848 93	11.863 3	10.071 1	58.927 9	61.925 5	11
12	1.195 62	0.836 39	13.041 2	10.907 5	68.964 6	72.833 0	12
13	1.213 55	0.824 03	14.236 8	11.731 5	79.676 9	84.564 5	13
14	1.231 76	0.811 85	15.450 4	12.543 4	91.042 8	97.107 9	14
15	1.250 23	0.799 85	16.682 1	13.343 2	103.040 6	110.451 1	15
16	1.268 99	0.788 03	17.932 4	14.131 3	115.649 1	124.582 4	16
17	1.288 02	0.776 39	19.201 4	14.907 6	128.847 6	139.490 0	17
18	1.307 34	0.764 91	20.489 4	15.672 6	142.616 0	155.162 6	18
19	1.326 95	0.753 61	21.796 7	16.426 2	156.934 6	171.588 8	19
20	1.346 86	0.742 47	23.123 7	17.168 6	171.784 0	188.757 4	20
21	1.367 06	0.731 50	24.470 5	17.900 1	187.145 5	206.657 6	21
22	1.387 56	0.720 69	25.837 6	18.620 8	203.000 6	225.278 4	22
23	1.408 38	0.710 04	27.225 1	19.330 9	219.331 4	244.609 2	23
24	1.429 50	0.699 54	28.633 5	20.030 4	236.120 5	264.639 6	24
25	1.450 95	0.689 21	30.063 0	20.719 6	253.350 6	285.359 3	25
26	1.472 71	0.679 02	31.514 0	21.398 6	271.005 2	306.757 9	26
27	1.494 80	0.668 99	32.986 7	22.067 6	289.067 8	328.825 5	27
28	1.517 22	0.659 10	34.481 5	22.726 7	307.522 6	351.552 2	28
29	1.539 98	0.649 36	35.998 7	23.376 1	326.354 0	374.928 3	29
30	1.563 08	0.639 76	37.538 7	24.015 8	345.546 8	398.944 1	30
31	1.586 53	0.630 31	39.101 8	24.646 1	365.086 4	423.590 3	31
32	1.610 32	0.620 99	40.688 3	25.267 1	384.958 2	448.857 4	32
33	1.634 48	0.611 82	42.298 6	25.879 0	405.148 1	474.736 4	33
34	1.659 00	0.602 77	43.933 1	26.481 7	425.642 4	501.218 1	34
35	1.683 88	0.593 87	45.592 1	27.075 6	446.427 7	528.293 7	35
36	1.709 14	0.585 09	47.276 0	27.660 7	467.490 9	555.954 4	36
37	1.734 78	0.576 44	48.985 1	28.237 1	488.819 3	584.191 5	37
38	1.760 80	0.567 92	50.719 9	28.805 1	510.400 5	612.996 6	38
39	1.787 21	0.559 53	52.480 7	29.364 6	532.222 2	642.361 1	39
40	1.814 02	0.551 26	54.267 9	29.915 8	554.272 7	672.277 0	40
41	1.841 23	0.543 12	56.081 9	30.459 0	576.540 4	702.735 9	41
42	1.868 85	0.535 09	57.923 1	30.994 1	599.014 2	733.730 0	42
43	1.896 88	0.527 18	59.792 0	31.521 2	621.683 0	765.251 2	43
44	1.925 33	0.519 39	61.688 9	32.040 6	644.536 1	797.291 9	44
45	1.954 21	0.511 71	63.614 2	32.552 3	667.563 3	829.844 2	45
46	1.983 53	0.504 15	65.568 4	33.056 5	690.754 3	862.900 7	46
47	2.013 28	0.496 70	67.551 9	33.553 2	714.099 3	896.453 9	47
48	2.043 48	0.489 36	69.565 2	34.042 6	737.588 7	930.496 4	48
49	2.074 13	0.482 13	71.608 7	34.524 7	761.213 1	965.021 1	49
50	2.105 24	0.475 00	73.682 8	34.999 7	784.963 3	1 000.020 8	50
60	2.443 22	0.409 30	96.214 7	39.380 3	1 027.547 7	1 374.648 7	60
70	2.835 46	0.352 68	122.363 8	43.154 9	1 274.320 7	1 789.675 2	70
80	3.290 66	0.303 89	152.710 9	46.407 3	1 519.481 4	2 239.511 8	80
90	3.818 95	0.261 85	187.929 9	49.209 9	1 758.753 7	2 719.343 0	90
100	4.432 05	0.225 63	228.803 0	51.624 7	1 989.075 3	3 225.019 8	100

Compound Interest

2%

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.020 00	0.980 39	1.000 0	0.980 4	0.980 4	0.980 4	1
2	1.040 40	0.961 17	2.020 0	1.941 6	2.902 7	2.922 0	2
3	1.061 21	0.942 32	3.060 4	2.883 9	5.729 7	5.805 8	3
4	1.082 43	0.923 85	4.121 6	3.807 7	9.425 1	9.613 6	4
5	1.104 08	0.905 73	5.204 0	4.713 5	13.953 7	14.327 0	5
6	1.126 16	0.887 97	6.308 1	5.601 4	19.281 6	19.928 5	6
7	1.148 69	0.870 56	7.434 3	6.472 0	25.375 5	26.400 4	7
8	1.171 66	0.853 49	8.583 0	7.325 5	32.203 4	33.725 9	8
9	1.195 09	0.836 76	9.754 6	8.162 2	39.734 2	41.888 2	9
10	1.218 99	0.820 35	10.949 7	8.982 6	47.937 7	50.870 7	10
11	1.243 37	0.804 26	12.168 7	9.786 8	56.784 6	60.657 6	11
12	1.268 24	0.788 49	13.412 1	10.575 3	66.246 5	71.232 9	12
13	1.293 61	0.773 03	14.680 3	11.348 4	76.295 9	82.581 3	13
14	1.319 48	0.757 88	15.973 9	12.106 2	86.906 2	94.687 6	14
15	1.345 87	0.743 01	17.293 4	12.849 3	98.051 4	107.536 8	15
16	1.372 79	0.728 45	18.639 3	13.577 7	109.706 5	121.114 5	16
17	1.400 24	0.714 16	20.012 1	14.291 9	121.847 3	135.406 4	17
18	1.428 25	0.700 16	21.412 3	14.992 0	134.450 2	150.398 4	18
19	1.456 81	0.686 43	22.840 6	15.678 5	147.492 3	166.076 9	19
20	1.485 95	0.672 97	24.297 4	16.351 4	160.951 8	182.428 3	20
21	1.515 67	0.659 78	25.783 3	17.011 2	174.807 1	199.439 5	21
22	1.545 98	0.646 84	27.299 0	17.658 0	189.037 5	217.097 6	22
23	1.576 90	0.634 16	28.845 0	18.292 2	203.623 1	235.389 8	23
24	1.608 44	0.621 72	30.421 9	18.913 9	218.544 4	254.303 7	24
25	1.640 61	0.609 53	32.030 3	19.523 5	233.782 7	273.827 2	25
26	1.673 42	0.597 58	33.670 9	20.121 0	249.319 8	293.948 2	26
27	1.706 89	0.585 86	35.344 3	20.706 9	265.138 0	314.655 1	27
28	1.741 02	0.574 37	37.051 2	21.281 3	281.220 5	335.936 4	28
29	1.775 84	0.563 11	38.792 2	21.844 4	297.550 8	357.780 8	29
30	1.811 36	0.552 07	40.568 1	22.396 5	314.112 9	380.177 2	30
31	1.847 59	0.541 25	42.379 4	22.937 7	330.891 5	403.114 9	31
32	1.884 54	0.530 63	44.227 0	23.468 3	347.871 8	426.583 3	32
33	1.922 23	0.520 23	46.111 6	23.988 6	365.039 3	450.571 8	33
34	1.960 68	0.510 03	48.033 8	24.498 6	382.380 3	475.070 4	34
35	1.999 89	0.500 03	49.994 5	24.998 6	399.881 3	500.069 0	35
36	2.039 89	0.490 22	51.994 4	25.488 8	417.529 3	525.557 9	36
37	2.080 69	0.480 61	54.034 3	25.969 5	435.311 9	551.527 3	37
38	2.122 30	0.471 19	56.114 9	26.440 6	453.217 0	577.968 0	38
39	2.164 74	0.461 95	58.237 2	26.902 6	471.233 0	604.870 6	39
40	2.208 04	0.452 89	60.402 0	27.355 5	489.348 6	632.226 0	40
41	2.252 20	0.444 01	62.610 0	27.799 5	507.553 0	660.025 5	41
42	2.297 24	0.435 30	64.862 2	28.234 8	525.835 8	688.260 3	42
43	2.343 19	0.426 77	67.159 5	28.661 6	544.186 9	716.921 9	43
44	2.390 05	0.418 40	69.502 7	29.080 0	562.596 5	746.001 8	44
45	2.437 85	0.410 20	71.892 7	29.490 2	581.055 3	775.492 0	45
46	2.486 61	0.402 15	74.330 6	29.892 3	599.554 4	805.384 3	46
47	2.536 34	0.394 27	76.817 2	30.286 6	618.085 0	835.670 9	47
48	2.587 07	0.386 54	79.353 5	30.673 1	636.638 8	866.344 0	48
49	2.638 81	0.378 96	81.940 6	31.052 1	655.207 8	897.396 1	49
50	2.691 59	0.371 53	84.579 4	31.423 6	673.784 2	928.819 7	50
60	3.281 03	0.304 78	114.051 5	34.760 9	858.458 4	1 261.955 7	60
70	3.999 56	0.250 03	149.977 9	37.498 6	1 037.332 9	1 625.069 0	70
80	4.875 44	0.205 11	193.772 0	39.744 5	1 206.531 3	2 012.774 3	80
90	5.943 13	0.168 26	247.156 7	41.586 9	1 363.757 0	2 420.653 5	90
100	7.244 65	0.138 03	312.232 3	43.098 4	1 507.851 1	2 845.082 4	100

i	0.020 000
$i^{(2)}$	0.019 901
$i^{(4)}$	0.019 852
$i^{(12)}$	0.019 819
δ	0.019 803
$(1+i)^{1/2}$	1.009 950
$(1+i)^{1/4}$	1.004 963
$(1+i)^{1/12}$	1.001 652
v	0.980 392
$v^{1/2}$	0.990 148
$v^{1/4}$	0.995 062
$v^{1/12}$	0.998 351
d	0.019 608
$d^{(2)}$	0.019 705
$d^{(4)}$	0.019 754
$d^{(12)}$	0.019 786
$i/i^{(2)}$	1.004 975
$i/i^{(4)}$	1.007 469
$i/i^{(12)}$	1.009 134
i/δ	1.009 967
$i/d^{(2)}$	1.014 975
$i/d^{(4)}$	1.012 469
$i/d^{(12)}$	1.010 801

Compound Interest

2½%

i	0.025 000
$i^{(2)}$	0.024 846
$i^{(4)}$	0.024 769
$i^{(12)}$	0.024 718
δ	0.024 693
$(1+i)^{1/2}$	1.012 423
$(1+i)^{1/4}$	1.006 192
$(1+i)^{1/12}$	1.002 060
v	0.975 610
$v^{1/2}$	0.987 730
$v^{1/4}$	0.993 846
$v^{1/12}$	0.997 944
d	0.024 390
$d^{(2)}$	0.024 541
$d^{(4)}$	0.024 617
$d^{(12)}$	0.024 667
$i/i^{(2)}$	1.006 211
$i/i^{(4)}$	1.009 327
$i/i^{(12)}$	1.011 407
i/δ	1.012 449
$i/d^{(2)}$	1.018 711
$i/d^{(4)}$	1.015 577
$i/d^{(12)}$	1.013 491

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.025 00	0.975 61	1.000 0	0.975 6	0.975 6	0.975 6	1
2	1.050 63	0.951 81	2.025 0	1.927 4	2.879 2	2.903 0	2
3	1.076 89	0.928 60	3.075 6	2.856 0	5.665 0	5.759 1	3
4	1.103 81	0.905 95	4.152 5	3.762 0	9.288 8	9.521 0	4
5	1.131 41	0.883 85	5.256 3	4.645 8	13.708 1	14.166 9	5
6	1.159 69	0.862 30	6.387 7	5.508 1	18.881 9	19.675 0	6
7	1.188 69	0.841 27	7.547 4	6.349 4	24.770 7	26.024 4	7
8	1.218 40	0.820 75	8.736 1	7.170 1	31.336 7	33.194 5	8
9	1.248 86	0.800 73	9.954 5	7.970 9	38.543 3	41.165 4	9
10	1.280 08	0.781 20	11.203 4	8.752 1	46.355 3	49.917 4	10
11	1.312 09	0.762 14	12.483 5	9.514 2	54.738 9	59.431 7	11
12	1.344 89	0.743 56	13.795 6	10.257 8	63.661 5	69.689 4	12
13	1.378 51	0.725 42	15.140 4	10.983 2	73.092 0	80.672 6	13
14	1.412 97	0.707 73	16.519 0	11.690 9	83.000 2	92.363 5	14
15	1.448 30	0.690 47	17.931 9	12.381 4	93.357 2	104.744 9	15
16	1.484 51	0.673 62	19.380 2	13.055 0	104.135 2	117.799 9	16
17	1.521 62	0.657 20	20.864 7	13.712 2	115.307 5	131.512 1	17
18	1.559 66	0.641 17	22.386 3	14.353 4	126.848 5	145.865 5	18
19	1.598 65	0.625 53	23.946 0	14.978 9	138.733 5	160.844 3	19
20	1.638 62	0.610 27	25.544 7	15.589 2	150.938 9	176.433 5	20
21	1.679 58	0.595 39	27.183 3	16.184 5	163.442 0	192.618 1	21
22	1.721 57	0.580 86	28.862 9	16.765 4	176.221 0	209.383 5	22
23	1.764 61	0.566 70	30.584 4	17.332 1	189.255 1	226.715 6	23
24	1.808 73	0.552 88	32.349 0	17.885 0	202.524 1	244.600 6	24
25	1.853 94	0.539 39	34.157 8	18.424 4	216.008 8	263.024 9	25
26	1.900 29	0.526 23	36.011 7	18.950 6	229.690 9	281.975 6	26
27	1.947 80	0.513 40	37.912 0	19.464 0	243.552 7	301.439 6	27
28	1.996 50	0.500 88	39.859 8	19.964 9	257.577 3	321.404 5	28
29	2.046 41	0.488 66	41.856 3	20.453 5	271.748 5	341.858 0	29
30	2.097 57	0.476 74	43.902 7	20.930 3	286.050 8	362.788 3	30
31	2.150 01	0.465 11	46.000 3	21.395 4	300.469 3	384.183 7	31
32	2.203 76	0.453 77	48.150 3	21.849 2	314.990 0	406.032 9	32
33	2.258 85	0.442 70	50.354 0	22.291 9	329.599 2	428.324 8	33
34	2.315 32	0.431 91	52.612 9	22.723 8	344.284 0	451.048 5	34
35	2.373 21	0.421 37	54.928 2	23.145 2	359.032 0	474.193 7	35
36	2.432 54	0.411 09	57.301 4	23.556 3	373.831 3	497.750 0	36
37	2.493 35	0.401 07	59.733 9	23.957 3	388.670 8	521.707 3	37
38	2.555 68	0.391 28	62.227 3	24.348 6	403.539 6	546.055 9	38
39	2.619 57	0.381 74	64.783 0	24.730 3	418.427 6	570.786 2	39
40	2.685 06	0.372 43	67.402 6	25.102 8	433.324 8	595.889 0	40
41	2.752 19	0.363 35	70.087 6	25.466 1	448.222 0	621.355 1	41
42	2.821 00	0.354 48	72.839 8	25.820 6	463.110 4	647.175 7	42
43	2.891 52	0.345 84	75.660 8	26.166 4	477.981 4	673.342 2	43
44	2.963 81	0.337 40	78.552 3	26.503 8	492.827 2	699.846 0	44
45	3.037 90	0.329 17	81.516 1	26.833 0	507.640 1	726.679 0	45
46	3.113 85	0.321 15	84.554 0	27.154 2	522.412 8	753.833 2	46
47	3.191 70	0.313 31	87.667 9	27.467 5	537.138 5	781.300 7	47
48	3.271 49	0.305 67	90.859 6	27.773 2	551.810 7	809.073 9	48
49	3.353 28	0.298 22	94.131 1	28.071 4	566.423 3	837.145 2	49
50	3.437 11	0.290 94	97.484 3	28.362 3	580.970 4	865.507 5	50
60	4.399 79	0.227 28	135.991 6	30.908 7	721.774 3	1 163.653 7	60
70	5.632 10	0.177 55	185.284 1	32.897 9	851.662 1	1 484.085 7	70
80	7.209 57	0.138 70	248.382 7	34.451 8	968.669 9	1 821.927 3	80
90	9.228 86	0.108 36	329.154 3	35.665 8	1 072.215 7	2 173.369 3	90
100	11.813 72	0.084 65	432.548 7	36.614 1	1 162.588 8	2 535.435 8	100

Compound Interest

3%

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.030 00	0.970 87	1.000 0	0.970 9	0.970 9	0.970 9	1
2	1.060 90	0.942 60	2.030 0	1.913 5	2.856 1	2.884 3	2
3	1.092 73	0.915 14	3.090 9	2.828 6	5.601 5	5.713 0	3
4	1.125 51	0.888 49	4.183 6	3.717 1	9.155 4	9.430 1	4
5	1.159 27	0.862 61	5.309 1	4.579 7	13.468 5	14.009 8	5
6	1.194 05	0.837 48	6.468 4	5.417 2	18.493 4	19.427 0	6
7	1.229 87	0.813 09	7.662 5	6.230 3	24.185 0	25.657 2	7
8	1.266 77	0.789 41	8.892 3	7.019 7	30.500 3	32.676 9	8
9	1.304 77	0.766 42	10.159 1	7.786 1	37.398 1	40.463 0	9
10	1.343 92	0.744 09	11.463 9	8.530 2	44.839 0	48.993 2	10
11	1.384 23	0.722 42	12.807 8	9.252 6	52.785 6	58.245 9	11
12	1.425 76	0.701 38	14.192 0	9.954 0	61.202 2	68.199 9	12
13	1.468 53	0.680 95	15.617 8	10.635 0	70.054 6	78.834 8	13
14	1.512 59	0.661 12	17.086 3	11.296 1	79.310 2	90.130 9	14
15	1.557 97	0.641 86	18.598 9	11.937 9	88.938 1	102.068 8	15
16	1.604 71	0.623 17	20.156 9	12.561 1	98.908 8	114.629 9	16
17	1.652 85	0.605 02	21.761 6	13.166 1	109.194 1	127.796 1	17
18	1.702 43	0.587 39	23.414 4	13.753 5	119.767 2	141.549 6	18
19	1.753 51	0.570 29	25.116 9	14.323 8	130.602 6	155.873 4	19
20	1.806 11	0.553 68	26.870 4	14.877 5	141.676 1	170.750 8	20
21	1.860 29	0.537 55	28.676 5	15.415 0	152.964 7	186.165 9	21
22	1.916 10	0.521 89	30.536 8	15.936 9	164.446 3	202.102 8	22
23	1.973 59	0.506 69	32.452 9	16.443 6	176.100 2	218.546 4	23
24	2.032 79	0.491 93	34.426 5	16.935 5	187.906 6	235.481 9	24
25	2.093 78	0.477 61	36.459 3	17.413 1	199.846 8	252.895 1	25
26	2.156 59	0.463 69	38.553 0	17.876 8	211.902 8	270.771 9	26
27	2.221 29	0.450 19	40.709 6	18.327 0	224.057 9	289.099 0	27
28	2.287 93	0.437 08	42.930 9	18.764 1	236.296 1	307.863 1	28
29	2.356 57	0.424 35	45.218 9	19.188 5	248.602 1	327.051 5	29
30	2.427 26	0.411 99	47.575 4	19.600 4	260.961 7	346.652 0	30
31	2.500 08	0.399 99	50.002 7	20.000 4	273.361 3	366.652 4	31
32	2.575 08	0.388 34	52.502 8	20.388 8	285.788 1	387.041 1	32
33	2.652 34	0.377 03	55.077 8	20.765 8	298.230 0	407.806 9	33
34	2.731 91	0.366 04	57.730 2	21.131 8	310.675 5	428.938 8	34
35	2.813 86	0.355 38	60.462 1	21.487 2	323.113 9	450.426 0	35
36	2.898 28	0.345 03	63.275 9	21.832 3	335.535 1	472.258 3	36
37	2.985 23	0.334 98	66.174 2	22.167 2	347.929 5	494.425 5	37
38	3.074 78	0.325 23	69.159 4	22.492 5	360.288 1	516.917 9	38
39	3.167 03	0.315 75	72.234 2	22.808 2	372.602 4	539.726 2	39
40	3.262 04	0.306 56	75.401 3	23.114 8	384.864 7	562.840 9	40
41	3.359 90	0.297 63	78.663 3	23.412 4	397.067 5	586.253 3	41
42	3.460 70	0.288 96	82.023 2	23.701 4	409.203 8	609.954 7	42
43	3.564 52	0.280 54	85.483 9	23.981 9	421.267 1	633.936 6	43
44	3.671 45	0.272 37	89.048 4	24.254 3	433.251 5	658.190 9	44
45	3.781 60	0.264 44	92.719 9	24.518 7	445.151 2	682.709 6	45
46	3.895 04	0.256 74	96.501 5	24.775 4	456.961 1	707.485 0	46
47	4.011 90	0.249 26	100.396 5	25.024 7	468.676 2	732.509 7	47
48	4.132 25	0.242 00	104.408 4	25.266 7	480.292 2	757.776 4	48
49	4.256 22	0.234 95	108.540 6	25.501 7	491.804 7	783.278 1	49
50	4.383 91	0.228 11	112.796 9	25.729 8	503.210 1	809.007 9	50
60	5.891 60	0.169 73	163.053 4	27.675 6	610.728 2	1 077.481 2	60
70	7.917 82	0.126 30	230.594 1	29.123 4	705.210 3	1 362.552 6	70
80	10.640 89	0.093 98	321.363 0	30.200 8	786.287 3	1 659.974 6	80
90	14.300 47	0.069 93	443.348 9	31.002 4	854.632 6	1 966.586 4	90
100	19.218 63	0.052 03	607.287 7	31.598 9	911.453 0	2 280.036 5	100

i	0.030 000
$i^{(2)}$	0.029 778
$i^{(4)}$	0.029 668
$i^{(12)}$	0.029 595
δ	0.029 559
$(1+i)^{1/2}$	1.014 889
$(1+i)^{1/4}$	1.007 417
$(1+i)^{1/12}$	1.002 466
v	0.970 874
$v^{1/2}$	0.985 329
$v^{1/4}$	0.992 638
$v^{1/12}$	0.997 540
d	0.029 126
$d^{(2)}$	0.029 341
$d^{(4)}$	0.029 450
$d^{(12)}$	0.029 522
$i/i^{(2)}$	1.007 445
$i/i^{(4)}$	1.011 181
$i/i^{(12)}$	1.013 677
i/δ	1.014 926
$i/d^{(2)}$	1.022 445
$i/d^{(4)}$	1.018 681
$i/d^{(12)}$	1.016 177

Compound Interest

4%

i	0.040 000
$i^{(2)}$	0.039 608
$i^{(4)}$	0.039 414
$i^{(12)}$	0.039 285
δ	0.039 221
$(1+i)^{1/2}$	1.019 804
$(1+i)^{1/4}$	1.009 853
$(1+i)^{1/12}$	1.003 274
v	0.961 538
$v^{1/2}$	0.980 581
$v^{1/4}$	0.990 243
$v^{1/12}$	0.996 737
d	0.038 462
$d^{(2)}$	0.038 839
$d^{(4)}$	0.039 029
$d^{(12)}$	0.039 157
$i/i^{(2)}$	1.009 902
$i/i^{(4)}$	1.014 877
$i/i^{(12)}$	1.018 204
i/δ	1.019 869
$i/d^{(2)}$	1.029 902
$i/d^{(4)}$	1.024 877
$i/d^{(12)}$	1.021 537

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.040 00	0.961 54	1.000 0	0.961 5	0.961 5	0.961 5	1
2	1.081 60	0.924 56	2.040 0	1.886 1	2.810 7	2.847 6	2
3	1.124 86	0.889 00	3.121 6	2.775 1	5.477 6	5.622 7	3
4	1.169 86	0.854 80	4.246 5	3.629 9	8.896 9	9.252 6	4
5	1.216 65	0.821 93	5.416 3	4.451 8	13.006 5	13.704 4	5
6	1.265 32	0.790 31	6.633 0	5.242 1	17.748 4	18.946 6	6
7	1.315 93	0.759 92	7.898 3	6.002 1	23.067 8	24.948 6	7
8	1.368 57	0.730 69	9.214 2	6.732 7	28.913 3	31.681 4	8
9	1.423 31	0.702 59	10.582 8	7.435 3	35.236 6	39.116 7	9
10	1.480 24	0.675 56	12.006 1	8.110 9	41.992 2	47.227 6	10
11	1.539 45	0.649 58	13.486 4	8.760 5	49.137 6	55.988 1	11
12	1.601 03	0.624 60	15.025 8	9.385 1	56.632 8	65.373 2	12
13	1.665 07	0.600 57	16.626 8	9.985 6	64.440 3	75.358 8	13
14	1.731 68	0.577 48	18.291 9	10.563 1	72.524 9	85.921 9	14
15	1.800 94	0.555 26	20.023 6	11.118 4	80.853 9	97.040 3	15
16	1.872 98	0.533 91	21.824 5	11.652 3	89.396 4	108.692 6	16
17	1.947 90	0.513 37	23.697 5	12.165 7	98.123 8	120.858 3	17
18	2.025 82	0.493 63	25.645 4	12.659 3	107.009 1	133.517 6	18
19	2.106 85	0.474 64	27.671 2	13.133 9	116.027 3	146.651 5	19
20	2.191 12	0.456 39	29.778 1	13.590 3	125.155 0	160.241 8	20
21	2.278 77	0.438 83	31.969 2	14.029 2	134.370 5	174.271 0	21
22	2.369 92	0.421 96	34.248 0	14.451 1	143.653 5	188.722 1	22
23	2.464 72	0.405 73	36.617 9	14.856 8	152.985 2	203.579 0	23
24	2.563 30	0.390 12	39.082 6	15.247 0	162.348 2	218.825 9	24
25	2.665 84	0.375 12	41.645 9	15.622 1	171.726 1	234.448 0	25
26	2.772 47	0.360 69	44.311 7	15.982 8	181.104 0	250.430 8	26
27	2.883 37	0.346 82	47.084 2	16.329 6	190.468 0	266.760 4	27
28	2.998 70	0.333 48	49.967 6	16.663 1	199.805 4	283.423 4	28
29	3.118 65	0.320 65	52.966 3	16.983 7	209.104 3	300.407 1	29
30	3.243 40	0.308 32	56.084 9	17.292 0	218.353 9	317.699 2	30
31	3.373 13	0.296 46	59.328 3	17.588 5	227.544 1	335.287 7	31
32	3.508 06	0.285 06	62.701 5	17.873 6	236.666 0	353.161 2	32
33	3.648 38	0.274 09	66.209 5	18.147 6	245.711 1	371.308 9	33
34	3.794 32	0.263 55	69.857 9	18.411 2	254.671 9	389.720 1	34
35	3.946 09	0.253 42	73.652 2	18.664 6	263.541 4	408.384 7	35
36	4.103 93	0.243 67	77.598 3	18.908 3	272.313 5	427.293 0	36
37	4.268 09	0.234 30	81.702 2	19.142 6	280.982 5	446.435 5	37
38	4.438 81	0.225 29	85.970 3	19.367 9	289.543 3	465.803 4	38
39	4.616 37	0.216 62	90.409 1	19.584 5	297.991 5	485.387 9	39
40	4.801 02	0.208 29	95.025 5	19.792 8	306.323 1	505.180 7	40
41	4.993 06	0.200 28	99.826 5	19.993 1	314.534 5	525.173 7	41
42	5.192 78	0.192 57	104.819 6	20.185 6	322.622 6	545.359 3	42
43	5.400 50	0.185 17	110.012 4	20.370 8	330.584 9	565.730 1	43
44	5.616 52	0.178 05	115.412 9	20.548 8	338.418 9	586.279 0	44
45	5.841 18	0.171 20	121.029 4	20.720 0	346.122 8	606.999 0	45
46	6.074 82	0.164 61	126.870 6	20.884 7	353.695 1	627.883 7	46
47	6.317 82	0.158 28	132.945 4	21.042 9	361.134 3	648.926 6	47
48	6.570 53	0.152 19	139.263 2	21.195 1	368.439 7	670.121 7	48
49	6.833 35	0.146 34	145.833 7	21.341 5	375.610 4	691.463 2	49
50	7.106 68	0.140 71	152.667 1	21.482 2	382.646 0	712.945 4	50
60	10.519 63	0.095 06	237.990 7	22.623 5	445.620 1	934.412 8	60
70	15.571 62	0.064 22	364.290 5	23.394 5	495.873 4	1 165.137 1	70
80	23.049 80	0.043 38	551.245 0	23.915 4	535.031 5	1 402.115 2	80
90	34.119 33	0.029 31	827.983 3	24.267 3	565.004 2	1 643.318 1	90
100	50.504 95	0.019 80	1 237.623 7	24.505 0	587.629 9	1 887.375 0	100

Compound Interest

5%

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.050 00	0.952 38	1.000 0	0.952 4	0.952 4	0.952 4	1
2	1.102 50	0.907 03	2.050 0	1.859 4	2.766 4	2.811 8	2
3	1.157 63	0.863 84	3.152 5	2.723 2	5.358 0	5.535 0	3
4	1.215 51	0.822 70	4.310 1	3.546 0	8.648 8	9.081 0	4
5	1.276 28	0.783 53	5.525 6	4.329 5	12.566 4	13.410 5	5
6	1.340 10	0.746 22	6.801 9	5.075 7	17.043 7	18.486 2	6
7	1.407 10	0.710 68	8.142 0	5.786 4	22.018 5	24.272 5	7
8	1.477 46	0.676 84	9.549 1	6.463 2	27.433 2	30.735 7	8
9	1.551 33	0.644 61	11.026 6	7.107 8	33.234 7	37.843 6	9
10	1.628 89	0.613 91	12.577 9	7.721 7	39.373 8	45.565 3	10
11	1.710 34	0.584 68	14.206 8	8.306 4	45.805 3	53.871 7	11
12	1.795 86	0.556 84	15.917 1	8.863 3	52.487 3	62.735 0	12
13	1.885 65	0.530 32	17.713 0	9.393 6	59.381 5	72.128 5	13
14	1.979 93	0.505 07	19.598 6	9.898 6	66.452 4	82.027 2	14
15	2.078 93	0.481 02	21.578 6	10.379 7	73.667 7	92.406 8	15
16	2.182 87	0.458 11	23.657 5	10.837 8	80.997 5	103.244 6	16
17	2.292 02	0.436 30	25.840 4	11.274 1	88.414 5	114.518 7	17
18	2.406 62	0.415 52	28.132 4	11.689 6	95.893 9	126.208 3	18
19	2.526 95	0.395 73	30.539 0	12.085 3	103.412 8	138.293 6	19
20	2.653 30	0.376 89	33.066 0	12.462 2	110.950 6	150.755 8	20
21	2.785 96	0.358 94	35.719 3	12.821 2	118.488 4	163.576 9	21
22	2.925 26	0.341 85	38.505 2	13.163 0	126.009 1	176.739 9	22
23	3.071 52	0.325 57	41.430 5	13.488 6	133.497 3	190.228 5	23
24	3.225 10	0.310 07	44.502 0	13.798 6	140.938 9	204.027 2	24
25	3.386 35	0.295 30	47.727 1	14.093 9	148.321 5	218.121 1	25
26	3.555 67	0.281 24	51.113 5	14.375 2	155.633 7	232.496 3	26
27	3.733 46	0.267 85	54.669 1	14.643 0	162.865 6	247.139 3	27
28	3.920 13	0.255 09	58.402 6	14.898 1	170.008 2	262.037 5	28
29	4.116 14	0.242 95	62.322 7	15.141 1	177.053 7	277.178 5	29
30	4.321 94	0.231 38	66.438 8	15.372 5	183.995 0	292.551 0	30
31	4.538 04	0.220 36	70.760 8	15.592 8	190.826 1	308.143 8	31
32	4.764 94	0.209 87	75.298 8	15.802 7	197.541 9	323.946 5	32
33	5.003 19	0.199 87	80.063 8	16.002 5	204.137 7	339.949 0	33
34	5.253 35	0.190 35	85.067 0	16.192 9	210.609 7	356.141 9	34
35	5.516 02	0.181 29	90.320 3	16.374 2	216.954 9	372.516 1	35
36	5.791 82	0.172 66	95.836 3	16.546 9	223.170 5	389.063 0	36
37	6.081 41	0.164 44	101.628 1	16.711 3	229.254 7	405.774 3	37
38	6.385 48	0.156 61	107.709 5	16.867 9	235.205 7	422.642 1	38
39	6.704 75	0.149 15	114.095 0	17.017 0	241.022 4	439.659 2	39
40	7.039 99	0.142 05	120.799 8	17.159 1	246.704 3	456.818 3	40
41	7.391 99	0.135 28	127.839 8	17.294 4	252.250 8	474.112 6	41
42	7.761 59	0.128 84	135.231 8	17.423 2	257.662 1	491.535 8	42
43	8.149 67	0.122 70	142.993 3	17.545 9	262.938 4	509.081 8	43
44	8.557 15	0.116 86	151.143 0	17.662 8	268.080 3	526.744 5	44
45	8.985 01	0.111 30	159.700 2	17.774 1	273.088 6	544.518 6	45
46	9.434 26	0.106 00	168.685 2	17.880 1	277.964 5	562.398 7	46
47	9.905 97	0.100 95	178.119 4	17.981 0	282.709 1	580.379 7	47
48	10.401 27	0.096 14	188.025 4	18.077 2	287.323 9	598.456 8	48
49	10.921 33	0.091 56	198.426 7	18.168 7	291.810 5	616.625 6	49
50	11.467 40	0.087 20	209.348 0	18.255 9	296.170 7	634.881 5	50
60	18.679 19	0.053 54	353.583 7	18.929 3	333.272 5	821.414 2	60
70	30.426 43	0.032 87	588.528 5	19.342 7	360.183 6	1 013.146 5	70
80	49.561 44	0.020 18	971.228 8	19.596 5	379.242 5	1 208.070 8	80
90	80.730 37	0.012 39	1 594.607 3	19.752 3	392.501 1	1 404.954 8	90
100	131.501 26	0.007 60	2 610.025 2	19.847 9	401.597 1	1 603.041 8	100

i	0.050 000
$i^{(2)}$	0.049 390
$i^{(4)}$	0.049 089
$i^{(12)}$	0.048 889
δ	0.048 790
$(1+i)^{1/2}$	1.024 695
$(1+i)^{1/4}$	1.012 272
$(1+i)^{1/12}$	1.004 074
v	0.952 381
$v^{1/2}$	0.975 900
$v^{1/4}$	0.987 877
$v^{1/12}$	0.995 942
d	0.047 619
$d^{(2)}$	0.048 200
$d^{(4)}$	0.048 494
$d^{(12)}$	0.048 691
$i/i^{(2)}$	1.012 348
$i/i^{(4)}$	1.018 559
$i/i^{(12)}$	1.022 715
i/δ	1.024 797
$i/d^{(2)}$	1.037 348
$i/d^{(4)}$	1.031 059
$i/d^{(12)}$	1.026 881

Compound Interest

6%

i	0.060 000
$i^{(2)}$	0.059 126
$i^{(4)}$	0.058 695
$i^{(12)}$	0.058 411
δ	0.058 269
$(1+i)^{1/2}$	1.029 563
$(1+i)^{1/4}$	1.014 674
$(1+i)^{1/12}$	1.004 868
v	0.943 396
$v^{1/2}$	0.971 286
$v^{1/4}$	0.985 538
$v^{1/12}$	0.995 156
d	0.056 604
$d^{(2)}$	0.057 428
$d^{(4)}$	0.057 847
$d^{(12)}$	0.058 128
$i/i^{(2)}$	1.014 782
$i/i^{(4)}$	1.022 227
$i/i^{(12)}$	1.027 211
i/δ	1.029 709
$i/d^{(2)}$	1.044 782
$i/d^{(4)}$	1.037 227
$i/d^{(12)}$	1.032 211

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.060 00	0.943 40	1.000 0	0.943 4	0.943 4	0.943 4	1
2	1.123 60	0.890 00	2.060 0	1.833 4	2.723 4	2.776 8	2
3	1.191 02	0.839 62	3.183 6	2.673 0	5.242 2	5.449 8	3
4	1.262 48	0.792 09	4.374 6	3.465 1	8.410 6	8.914 9	4
5	1.338 23	0.747 26	5.637 1	4.212 4	12.146 9	13.127 3	5
6	1.418 52	0.704 96	6.975 3	4.917 3	16.376 7	18.044 6	6
7	1.503 63	0.665 06	8.393 8	5.582 4	21.032 1	23.627 0	7
8	1.593 85	0.627 41	9.897 5	6.209 8	26.051 4	29.836 8	8
9	1.689 48	0.591 90	11.491 3	6.801 7	31.378 5	36.638 5	9
10	1.790 85	0.558 39	13.180 8	7.360 1	36.962 4	43.998 5	10
11	1.898 30	0.526 79	14.971 6	7.886 9	42.757 1	51.885 4	11
12	2.012 20	0.496 97	16.869 9	8.383 8	48.720 7	60.269 3	12
13	2.132 93	0.468 84	18.882 1	8.852 7	54.815 6	69.122 0	13
14	2.260 90	0.442 30	21.015 1	9.295 0	61.007 8	78.416 9	14
15	2.396 56	0.417 27	23.276 0	9.712 2	67.266 8	88.129 2	15
16	2.540 35	0.393 65	25.672 5	10.105 9	73.565 1	98.235 1	16
17	2.692 77	0.371 36	28.212 9	10.477 3	79.878 3	108.712 3	17
18	2.854 34	0.350 34	30.905 7	10.827 6	86.184 5	119.539 9	18
19	3.025 60	0.330 51	33.760 0	11.158 1	92.464 3	130.698 1	19
20	3.207 14	0.311 80	36.785 6	11.469 9	98.700 4	142.168 0	20
21	3.399 56	0.294 16	39.992 7	11.764 1	104.877 6	153.932 1	21
22	3.603 54	0.277 51	43.392 3	12.041 6	110.982 7	165.973 6	22
23	3.819 75	0.261 80	46.995 8	12.303 4	117.004 1	178.277 0	23
24	4.048 93	0.246 98	50.815 6	12.550 4	122.931 6	190.827 4	24
25	4.291 87	0.233 00	54.864 5	12.783 4	128.756 5	203.610 7	25
26	4.549 38	0.219 81	59.156 4	13.003 2	134.471 6	216.613 9	26
27	4.822 35	0.207 37	63.705 8	13.210 5	140.070 5	229.824 4	27
28	5.111 69	0.195 63	68.528 1	13.406 2	145.548 2	243.230 6	28
29	5.418 39	0.184 56	73.639 8	13.590 7	150.900 3	256.821 3	29
30	5.743 49	0.174 11	79.058 2	13.764 8	156.123 6	270.586 1	30
31	6.088 10	0.164 25	84.801 7	13.929 1	161.215 5	284.515 2	31
32	6.453 39	0.154 96	90.889 8	14.084 0	166.174 2	298.599 3	32
33	6.840 59	0.146 19	97.343 2	14.230 2	170.998 3	312.829 5	33
34	7.251 03	0.137 91	104.183 8	14.368 1	175.687 3	327.197 6	34
35	7.686 09	0.130 11	111.434 8	14.498 2	180.241 0	341.695 9	35
36	8.147 25	0.122 74	119.120 9	14.621 0	184.659 6	356.316 9	36
37	8.636 09	0.115 79	127.268 1	14.736 8	188.944 0	371.053 7	37
38	9.154 25	0.109 24	135.904 2	14.846 0	193.095 1	385.899 7	38
39	9.703 51	0.103 06	145.058 5	14.949 1	197.114 2	400.848 8	39
40	10.285 72	0.097 22	154.762 0	15.046 3	201.003 1	415.895 1	40
41	10.902 86	0.091 72	165.047 7	15.138 0	204.763 6	431.033 1	41
42	11.557 03	0.086 53	175.950 5	15.224 5	208.397 8	446.257 6	42
43	12.250 45	0.081 63	187.507 6	15.306 2	211.907 8	461.563 8	43
44	12.985 48	0.077 01	199.758 0	15.383 2	215.296 2	476.947 0	44
45	13.764 61	0.072 65	212.743 5	15.455 8	218.565 5	492.402 8	45
46	14.590 49	0.068 54	226.508 1	15.524 4	221.718 2	507.927 2	46
47	15.465 92	0.064 66	241.098 6	15.589 0	224.757 2	523.516 2	47
48	16.393 87	0.061 00	256.564 5	15.650 0	227.685 1	539.166 2	48
49	17.377 50	0.057 55	272.958 4	15.707 6	230.504 8	554.873 8	49
50	18.420 15	0.054 29	290.335 9	15.761 9	233.219 2	570.635 7	50
60	32.987 69	0.030 31	533.128 2	16.161 4	255.204 2	730.642 9	60
70	59.075 93	0.016 93	967.932 2	16.384 5	269.711 7	893.590 9	70
80	105.795 99	0.009 45	1 746.599 9	16.509 1	279.058 4	1 058.181 2	80
90	189.464 51	0.005 28	3 141.075 2	16.578 7	284.973 3	1 223.688 3	90
100	339.302 08	0.002 95	5 638.368 1	16.617 5	288.664 6	1 389.707 6	100

Compound Interest

7%

n	$(1+i)^n$	v^n	$s_{\overline{n}\|}$	$a_{\overline{n}\|}$	$(Ia)_{\overline{n}\|}$	$(Da)_{\overline{n}\|}$	n
1	1.070 00	0.934 58	1.000 0	0.934 6	0.934 6	0.934 6	1
2	1.144 90	0.873 44	2.070 0	1.808 0	2.681 5	2.742 6	2
3	1.225 04	0.816 30	3.214 9	2.624 3	5.130 4	5.366 9	3
4	1.310 80	0.762 90	4.439 9	3.387 2	8.181 9	8.754 1	4
5	1.402 55	0.712 99	5.750 7	4.100 2	11.746 9	12.854 3	5
6	1.500 73	0.666 34	7.153 3	4.766 5	15.744 9	17.620 9	6
7	1.605 78	0.622 75	8.654 0	5.389 3	20.104 2	23.010 2	7
8	1.718 19	0.582 01	10.259 8	5.971 3	24.760 2	28.981 4	8
9	1.838 46	0.543 93	11.978 0	6.515 2	29.655 6	35.496 7	9
10	1.967 15	0.508 35	13.816 4	7.023 6	34.739 1	42.520 3	10
11	2.104 85	0.475 09	15.783 6	7.498 7	39.965 2	50.018 9	11
12	2.252 19	0.444 01	17.888 5	7.942 7	45.293 3	57.961 6	12
13	2.409 85	0.414 96	20.140 6	8.357 7	50.687 8	66.319 3	13
14	2.578 53	0.387 82	22.550 5	8.745 5	56.117 3	75.064 7	14
15	2.759 03	0.362 45	25.129 0	9.107 9	61.554 0	84.172 7	15
16	2.952 16	0.338 73	27.888 1	9.446 6	66.973 7	93.619 3	16
17	3.158 82	0.316 57	30.840 2	9.763 2	72.355 5	103.382 5	17
18	3.379 93	0.295 86	33.999 0	10.059 1	77.681 0	113.441 6	18
19	3.616 53	0.276 51	37.379 0	10.335 6	82.934 7	123.777 2	19
20	3.869 68	0.258 42	40.995 5	10.594 0	88.103 1	134.371 2	20
21	4.140 56	0.241 51	44.865 2	10.835 5	93.174 8	145.206 8	21
22	4.430 40	0.225 71	49.005 7	11.061 2	98.140 5	156.268 0	22
23	4.740 53	0.210 95	53.436 1	11.272 2	102.992 3	167.540 2	23
24	5.072 37	0.197 15	58.176 7	11.469 3	107.723 8	179.009 5	24
25	5.427 43	0.184 25	63.249 0	11.653 6	112.330 1	190.663 1	25
26	5.807 35	0.172 20	68.676 5	11.825 8	116.807 1	202.488 9	26
27	6.213 87	0.160 93	74.483 8	11.986 7	121.152 3	214.475 6	27
28	6.648 84	0.150 40	80.697 7	12.137 1	125.363 5	226.612 7	28
29	7.114 26	0.140 56	87.346 5	12.277 7	129.439 9	238.890 4	29
30	7.612 26	0.131 37	94.460 8	12.409 0	133.380 9	251.299 4	30
31	8.145 11	0.122 77	102.073 0	12.531 8	137.186 8	263.831 2	31
32	8.715 27	0.114 74	110.218 2	12.646 6	140.858 5	276.477 8	32
33	9.325 34	0.107 23	118.933 4	12.753 8	144.397 3	289.231 6	33
34	9.978 11	0.100 22	128.258 8	12.854 0	147.804 7	302.085 6	34
35	10.676 58	0.093 66	138.236 9	12.947 7	151.082 9	315.033 3	35
36	11.423 94	0.087 54	148.913 5	13.035 2	154.234 2	328.068 5	36
37	12.223 62	0.081 81	160.337 4	13.117 0	157.261 2	341.185 5	37
38	13.079 27	0.076 46	172.561 0	13.193 5	160.166 5	354.379 0	38
39	13.994 82	0.071 46	185.640 3	13.264 9	162.953 3	367.643 9	39
40	14.974 46	0.066 78	199.635 1	13.331 7	165.624 5	380.975 6	40
41	16.022 67	0.062 41	214.609 6	13.394 1	168.183 3	394.369 7	41
42	17.144 26	0.058 33	230.632 2	13.452 4	170.633 1	407.822 2	42
43	18.344 35	0.054 51	247.776 5	13.507 0	172.977 2	421.329 1	43
44	19.628 46	0.050 95	266.120 9	13.557 9	175.218 8	434.887 0	44
45	21.002 45	0.047 61	285.749 3	13.605 5	177.361 4	448.492 5	45
46	22.472 62	0.044 50	306.751 8	13.650 0	179.408 4	462.142 6	46
47	24.045 71	0.041 59	329.224 4	13.691 6	181.363 0	475.834 2	47
48	25.728 91	0.038 87	353.270 1	13.730 5	183.228 6	489.564 7	48
49	27.529 93	0.036 32	378.999 0	13.766 8	185.008 5	503.331 4	49
50	29.457 03	0.033 95	406.528 9	13.800 7	186.705 9	517.132 2	50
60	57.946 43	0.017 26	813.520 4	14.039 2	199.806 9	656.583 1	60
70	113.989 39	0.008 77	1 614.134 2	14.160 4	207.678 9	797.708 7	70
80	224.234 39	0.004 46	3 189.062 7	14.222 0	212.296 8	939.685 6	80
90	441.102 98	0.002 27	6 287.185 4	14.253 3	214.957 5	1 082.095 3	90
100	867.716 33	0.001 15	12 381.661 8	14.269 3	216.469 3	1 224.725 0	100

i	0.070 000
$i^{(2)}$	0.068 816
$i^{(4)}$	0.068 234
$i^{(12)}$	0.067 850
δ	0.067 659
$(1+i)^{1/2}$	1.034 408
$(1+i)^{1/4}$	1.017 059
$(1+i)^{1/12}$	1.005 654
v	0.934 579
$v^{1/2}$	0.966 736
$v^{1/4}$	0.983 228
$v^{1/12}$	0.994 378
d	0.065 421
$d^{(2)}$	0.066 527
$d^{(4)}$	0.067 090
$d^{(12)}$	0.067 468
$i/i^{(2)}$	1.017 204
$i/i^{(4)}$	1.025 880
$i/i^{(12)}$	1.031 691
i/δ	1.034 605
$i/d^{(2)}$	1.052 204
$i/d^{(4)}$	1.043 380
$i/d^{(12)}$	1.037 525

Compound Interest

8%

i	0.080 000
$i^{(2)}$	0.078 461
$i^{(4)}$	0.077 706
$i^{(12)}$	0.077 208
δ	0.076 961
$(1+i)^{1/2}$	1.039 230
$(1+i)^{1/4}$	1.019 427
$(1+i)^{1/12}$	1.006 434
v	0.925 926
$v^{1/2}$	0.962 250
$v^{1/4}$	0.980 944
$v^{1/12}$	0.993 607
d	0.074 074
$d^{(2)}$	0.075 499
$d^{(4)}$	0.076 225
$d^{(12)}$	0.076 715
$i/i^{(2)}$	1.019 615
$i/i^{(4)}$	1.029 519
$i/i^{(12)}$	1.036 157
i/δ	1.039 487
$i/d^{(2)}$	1.059 615
$i/d^{(4)}$	1.049 519
$i/d^{(12)}$	1.042 824

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.080 00	0.925 93	1.000 0	0.925 9	0.925 9	0.925 9	1
2	1.166 40	0.857 34	2.080 0	1.783 3	2.640 6	2.709 2	2
3	1.259 71	0.793 83	3.246 4	2.577 1	5.022 1	5.286 3	3
4	1.360 49	0.735 03	4.506 1	3.312 1	7.962 2	8.598 4	4
5	1.469 33	0.680 58	5.866 6	3.992 7	11.365 1	12.591 1	5
6	1.586 87	0.630 17	7.335 9	4.622 9	15.146 2	17.214 0	6
7	1.713 82	0.583 49	8.922 8	5.206 4	19.230 6	22.420 4	7
8	1.850 93	0.540 27	10.636 6	5.746 6	23.552 7	28.167 0	8
9	1.999 00	0.500 25	12.487 6	6.246 9	28.055 0	34.413 9	9
10	2.158 92	0.463 19	14.486 6	6.710 1	32.686 9	41.124 0	10
11	2.331 64	0.428 88	16.645 5	7.139 0	37.404 6	48.262 9	11
12	2.518 17	0.397 11	18.977 1	7.536 1	42.170 0	55.799 0	12
13	2.719 62	0.367 70	21.495 3	7.903 8	46.950 1	63.702 8	13
14	2.937 19	0.340 46	24.214 9	8.244 2	51.716 5	71.947 0	14
15	3.172 17	0.315 24	27.152 1	8.559 5	56.445 1	80.506 5	15
16	3.425 94	0.291 89	30.324 3	8.851 4	61.115 4	89.357 9	16
17	3.700 02	0.270 27	33.750 2	9.121 6	65.710 0	98.479 5	17
18	3.996 02	0.250 25	37.450 2	9.371 9	70.214 4	107.851 4	18
19	4.315 70	0.231 71	41.446 3	9.603 6	74.617 0	117.455 0	19
20	4.660 96	0.214 55	45.762 0	9.818 1	78.907 9	127.273 2	20
21	5.033 83	0.198 66	50.422 9	10.016 8	83.079 7	137.290 0	21
22	5.436 54	0.183 94	55.456 8	10.200 7	87.126 4	147.490 7	22
23	5.871 46	0.170 32	60.893 3	10.371 1	91.043 7	157.861 8	23
24	6.341 18	0.157 70	66.764 8	10.528 8	94.828 4	168.390 5	24
25	6.848 48	0.146 02	73.105 9	10.674 8	98.478 9	179.065 3	25
26	7.396 35	0.135 20	79.954 4	10.810 0	101.994 1	189.875 3	26
27	7.988 06	0.125 19	87.350 8	10.935 2	105.374 2	200.810 4	27
28	8.627 11	0.115 91	95.338 8	11.051 1	108.619 8	211.861 5	28
29	9.317 27	0.107 33	103.965 9	11.158 4	111.732 3	223.019 9	29
30	10.062 66	0.099 38	113.283 2	11.257 8	114.713 6	234.277 7	30
31	10.867 67	0.092 02	123.345 9	11.349 8	117.566 1	245.627 5	31
32	11.737 08	0.085 20	134.213 5	11.435 0	120.292 5	257.062 5	32
33	12.676 05	0.078 89	145.950 6	11.513 9	122.895 8	268.576 4	33
34	13.690 13	0.073 05	158.626 7	11.586 9	125.379 3	280.163 3	34
35	14.785 34	0.067 63	172.316 8	11.654 6	127.746 6	291.817 9	35
36	15.968 17	0.062 62	187.102 1	11.717 2	130.001 0	303.535 1	36
37	17.245 63	0.057 99	203.070 3	11.775 2	132.146 5	315.310 3	37
38	18.625 28	0.053 69	220.315 9	11.828 9	134.186 8	327.139 1	38
39	20.115 30	0.049 71	238.941 2	11.878 6	136.125 6	339.017 7	39
40	21.724 52	0.046 03	259.056 5	11.924 6	137.966 8	350.942 3	40
41	23.462 48	0.042 62	280.781 0	11.967 2	139.714 3	362.909 6	41
42	25.339 48	0.039 46	304.243 5	12.006 7	141.371 8	374.916 3	42
43	27.366 64	0.036 54	329.583 0	12.043 2	142.943 0	386.959 5	43
44	29.555 97	0.033 83	356.949 6	12.077 1	144.431 7	399.036 6	44
45	31.920 45	0.031 33	386.505 6	12.108 4	145.841 5	411.145 0	45
46	34.474 09	0.029 01	418.426 1	12.137 4	147.175 8	423.282 4	46
47	37.232 01	0.026 86	452.900 2	12.164 3	148.438 2	435.446 7	47
48	40.210 57	0.024 87	490.132 2	12.189 1	149.631 9	447.635 8	48
49	43.427 42	0.023 03	530.342 7	12.212 2	150.760 2	459.848 0	49
50	46.901 61	0.021 32	573.770 2	12.233 5	151.826 3	472.081 4	50
60	101.257 06	0.009 88	1 253.213 3	12.376 6	159.676 6	595.293 1	60
70	218.606 41	0.004 57	2 720.080 1	12.442 8	163.975 4	719.464 8	70
80	471.954 83	0.002 12	5 886.935 4	12.473 5	166.273 6	844.081 1	80
90	1 018.915 09	0.000 98	12 723.938 6	12.487 7	167.480 3	968.903 3	90
100	2 199.761 26	0.000 45	27 484.515 7	12.494 3	168.105 0	1 093.821 0	100

Compound Interest

9%

n	$(1+i)^n$	v^n	$s_{\overline{n}|}$	$a_{\overline{n}|}$	$(Ia)_{\overline{n}|}$	$(Da)_{\overline{n}|}$	n
1	1.090 00	0.917 43	1.000 0	0.917 4	0.917 4	0.917 4	1
2	1.188 10	0.841 68	2.090 0	1.759 1	2.600 8	2.676 5	2
3	1.295 03	0.772 18	3.278 1	2.531 3	4.917 3	5.207 8	3
4	1.411 58	0.708 43	4.573 1	3.239 7	7.751 0	8.447 6	4
5	1.538 62	0.649 93	5.984 7	3.889 7	11.000 7	12.337 2	5
6	1.677 10	0.596 27	7.523 3	4.485 9	14.578 3	16.823 1	6
7	1.828 04	0.547 03	9.200 4	5.033 0	18.407 5	21.856 1	7
8	1.992 56	0.501 87	11.028 5	5.534 8	22.422 5	27.390 9	8
9	2.171 89	0.460 43	13.021 0	5.995 2	26.566 3	33.386 1	9
10	2.367 36	0.422 41	15.192 9	6.417 7	30.790 4	39.803 8	10
11	2.580 43	0.387 53	17.560 3	6.805 2	35.053 3	46.609 0	11
12	2.812 66	0.355 53	20.140 7	7.160 7	39.319 7	53.769 7	12
13	3.065 80	0.326 18	22.953 4	7.486 9	43.560 0	61.256 6	13
14	3.341 73	0.299 25	26.019 2	7.786 2	47.749 5	69.042 8	14
15	3.642 48	0.274 54	29.360 9	8.060 7	51.867 6	77.103 5	15
16	3.970 31	0.251 87	33.003 4	8.312 6	55.897 5	85.416 0	16
17	4.327 63	0.231 07	36.973 7	8.543 6	59.825 7	93.959 7	17
18	4.717 12	0.211 99	41.301 3	8.755 6	63.641 6	102.715 3	18
19	5.141 66	0.194 49	46.018 5	8.950 1	67.336 9	111.665 4	19
20	5.604 41	0.178 43	51.160 1	9.128 5	70.905 5	120.793 9	20
21	6.108 81	0.163 70	56.764 5	9.292 2	74.343 2	130.086 2	21
22	6.658 60	0.150 18	62.873 3	9.442 4	77.647 2	139.528 6	22
23	7.257 87	0.137 78	69.531 9	9.580 2	80.816 2	149.108 8	23
24	7.911 08	0.126 40	76.789 8	9.706 6	83.849 9	158.815 4	24
25	8.623 08	0.115 97	84.700 9	9.822 6	86.749 1	168.638 0	25
26	9.399 16	0.106 39	93.324 0	9.929 0	89.515 3	178.567 0	26
27	10.245 08	0.097 61	102.723 1	10.026 6	92.150 7	188.593 6	27
28	11.167 14	0.089 55	112.968 2	10.116 1	94.658 0	198.709 7	28
29	12.172 18	0.082 15	124.135 4	10.198 3	97.040 5	208.908 0	29
30	13.267 68	0.075 37	136.307 5	10.273 7	99.301 7	219.181 6	30
31	14.461 77	0.069 15	149.575 2	10.342 8	101.445 2	229.524 4	31
32	15.763 33	0.063 44	164.037 0	10.406 2	103.475 3	239.930 7	32
33	17.182 03	0.058 20	179.800 3	10.464 4	105.395 9	250.395 1	33
34	18.728 41	0.053 39	196.982 3	10.517 8	107.211 3	260.912 9	34
35	20.413 97	0.048 99	215.710 8	10.566 8	108.925 8	271.479 8	35
36	22.251 23	0.044 94	236.124 7	10.611 8	110.543 7	282.091 5	36
37	24.253 84	0.041 23	258.375 9	10.653 0	112.069 2	292.744 5	37
38	26.436 68	0.037 83	282.629 8	10.690 8	113.506 6	303.435 3	38
39	28.815 98	0.034 70	309.066 5	10.725 5	114.860 0	314.160 9	39
40	31.409 42	0.031 84	337.882 4	10.757 4	116.133 5	324.918 2	40
41	34.236 27	0.029 21	369.291 9	10.786 6	117.331 1	335.704 8	41
42	37.317 53	0.026 80	403.528 1	10.813 4	118.456 6	346.518 2	42
43	40.676 11	0.024 58	440.845 7	10.838 0	119.513 7	357.356 1	43
44	44.336 96	0.022 55	481.521 8	10.860 5	120.506 1	368.216 6	44
45	48.327 29	0.020 69	525.858 7	10.881 2	121.437 3	379.097 8	45
46	52.676 74	0.018 98	574.186 0	10.900 2	122.310 5	389.998 0	46
47	57.417 65	0.017 42	626.862 8	10.917 6	123.129 1	400.915 6	47
48	62.585 24	0.015 98	684.280 4	10.933 6	123.896 0	411.849 2	48
49	68.217 91	0.014 66	746.865 6	10.948 2	124.614 3	422.797 4	49
50	74.357 52	0.013 45	815.083 6	10.961 7	125.286 7	433.759 1	50
60	176.031 29	0.005 68	1 944.792 1	11.048 0	130.016 2	543.911 2	60
70	416.730 09	0.002 40	4 619.223 2	11.084 4	132.378 6	654.617 2	70
80	986.551 67	0.001 01	10 950.574 1	11.099 8	133.530 5	765.557 2	80
90	2 335.526 58	0.000 43	25 939.184 2	11.106 4	134.082 1	876.596 1	90
100	5 529.040 79	0.000 18	61 422.675 5	11.109 1	134.342 6	987.676 6	100

i	0.090 000
$i^{(2)}$	0.088 061
$i^{(4)}$	0.087 113
$i^{(12)}$	0.086 488
δ	0.086 178
$(1+i)^{1/2}$	1.044 031
$(1+i)^{1/4}$	1.021 778
$(1+i)^{1/12}$	1.007 207
v	0.917 431
$v^{1/2}$	0.957 826
$v^{1/4}$	0.978 686
$v^{1/12}$	0.992 844
d	0.082 569
$d^{(2)}$	0.084 347
$d^{(4)}$	0.085 256
$d^{(12)}$	0.085 869
$i/i^{(2)}$	1.022 015
$i/i^{(4)}$	1.033 144
$i/i^{(12)}$	1.040 608
i/δ	1.044 354
$i/d^{(2)}$	1.067 015
$i/d^{(4)}$	1.055 644
$i/d^{(12)}$	1.048 108

Compound Interest

10%

i	0.100 000
$i^{(2)}$	0.097 618
$i^{(4)}$	0.096 455
$i^{(12)}$	0.095 690
δ	0.095 310
$(1+i)^{1/2}$	1.048 809
$(1+i)^{1/4}$	1.024 114
$(1+i)^{1/12}$	1.007 974
v	0.909 091
$v^{1/2}$	0.953 463
$v^{1/4}$	0.976 454
$v^{1/12}$	0.992 089
d	0.090 909
$d^{(2)}$	0.093 075
$d^{(4)}$	0.094 184
$d^{(12)}$	0.094 933
$i/i^{(2)}$	1.024 404
$i/i^{(4)}$	1.036 756
$i/i^{(12)}$	1.045 045
i/δ	1.049 206
$i/d^{(2)}$	1.074 404
$i/d^{(4)}$	1.061 756
$i/d^{(12)}$	1.053 378

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.100 00	0.909 09	1.000 0	0.909 1	0.909 1	0.909 1	1
2	1.210 00	0.826 45	2.100 0	1.735 5	2.562 0	2.644 6	2
3	1.331 00	0.751 31	3.310 0	2.486 9	4.815 9	5.131 5	3
4	1.464 10	0.683 01	4.641 0	3.169 9	7.548 0	8.301 3	4
5	1.610 51	0.620 92	6.105 1	3.790 8	10.652 6	12.092 1	5
6	1.771 56	0.564 47	7.715 6	4.355 3	14.039 4	16.447 4	6
7	1.948 72	0.513 16	9.487 2	4.868 4	17.631 5	21.315 8	7
8	2.143 59	0.466 51	11.435 9	5.334 9	21.363 6	26.650 7	8
9	2.357 95	0.424 10	13.579 5	5.759 0	25.180 5	32.409 8	9
10	2.593 74	0.385 54	15.937 4	6.144 6	29.035 9	38.554 3	10
11	2.853 12	0.350 49	18.531 2	6.495 1	32.891 3	45.049 4	11
12	3.138 43	0.318 63	21.384 3	6.813 7	36.714 9	51.863 1	12
13	3.452 27	0.289 66	24.522 7	7.103 4	40.480 5	58.966 4	13
14	3.797 50	0.263 33	27.975 0	7.366 7	44.167 2	66.333 1	14
15	4.177 25	0.239 39	31.772 5	7.606 1	47.758 1	73.939 2	15
16	4.594 97	0.217 63	35.949 7	7.823 7	51.240 1	81.762 9	16
17	5.054 47	0.197 84	40.544 7	8.021 6	54.603 5	89.784 5	17
18	5.559 92	0.179 86	45.599 2	8.201 4	57.841 0	97.985 9	18
19	6.115 91	0.163 51	51.159 1	8.364 9	60.947 6	106.350 8	19
20	6.727 50	0.148 64	57.275 0	8.513 6	63.920 5	114.864 4	20
21	7.400 25	0.135 13	64.002 5	8.648 7	66.758 2	123.513 1	21
22	8.140 27	0.122 85	71.402 7	8.771 5	69.460 8	132.284 6	22
23	8.954 30	0.111 68	79.543 0	8.883 2	72.029 4	141.167 8	23
24	9.849 73	0.101 53	88.497 3	8.984 7	74.466 0	150.152 6	24
25	10.834 71	0.092 30	98.347 1	9.077 0	76.773 4	159.229 6	25
26	11.918 18	0.083 91	109.181 8	9.160 9	78.955 0	168.390 5	26
27	13.109 99	0.076 28	121.099 9	9.237 2	81.014 5	177.627 8	27
28	14.420 99	0.069 34	134.209 9	9.306 6	82.956 1	186.934 3	28
29	15.863 09	0.063 04	148.630 9	9.369 6	84.784 2	196.303 9	29
30	17.449 40	0.057 31	164.494 0	9.426 9	86.503 5	205.730 9	30
31	19.194 34	0.052 10	181.943 4	9.479 0	88.118 6	215.209 9	31
32	21.113 78	0.047 36	201.137 8	9.526 4	89.634 2	224.736 2	32
33	23.225 15	0.043 06	222.251 5	9.569 4	91.055 0	234.305 7	33
34	25.547 67	0.039 14	245.476 7	9.608 6	92.385 9	243.914 3	34
35	28.102 44	0.035 58	271.024 4	9.644 2	93.631 3	253.558 4	35
36	30.912 68	0.032 35	299.126 8	9.676 5	94.795 9	263.234 9	36
37	34.003 95	0.029 41	330.039 5	9.705 9	95.884 0	272.940 8	37
38	37.404 34	0.026 73	364.043 4	9.732 7	96.899 9	282.673 5	38
39	41.144 78	0.024 30	401.447 8	9.757 0	97.847 8	292.430 4	39
40	45.259 26	0.022 09	442.592 6	9.779 1	98.731 6	302.209 5	40
41	49.785 18	0.020 09	487.851 8	9.799 1	99.555 1	312.008 6	41
42	54.763 70	0.018 26	537.637 0	9.817 4	100.322 1	321.826 0	42
43	60.240 07	0.016 60	592.400 7	9.834 0	101.035 9	331.660 0	43
44	66.264 08	0.015 09	652.640 8	9.849 1	101.699 9	341.509 1	44
45	72.890 48	0.013 72	718.904 8	9.862 8	102.317 2	351.371 9	45
46	80.179 53	0.012 47	791.795 3	9.875 3	102.891 0	361.247 2	46
47	88.197 49	0.011 34	871.974 9	9.886 6	103.423 8	371.133 8	47
48	97.017 23	0.010 31	960.172 3	9.896 9	103.918 6	381.030 7	48
49	106.718 96	0.009 37	1 057.189 6	9.906 3	104.377 8	390.937 0	49
50	117.390 85	0.008 52	1 163.908 5	9.914 8	104.803 7	400.851 9	50
60	304.481 64	0.003 28	3 034.816 4	9.967 2	107.668 2	500.328 4	60
70	789.746 96	0.001 27	7 887.469 6	9.987 3	108.974 4	600.126 6	70
80	2 048.400 21	0.000 49	20 474.002 1	9.995 1	109.555 8	700.048 8	80
90	5 313.022 61	0.000 19	53 120.226 1	9.998 1	109.809 9	800.018 8	90
100	13 780.612 34	0.000 07	137 796.123 4	9.999 3	109.919 5	900.007 3	100

Compound Interest

12%

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.120 00	0.892 86	1.000 0	0.892 9	0.892 9	0.892 9	1
2	1.254 40	0.797 19	2.120 0	1.690 1	2.487 2	2.582 9	2
3	1.404 93	0.711 78	3.374 4	2.401 8	4.622 6	4.984 7	3
4	1.573 52	0.635 52	4.779 3	3.037 3	7.164 7	8.022 1	4
5	1.762 34	0.567 43	6.352 8	3.604 8	10.001 8	11.626 9	5
6	1.973 82	0.506 63	8.115 2	4.111 4	13.041 6	15.738 3	6
7	2.210 68	0.452 35	10.089 0	4.563 8	16.208 0	20.302 0	7
8	2.475 96	0.403 88	12.299 7	4.967 6	19.439 1	25.269 7	8
9	2.773 08	0.360 61	14.775 7	5.328 2	22.684 6	30.597 9	9
10	3.105 85	0.321 97	17.548 7	5.650 2	25.904 3	36.248 1	10
11	3.478 55	0.287 48	20.654 6	5.937 7	29.066 5	42.185 8	11
12	3.895 98	0.256 68	24.133 1	6.194 4	32.146 7	48.380 2	12
13	4.363 49	0.229 17	28.029 1	6.423 5	35.125 9	54.803 8	13
14	4.887 11	0.204 62	32.392 6	6.628 2	37.990 6	61.431 9	14
15	5.473 57	0.182 70	37.279 7	6.810 9	40.731 0	68.242 8	15
16	6.130 39	0.163 12	42.753 3	6.974 0	43.341 0	75.216 8	16
17	6.866 04	0.145 64	48.883 7	7.119 6	45.816 9	82.336 4	17
18	7.689 97	0.130 04	55.749 7	7.249 7	48.157 6	89.586 1	18
19	8.612 76	0.116 11	63.439 7	7.365 8	50.363 7	96.951 9	19
20	9.646 29	0.103 67	72.052 4	7.469 4	52.437 0	104.421 3	20
21	10.803 85	0.092 56	81.698 7	7.562 0	54.380 8	111.983 3	21
22	12.100 31	0.082 64	92.502 6	7.644 6	56.198 9	119.628 0	22
23	13.552 35	0.073 79	104.602 9	7.718 4	57.896 0	127.346 4	23
24	15.178 63	0.065 88	118.155 2	7.784 3	59.477 2	135.130 7	24
25	17.000 06	0.058 82	133.333 9	7.843 1	60.947 8	142.973 8	25
26	19.040 07	0.052 52	150.333 9	7.895 7	62.313 3	150.869 5	26
27	21.324 88	0.046 89	169.374 0	7.942 6	63.579 4	158.812 1	27
28	23.883 87	0.041 87	190.698 9	7.984 4	64.751 8	166.796 5	28
29	26.749 93	0.037 38	214.582 8	8.021 8	65.835 9	174.818 3	29
30	29.959 92	0.033 38	241.332 7	8.055 2	66.837 2	182.873 5	30
31	33.555 11	0.029 80	271.292 6	8.085 0	67.761 1	190.958 5	31
32	37.581 73	0.026 61	304.847 7	8.111 6	68.612 6	199.070 0	32
33	42.091 53	0.023 76	342.429 4	8.135 4	69.396 6	207.205 4	33
34	47.142 52	0.021 21	384.521 0	8.156 6	70.117 8	215.362 0	34
35	52.799 62	0.018 94	431.663 5	8.175 5	70.780 7	223.537 5	35
36	59.135 57	0.016 91	484.463 1	8.192 4	71.389 4	231.729 9	36
37	66.231 84	0.015 10	543.598 7	8.207 5	71.948 1	239.937 4	37
38	74.179 66	0.013 48	609.830 5	8.221 0	72.460 4	248.158 4	38
39	83.081 22	0.012 04	684.010 2	8.233 0	72.929 8	256.391 4	39
40	93.050 97	0.010 75	767.091 4	8.243 8	73.359 6	264.635 2	40
41	104.217 09	0.009 60	860.142 4	8.253 4	73.753 1	272.888 6	41
42	116.723 14	0.008 57	964.359 5	8.261 9	74.112 9	281.150 5	42
43	130.729 91	0.007 65	1 081.082 6	8.269 6	74.441 8	289.420 1	43
44	146.417 50	0.006 83	1 211.812 5	8.276 4	74.742 3	297.696 5	44
45	163.987 60	0.006 10	1 358.230 0	8.282 5	75.016 7	305.979 0	45
46	183.666 12	0.005 44	1 522.217 6	8.288 0	75.267 2	314.267 0	46
47	205.706 05	0.004 86	1 705.883 8	8.292 8	75.495 7	322.559 8	47
48	230.390 78	0.004 34	1 911.589 8	8.297 2	75.704 0	330.857 0	48
49	258.037 67	0.003 88	2 141.980 6	8.301 0	75.893 9	339.158 0	49
50	289.002 19	0.003 46	2 400.018 2	8.304 5	76.066 9	347.462 5	50
60	897.596 93	0.001 11	7 471.641 1	8.324 0	77.134 1	430.632 9	60
70	2 787.799 83	0.000 36	23 223.331 9	8.330 3	77.540 6	513.913 8	70
80	8 658.483 10	0.000 12	72 145.692 5	8.332 4	77.691 8	597.230 2	80
90	26 891.934 22	0.000 04	224 091.118 5	8.333 0	77.747 0	680.558 1	90
100	83 522.265 73	0.000 01	696 010.547 7	8.333 2	77.766 9	763.889 7	100

i	0.120 000
$i^{(2)}$	0.116 601
$i^{(4)}$	0.114 949
$i^{(12)}$	0.113 866
δ	0.113 329
$(1+i)^{1/2}$	1.058 301
$(1+i)^{1/4}$	1.028 737
$(1+i)^{1/12}$	1.009 489
v	0.892 857
$v^{1/2}$	0.944 911
$v^{1/4}$	0.972 065
$v^{1/12}$	0.990 600
d	0.107 143
$d^{(2)}$	0.110 178
$d^{(4)}$	0.111 738
$d^{(12)}$	0.112 795
$i / i^{(2)}$	1.029 150
$i / i^{(4)}$	1.043 938
$i / i^{(12)}$	1.053 875
i / δ	1.058 867
$i / d^{(2)}$	1.089 150
$i / d^{(4)}$	1.073 938
$i / d^{(12)}$	1.063 875

Compound Interest

15%

i	0.150 000
$i^{(2)}$	0.144 761
$i^{(4)}$	0.142 232
$i^{(12)}$	0.140 579
δ	0.139 762
$(1+i)^{1/2}$	1.072 381
$(1+i)^{1/4}$	1.035 558
$(1+i)^{1/12}$	1.011 715
v	0.869 565
$v^{1/2}$	0.932 505
$v^{1/4}$	0.965 663
$v^{1/12}$	0.988 421
d	0.130 435
$d^{(2)}$	0.134 990
$d^{(4)}$	0.137 348
$d^{(12)}$	0.138 951
$i/i^{(2)}$	1.036 190
$i/i^{(4)}$	1.054 613
$i/i^{(12)}$	1.067 016
i/δ	1.073 254
$i/d^{(2)}$	1.111 190
$i/d^{(4)}$	1.092 113
$i/d^{(12)}$	1.079 516

n	$(1+i)^n$	v^n	$s_{\overline{n}\|}$	$a_{\overline{n}\|}$	$(Ia)_{\overline{n}\|}$	$(Da)_{\overline{n}\|}$	n
1	1.150 00	0.869 57	1.000 0	0.869 6	0.869 6	0.869 6	1
2	1.322 50	0.756 14	2.150 0	1.625 7	2.381 9	2.495 3	2
3	1.520 88	0.657 52	3.472 5	2.283 2	4.354 4	4.778 5	3
4	1.749 01	0.571 75	4.993 4	2.855 0	6.641 4	7.633 5	4
5	2.011 36	0.497 18	6.742 4	3.352 2	9.127 3	10.985 6	5
6	2.313 06	0.432 33	8.753 7	3.784 5	11.721 3	14.770 1	6
7	2.660 02	0.375 94	11.066 8	4.160 4	14.352 8	18.930 5	7
8	3.059 02	0.326 90	13.726 8	4.487 3	16.968 0	23.417 9	8
9	3.517 88	0.284 26	16.785 8	4.771 6	19.526 4	28.189 4	9
10	4.045 56	0.247 18	20.303 7	5.018 8	21.998 2	33.208 2	10
11	4.652 39	0.214 94	24.349 3	5.233 7	24.362 6	38.441 9	11
12	5.350 25	0.186 91	29.001 7	5.420 6	26.605 5	43.862 5	12
13	6.152 79	0.162 53	34.351 9	5.583 1	28.718 4	49.445 7	13
14	7.075 71	0.141 33	40.504 7	5.724 5	30.697 0	55.170 2	14
15	8.137 06	0.122 89	47.580 4	5.847 4	32.540 4	61.017 5	15
16	9.357 62	0.106 86	55.717 5	5.954 2	34.250 2	66.971 8	16
17	10.761 26	0.092 93	65.075 1	6.047 2	35.830 0	73.018 9	17
18	12.375 45	0.080 81	75.836 4	6.128 0	37.284 5	79.146 9	18
19	14.231 77	0.070 27	88.211 8	6.198 2	38.619 5	85.345 1	19
20	16.366 54	0.061 10	102.443 6	6.259 3	39.841 5	91.604 5	20
21	18.821 52	0.053 13	118.810 1	6.312 5	40.957 2	97.916 9	21
22	21.644 75	0.046 20	137.631 6	6.358 7	41.973 7	104.275 6	22
23	24.891 46	0.040 17	159.276 4	6.398 8	42.897 7	110.674 4	23
24	28.625 18	0.034 93	184.167 8	6.433 8	43.736 1	117.108 2	24
25	32.918 95	0.030 38	212.793 0	6.464 1	44.495 5	123.572 3	25
26	37.856 80	0.026 42	245.712 0	6.490 6	45.182 3	130.062 9	26
27	43.535 31	0.022 97	283.568 8	6.513 5	45.802 5	136.576 4	27
28	50.065 61	0.019 97	327.104 1	6.533 5	46.361 8	143.109 9	28
29	57.575 45	0.017 37	377.169 7	6.550 9	46.865 5	149.660 8	29
30	66.211 77	0.015 10	434.745 1	6.566 0	47.318 6	156.226 8	30
31	76.143 54	0.013 13	500.956 9	6.579 1	47.725 7	162.805 9	31
32	87.565 07	0.011 42	577.100 5	6.590 5	48.091 1	169.396 4	32
33	100.699 83	0.009 93	664.665 5	6.600 5	48.418 8	175.996 9	33
34	115.804 80	0.008 64	765.365 4	6.609 1	48.712 4	182.606 0	34
35	133.175 52	0.007 51	881.170 2	6.616 6	48.975 2	189.222 6	35
36	153.151 85	0.006 53	1 014.345 7	6.623 1	49.210 3	195.845 8	36
37	176.124 63	0.005 68	1 167.497 5	6.628 8	49.420 4	202.474 6	37
38	202.543 32	0.004 94	1 343.622 2	6.633 8	49.608 0	209.108 3	38
39	232.924 82	0.004 29	1 546.165 5	6.638 0	49.775 4	215.746 4	39
40	267.863 55	0.003 73	1 779.090 3	6.641 8	49.924 8	222.388 1	40
41	308.043 08	0.003 25	2 046.953 9	6.645 0	50.057 9	229.033 2	41
42	354.249 54	0.002 82	2 354.996 9	6.647 8	50.176 4	235.681 0	42
43	407.386 97	0.002 45	2 709.246 5	6.650 3	50.282 0	242.331 3	43
44	468.495 02	0.002 13	3 116.633 4	6.652 4	50.375 9	248.983 8	44
45	538.769 27	0.001 86	3 585.128 5	6.654 3	50.459 4	255.638 0	45
46	619.584 66	0.001 61	4 123.897 7	6.655 9	50.533 7	262.294 0	46
47	712.522 36	0.001 40	4 743.482 4	6.657 3	50.599 6	268.951 3	47
48	819.400 71	0.001 22	5 456.004 7	6.658 5	50.658 2	275.609 8	48
49	942.310 82	0.001 06	6 275.405 5	6.659 6	50.710 2	282.269 4	49
50	1 083.657 44	0.000 92	7 217.716 3	6.660 5	50.756 3	288.929 9	50

Compound Interest

20%

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.200 00	0.833 33	1.000 0	0.833 3	0.833 3	0.833 3	1
2	1.440 00	0.694 44	2.200 0	1.527 8	2.222 2	2.361 1	2
3	1.728 00	0.578 70	3.640 0	2.106 5	3.958 3	4.467 6	3
4	2.073 60	0.482 25	5.368 0	2.588 7	5.887 3	7.056 3	4
5	2.488 32	0.401 88	7.441 6	2.990 6	7.896 7	10.046 9	5
6	2.985 98	0.334 90	9.929 9	3.325 5	9.906 1	13.372 4	6
7	3.583 18	0.279 08	12.915 9	3.604 6	11.859 7	16.977 0	7
8	4.299 82	0.232 57	16.499 1	3.837 2	13.720 2	20.814 2	8
9	5.159 78	0.193 81	20.798 9	4.031 0	15.464 5	24.845 2	9
10	6.191 74	0.161 51	25.958 7	4.192 5	17.079 6	29.037 6	10
11	7.430 08	0.134 59	32.150 4	4.327 1	18.560 0	33.364 7	11
12	8.916 10	0.112 16	39.580 5	4.439 2	19.905 9	37.803 9	12
13	10.699 32	0.093 46	48.496 6	4.532 7	21.120 9	42.336 6	13
14	12.839 18	0.077 89	59.195 9	4.610 6	22.211 3	46.947 2	14
15	15.407 02	0.064 91	72.035 1	4.675 5	23.184 9	51.622 6	15
16	18.488 43	0.054 09	87.442 1	4.729 6	24.050 3	56.352 2	16
17	22.186 11	0.045 07	105.930 6	4.774 6	24.816 6	61.126 8	17
18	26.623 33	0.037 56	128.116 7	4.812 2	25.492 7	65.939 0	18
19	31.948 00	0.031 30	154.740 0	4.843 5	26.087 4	70.782 5	19
20	38.337 60	0.026 08	186.688 0	4.869 6	26.609 1	75.652 1	20
21	46.005 12	0.021 74	225.025 6	4.891 3	27.065 5	80.543 4	21
22	55.206 14	0.018 11	271.030 7	4.909 4	27.464 1	85.452 8	22
23	66.247 37	0.015 09	326.236 9	4.924 5	27.811 2	90.377 4	23
24	79.496 85	0.012 58	392.484 2	4.937 1	28.113 1	95.314 5	24
25	95.396 22	0.010 48	471.981 1	4.947 6	28.375 2	100.262 1	25
26	114.475 46	0.008 74	567.377 3	4.956 3	28.602 3	105.218 4	26
27	137.370 55	0.007 28	681.852 8	4.963 6	28.798 9	110.182 0	27
28	164.844 66	0.006 07	819.223 3	4.969 7	28.968 7	115.151 7	28
29	197.813 59	0.005 06	984.068 0	4.974 7	29.115 3	120.126 4	29
30	237.376 31	0.004 21	1 181.881 6	4.978 9	29.241 7	125.105 3	30
31	284.851 58	0.003 51	1 419.257 9	4.982 4	29.350 5	130.087 8	31
32	341.821 89	0.002 93	1 704.109 5	4.985 4	29.444 2	135.073 1	32
33	410.186 27	0.002 44	2 045.931 4	4.987 8	29.524 6	140.060 9	33
34	492.223 52	0.002 03	2 456.117 6	4.989 8	29.593 7	145.050 8	34
35	590.668 23	0.001 69	2 948.341 1	4.991 5	29.652 9	150.042 3	35
36	708.801 87	0.001 41	3 539.009 4	4.992 9	29.703 7	155.035 3	36
37	850.562 25	0.001 18	4 247.811 2	4.994 1	29.747 2	160.029 4	37
38	1 020.674 70	0.000 98	5 098.373 5	4.995 1	29.784 5	165.024 5	38
39	1 224.809 64	0.000 82	6 119.048 2	4.995 9	29.816 3	170.020 4	39
40	1 469.771 57	0.000 68	7 343.857 8	4.996 6	29.843 5	175.017 0	40
41	1 763.725 88	0.000 57	8 813.629 4	4.997 2	29.866 8	180.014 2	41
42	2 116.471 06	0.000 47	10 577.355 3	4.997 6	29.886 6	185.011 8	42
43	2 539.765 27	0.000 39	12 693.826 3	4.998 0	29.903 5	190.009 8	43
44	3 047.718 32	0.000 33	15 233.591 6	4.998 4	29.918 0	195.008 2	44
45	3 657.261 99	0.000 27	18 281.309 9	4.998 6	29.930 3	200.006 8	45
46	4 388.714 39	0.000 23	21 938.571 9	4.998 9	29.940 8	205.005 7	46
47	5 266.457 26	0.000 19	26 327.286 3	4.999 1	29.949 7	210.004 7	47
48	6 319.748 72	0.000 16	31 593.743 6	4.999 2	29.957 3	215.004 0	48
49	7 583.698 46	0.000 13	37 913.492 3	4.999 3	29.963 7	220.003 3	49
50	9 100.438 15	0.000 11	45 497.190 8	4.999 5	29.969 2	225.002 7	50

i	0.200 000
$i^{(2)}$	0.190 890
$i^{(4)}$	0.186 541
$i^{(12)}$	0.183 714
δ	0.182 322
$(1+i)^{1/2}$	1.095 445
$(1+i)^{1/4}$	1.046 635
$(1+i)^{1/12}$	1.015 309
v	0.833 333
$v^{1/2}$	0.912 871
$v^{1/4}$	0.955 443
$v^{1/12}$	0.984 921
d	0.166 667
$d^{(2)}$	0.174 258
$d^{(4)}$	0.178 229
$d^{(12)}$	0.180 943
$i/i^{(2)}$	1.047 723
$i/i^{(4)}$	1.072 153
$i/i^{(12)}$	1.088 651
i/δ	1.096 963
$i/d^{(2)}$	1.147 723
$i/d^{(4)}$	1.122 153
$i/d^{(12)}$	1.105 317

Compound Interest

25%

i	0.250 000
$i^{(2)}$	0.236 068
$i^{(4)}$	0.229 485
$i^{(12)}$	0.225 231
δ	0.223 144
$(1+i)^{1/2}$	1.118 034
$(1+i)^{1/4}$	1.057 371
$(1+i)^{1/12}$	1.018 769
v	0.800 000
$v^{1/2}$	0.894 427
$v^{1/4}$	0.945 742
$v^{1/12}$	0.981 577
d	0.200 000
$d^{(2)}$	0.211 146
$d^{(4)}$	0.217 034
$d^{(12)}$	0.221 082
$i/i^{(2)}$	1.059 017
$i/i^{(4)}$	1.089 396
$i/i^{(12)}$	1.109 971
i/δ	1.120 355
$i/d^{(2)}$	1.184 017
$i/d^{(4)}$	1.151 896
$i/d^{(12)}$	1.130 804

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.250 00	0.800 00	1.000 0	0.800 0	0.800 0	0.800 0	1
2	1.562 50	0.640 00	2.250 0	1.440 0	2.080 0	2.240 0	2
3	1.953 13	0.512 00	3.812 5	1.952 0	3.616 0	4.192 0	3
4	2.441 41	0.409 60	5.765 6	2.361 6	5.254 4	6.553 6	4
5	3.051 76	0.327 68	8.207 0	2.689 3	6.892 8	9.242 9	5
6	3.814 70	0.262 14	11.258 8	2.951 4	8.465 7	12.194 3	6
7	4.768 37	0.209 72	15.073 5	3.161 1	9.933 7	15.355 4	7
8	5.960 46	0.167 77	19.841 9	3.328 9	11.275 8	18.684 4	8
9	7.450 58	0.134 22	25.802 3	3.463 1	12.483 8	22.147 5	9
10	9.313 23	0.107 37	33.252 9	3.570 5	13.557 5	25.718 0	10
11	11.641 53	0.085 90	42.566 1	3.656 4	14.502 4	29.374 4	11
12	14.551 92	0.068 72	54.207 7	3.725 1	15.327 1	33.099 5	12
13	18.189 89	0.054 98	68.759 6	3.780 1	16.041 8	36.879 6	13
14	22.737 37	0.043 98	86.949 5	3.824 1	16.657 5	40.703 7	14
15	28.421 71	0.035 18	109.686 8	3.859 3	17.185 3	44.562 9	15
16	35.527 14	0.028 15	138.108 5	3.887 4	17.635 6	48.450 4	16
17	44.408 92	0.022 52	173.635 7	3.909 9	18.018 4	52.360 3	17
18	55.511 15	0.018 01	218.044 6	3.927 9	18.342 7	56.288 2	18
19	69.388 94	0.014 41	273.555 8	3.942 4	18.616 5	60.230 6	19
20	86.736 17	0.011 53	342.944 7	3.953 9	18.847 1	64.184 5	20
21	108.420 22	0.009 22	429.680 9	3.963 1	19.040 8	68.147 6	21
22	135.525 27	0.007 38	538.101 1	3.970 5	19.203 1	72.118 1	22
23	169.406 59	0.005 90	673.626 4	3.976 4	19.338 9	76.094 4	23
24	211.758 24	0.004 72	843.032 9	3.981 1	19.452 2	80.075 6	24
25	264.697 80	0.003 78	1 054.791 2	3.984 9	19.546 7	84.060 4	25
26	330.872 25	0.003 02	1 319.489 0	3.987 9	19.625 2	88.048 4	26
27	413.590 31	0.002 42	1 650.361 2	3.990 3	19.690 5	92.038 7	27
28	516.987 88	0.001 93	2 063.951 5	3.992 3	19.744 7	96.030 9	28
29	646.234 85	0.001 55	2 580.939 4	3.993 8	19.789 6	100.024 8	29
30	807.793 57	0.001 24	3 227.174 3	3.995 0	19.826 7	104.019 8	30
31	1 009.741 96	0.000 99	4 034.967 8	3.996 0	19.857 4	108.015 8	31
32	1 262.177 45	0.000 79	5 044.709 8	3.996 8	19.882 7	112.012 7	32
33	1 577.721 81	0.000 63	6 306.887 2	3.997 5	19.903 7	116.010 1	33
34	1 972.152 26	0.000 51	7 884.609 1	3.998 0	19.920 9	120.008 1	34
35	2 465.190 33	0.000 41	9 856.761 3	3.998 4	19.935 1	124.006 5	35
36	3 081.487 91	0.000 32	12 321.951 6	3.998 7	19.946 8	128.005 2	36
37	3 851.859 89	0.000 26	15 403.439 6	3.999 0	19.956 4	132.004 2	37
38	4 814.824 86	0.000 21	19 255.299 4	3.999 2	19.964 3	136.003 3	38
39	6 018.531 08	0.000 17	24 070.124 3	3.999 3	19.970 8	140.002 7	39
40	7 523.163 85	0.000 13	30 088.655 4	3.999 5	19.976 1	144.002 1	40
41	9 403.954 81	0.000 11	37 611.819 2	3.999 6	19.980 4	148.001 7	41
42	11 754.943 51	0.000 09	47 015.774 0	3.999 7	19.984 0	152.001 4	42
43	14 693.679 39	0.000 07	58 770.717 5	3.999 7	19.986 9	156.001 1	43
44	18 367.099 23	0.000 05	73 464.396 9	3.999 8	19.989 3	160.000 9	44
45	22 958.874 04	0.000 04	91 831.496 2	3.999 8	19.991 3	164.000 7	45
46	28 698.592 55	0.000 03	114 790.370 2	3.999 9	19.992 9	168.000 6	46
47	35 873.240 69	0.000 03	143 488.962 7	3.999 9	19.994 2	172.000 4	47
48	44 841.550 86	0.000 02	179 362.203 4	3.999 9	19.995 3	176.000 4	48
49	56 051.938 57	0.000 02	224 203.754 3	3.999 9	19.996 1	180.000 3	49
50	70 064.923 22	0.000 01	280 255.692 9	3.999 9	19.996 9	184.000 2	50

POPULATION MORTALITY TABLE

ELT15 (Males) and ELT15 (Females)

This table is based on the mortality of the population of England and Wales during the years 1990, 1991, and 1992. Full details are given in *English Life Tables No. 15* published by The Stationery Office.

Note that no μ_0 values have been included because of the difficulty of calculating reasonable estimates from observed data.

ELT15 (Males)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
0	100 000	814	0.008 14		73.413	0
1	99 186	62	0.000 62	0.000 80	73.019	1
2	99 124	38	0.000 38	0.000 43	72.064	2
3	99 086	30	0.000 30	0.000 33	71.091	3
4	99 056	24	0.000 24	0.000 27	70.113	4
5	99 032	22	0.000 22	0.000 23	69.130	5
6	99 010	20	0.000 20	0.000 21	68.145	6
7	98 990	18	0.000 19	0.000 19	67.158	7
8	98 972	19	0.000 18	0.000 18	66.171	8
9	98 953	18	0.000 18	0.000 18	65.183	9
10	98 935	18	0.000 18	0.000 18	64.195	10
11	98 917	18	0.000 18	0.000 18	63.206	11
12	98 899	19	0.000 19	0.000 19	62.218	12
13	98 880	23	0.000 23	0.000 21	61.230	13
14	98 857	29	0.000 29	0.000 26	60.244	14
15	98 828	39	0.000 40	0.000 34	59.261	15
16	98 789	52	0.000 52	0.000 45	58.285	16
17	98 737	74	0.000 75	0.000 64	57.315	17
18	98 663	86	0.000 87	0.000 83	56.358	18
19	98 577	81	0.000 83	0.000 85	55.406	19
20	98 496	83	0.000 84	0.000 83	54.452	20
21	98 413	85	0.000 86	0.000 85	53.497	21
22	98 328	87	0.000 89	0.000 88	52.543	22
23	98 241	87	0.000 89	0.000 89	51.589	23
24	98 154	87	0.000 88	0.000 89	50.635	24
25	98 067	84	0.000 86	0.000 87	49.679	25
26	97 983	83	0.000 85	0.000 85	48.721	26
27	97 900	83	0.000 85	0.000 84	47.762	27
28	97 817	85	0.000 87	0.000 86	46.802	28
29	97 732	87	0.000 90	0.000 88	45.842	29
30	97 645	89	0.000 91	0.000 90	44.883	30
31	97 556	91	0.000 94	0.000 92	43.923	31
32	97 465	95	0.000 97	0.000 96	42.964	32
33	97 370	97	0.000 99	0.000 98	42.005	33
34	97 273	103	0.001 06	0.001 02	41.046	34
35	97 170	113	0.001 16	0.001 11	40.090	35
36	97 057	124	0.001 27	0.001 22	39.136	36
37	96 933	133	0.001 38	0.001 33	38.185	37
38	96 800	145	0.001 49	0.001 44	37.237	38
39	96 655	155	0.001 60	0.001 55	36.292	39
40	96 500	166	0.001 72	0.001 66	35.349	40
41	96 334	179	0.001 86	0.001 79	34.409	41
42	96 155	194	0.002 01	0.001 93	33.473	42
43	95 961	210	0.002 19	0.002 10	32.539	43
44	95 751	230	0.002 40	0.002 29	31.609	44
45	95 521	255	0.002 66	0.002 53	30.684	45
46	95 266	283	0.002 97	0.002 81	29.765	46
47	94 983	315	0.003 32	0.003 14	28.852	47
48	94 668	352	0.003 71	0.003 52	27.947	48
49	94 316	391	0.004 15	0.003 93	27.049	49
50	93 925	436	0.004 64	0.004 40	26.159	50
51	93 489	485	0.005 19	0.004 92	25.279	51
52	93 004	537	0.005 77	0.005 49	24.408	52
53	92 467	594	0.006 42	0.006 10	23.547	53
54	91 873	656	0.007 14	0.006 79	22.696	54

ELT15 (Males)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
55	91 217	727	0.007 97	0.007 57	21.856	55
56	90 490	806	0.008 90	0.008 45	21.027	56
57	89 684	892	0.009 95	0.009 45	20.211	57
58	88 792	987	0.011 12	0.010 57	19.409	58
59	87 805	1 091	0.012 43	0.011 82	18.622	59
60	86 714	1 207	0.013 92	0.013 23	17.850	60
61	85 507	1 334	0.015 60	0.014 83	17.095	61
62	84 173	1 472	0.017 49	0.016 64	16.357	62
63	82 701	1 625	0.019 65	0.018 70	15.640	63
64	81 076	1 783	0.021 99	0.021 01	14.943	64
65	79 293	1 940	0.024 47	0.023 48	14.267	65
66	77 353	2 097	0.027 11	0.026 10	13.612	66
67	75 256	2 255	0.029 97	0.028 93	12.978	67
68	73 001	2 403	0.032 92	0.031 92	12.363	68
69	70 598	2 543	0.036 02	0.035 05	11.767	69
70	68 055	2 674	0.039 30	0.038 33	11.187	70
71	65 381	2 819	0.043 11	0.041 98	10.624	71
72	62 562	2 969	0.047 45	0.046 26	10.080	72
73	59 593	3 109	0.052 17	0.051 05	9.557	73
74	56 484	3 218	0.056 97	0.056 09	9.056	74
75	53 266	3 301	0.061 97	0.061 23	8.572	75
76	49 965	3 386	0.067 77	0.066 94	8.106	76
77	46 579	3 455	0.074 18	0.073 52	7.658	77
78	43 124	3 494	0.081 01	0.080 68	7.232	78
79	39 630	3 502	0.088 38	0.088 40	6.825	79
80	36 128	3 474	0.096 16	0.096 75	6.438	80
81	32 654	3 400	0.104 11	0.105 44	6.070	81
82	29 254	3 300	0.112 79	0.114 64	5.718	82
83	25 954	3 175	0.122 35	0.124 91	5.382	83
84	22 779	3 023	0.132 70	0.136 27	5.063	84
85	19 756	2 839	0.143 72	0.148 57	4.762	85
86	16 917	2 637	0.155 85	0.162 08	4.478	86
87	14 280	2 406	0.168 48	0.176 89	4.213	87
88	11 874	2 144	0.180 61	0.191 90	3.968	88
89	9 730	1 873	0.192 46	0.206 47	3.734	89
90	7 857	1 608	0.204 65	0.221 14	3.508	90
91	6 249	1 369	0.219 11	0.237 54	3.285	91
92	4 880	1 154	0.236 55	0.257 93	3.071	92
93	3 726	953	0.255 75	0.282 26	2.872	93
94	2 773	762	0.274 83	0.308 37	2.693	94
95	2 011	590	0.293 11	0.334 24	2.531	95
96	1 421	442	0.311 04	0.359 74	2.383	96
97	979	322	0.329 19	0.385 79	2.244	97
98	657	229	0.347 83	0.413 13	2.114	98
99	428	157	0.367 12	0.442 16	1.991	99
100	271	105	0.387 05	0.473 12	1.874	100
101	166	68	0.407 60	0.506 09	1.764	101
102	98	42	0.428 70	0.541 17	1.660	102
103	56	25	0.450 30	0.578 32	1.562	103
104	31	15	0.474 28	0.619 01	1.468	104
105	16	8	0.496 34	0.664 18	1.384	105
106	8	4	0.518 41	0.706 30	1.306	106
107	4	2	0.540 41	0.751 11	1.234	107
108	2	1	0.562 25	0.797 41	1.166	108
109	1	1	0.583 85	0.844 99	1.104	109

ELT15 (Females)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
0	100 000	632	0.006 32		78.956	0
1	99 368	55	0.000 55	0.000 73	78.462	1
2	99 313	30	0.000 30	0.000 35	77.505	2
3	99 283	22	0.000 22	0.000 25	76.528	3
4	99 261	18	0.000 18	0.000 20	75.545	4
5	99 243	15	0.000 16	0.000 17	74.559	5
6	99 228	15	0.000 15	0.000 15	73.570	6
7	99 213	14	0.000 14	0.000 14	72.581	7
8	99 199	14	0.000 14	0.000 14	71.591	8
9	99 185	13	0.000 13	0.000 14	70.601	9
10	99 172	13	0.000 13	0.000 13	69.610	10
11	99 159	14	0.000 14	0.000 14	68.620	11
12	99 145	14	0.000 14	0.000 14	67.629	12
13	99 131	15	0.000 15	0.000 14	66.638	13
14	99 116	18	0.000 18	0.000 17	65.649	14
15	99 098	21	0.000 22	0.000 20	64.660	15
16	99 077	26	0.000 26	0.000 24	63.674	16
17	99 051	31	0.000 31	0.000 29	62.691	17
18	99 020	31	0.000 31	0.000 31	61.710	18
19	98 989	32	0.000 32	0.000 32	60.729	19
20	98 957	31	0.000 31	0.000 32	59.748	20
21	98 926	32	0.000 32	0.000 32	58.767	21
22	98 894	32	0.000 33	0.000 32	57.786	22
23	98 862	33	0.000 33	0.000 33	56.805	23
24	98 829	32	0.000 33	0.000 33	55.823	24
25	98 797	34	0.000 34	0.000 33	54.842	25
26	98 763	34	0.000 35	0.000 34	53.860	26
27	98 729	35	0.000 36	0.000 35	52.878	27
28	98 694	38	0.000 38	0.000 37	51.897	28
29	98 656	39	0.000 40	0.000 39	50.917	29
30	98 617	43	0.000 43	0.000 42	49.937	30
31	98 574	46	0.000 47	0.000 45	48.958	31
32	98 528	51	0.000 52	0.000 50	47.981	32
33	98 477	57	0.000 57	0.000 54	47.006	33
34	98 420	61	0.000 63	0.000 60	46.032	34
35	98 359	68	0.000 69	0.000 66	45.061	35
36	98 291	74	0.000 75	0.000 72	44.092	36
37	98 217	81	0.000 82	0.000 79	43.124	37
38	98 136	88	0.000 90	0.000 86	42.160	38
39	98 048	96	0.000 98	0.000 94	41.197	39
40	97 952	105	0.001 07	0.001 02	40.237	40
41	97 847	114	0.001 17	0.001 12	39.279	41
42	97 733	126	0.001 29	0.001 23	38.325	42
43	97 607	138	0.001 42	0.001 35	37.374	43
44	97 469	154	0.001 58	0.001 49	36.426	44
45	97 315	173	0.001 77	0.001 67	35.483	45
46	97 142	192	0.001 98	0.001 87	34.545	46
47	96 950	212	0.002 19	0.002 08	33.612	47
48	96 738	234	0.002 41	0.002 30	32.685	48
49	96 504	257	0.002 66	0.002 53	31.763	49
50	96 247	283	0.002 94	0.002 80	30.846	50
51	95 964	312	0.003 26	0.003 10	29.936	51
52	95 652	342	0.003 57	0.003 42	29.032	52
53	95 310	372	0.003 90	0.003 74	28.134	53
54	94 938	406	0.004 28	0.004 08	27.242	54

ELT15 (Females)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
55	94 532	450	0.004 75	0.004 51	26.357	55
56	94 082	499	0.005 31	0.005 03	25.481	56
57	93 583	554	0.005 92	0.005 62	24.614	57
58	93 029	614	0.006 60	0.006 26	23.757	58
59	92 415	683	0.007 39	0.007 00	22.912	59
60	91 732	761	0.008 30	0.007 86	22.079	60
61	90 971	839	0.009 22	0.008 80	21.259	61
62	90 132	915	0.010 15	0.009 72	20.452	62
63	89 217	1007	0.011 29	0.010 74	19.657	63
64	88 210	1117	0.012 66	0.012 03	18.875	64
65	87 093	1218	0.013 99	0.013 42	18.111	65
66	85 875	1308	0.015 23	0.014 70	17.361	66
67	84 567	1417	0.016 76	0.016 09	16.621	67
68	83 150	1533	0.018 44	0.017 74	15.896	68
69	81 617	1647	0.020 17	0.019 49	15.185	69
70	79 970	1751	0.021 90	0.021 23	14.487	70
71	78 219	1876	0.023 99	0.023 11	13.800	71
72	76 343	2056	0.026 93	0.025 69	13.127	72
73	74 287	2239	0.030 14	0.028 97	12.476	73
74	72 048	2366	0.032 84	0.032 03	11.848	74
75	69 682	2487	0.035 69	0.034 80	11.234	75
76	67 195	2634	0.039 19	0.038 03	10.631	76
77	64 561	2812	0.043 56	0.042 14	10.044	77
78	61 749	2984	0.048 33	0.046 94	9.478	78
79	58 765	3158	0.053 73	0.052 28	8.934	79
80	55 607	3314	0.059 61	0.058 27	8.413	80
81	52 293	3435	0.065 68	0.064 64	7.914	81
82	48 858	3526	0.072 16	0.071 31	7.435	82
83	45 332	3596	0.079 33	0.078 61	6.974	83
84	41 736	3655	0.087 57	0.086 91	6.532	84
85	38 081	3706	0.097 31	0.096 74	6.111	85
86	34 375	3724	0.108 33	0.108 41	5.715	86
87	30 651	3634	0.118 59	0.120 52	5.349	87
88	27 017	3475	0.128 60	0.131 74	5.002	88
89	23 542	3330	0.141 46	0.144 62	4.667	89
90	20 212	3143	0.155 50	0.160 53	4.354	90
91	17 069	2903	0.170 06	0.177 51	4.065	91
92	14 166	2631	0.185 73	0.195 73	3.797	92
93	11 535	2321	0.201 26	0.214 98	3.551	93
94	9 214	2008	0.217 90	0.234 90	3.322	94
95	7 206	1702	0.236 19	0.257 32	3.112	95
96	5 504	1395	0.253 44	0.281 14	2.925	96
97	4 109	1102	0.268 20	0.302 67	2.754	97
98	3 007	853	0.283 52	0.322 41	2.588	98
99	2 154	653	0.303 31	0.346 28	2.422	99
100	1 501	488	0.324 89	0.376 71	2.269	100
101	1 013	350	0.345 62	0.408 87	2.133	101
102	663	240	0.361 86	0.437 69	2.011	102
103	423	161	0.379 92	0.462 73	1.887	103
104	262	105	0.400 45	0.493 00	1.758	104
105	157	68	0.436 18	0.537 29	1.621	105
106	89	41	0.459 94	0.599 08	1.518	106
107	48	23	0.483 89	0.637 85	1.425	107
108	25	13	0.507 91	0.683 88	1.338	108
109	12	6	0.531 90	0.731 91	1.257	109
110	6	3	0.555 74	0.781 81	1.183	110
111	3	2	0.579 32	0.833 37	1.114	111
112	1	1	0.602 55	0.886 29	1.050	112

ASSURED LIVES MORTALITY TABLE

AM92

This table is based on the mortality of assured male lives in the UK during the years 1991, 1992, 1993, and 1994. Full details are given in *C.M.I.R.* **17**.

Due to potential rounding errors at high ages, the commutation functions (D_x, N_x, S_x, C_x, M_x and R_x) are tabulated here to age 110 only.

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
17	9 997.809 1		10 000.000 0	17
18	9 991.890 4	9 993.540 0	9 994.000 0	18
19	9 986.035 1	9 987.633 8	9 988.063 6	19
20	9 980.243 2	9 981.791 1	9 982.200 6	20
21	9 974.504 6	9 976.001 6	9 976.390 9	21
22	9 968.839 1	9 970.265 4	9 970.634 6	22
23	9 963.196 7	9 964.582 4	9 964.931 3	23
24	9 957.577 5	9 958.922 5	9 959.261 3	24
25	9 951.991 3	9 953.285 8	9 953.614 4	25
26	9 946.398 2	9 947.662 2	9 947.980 7	26
27	9 940.798 4	9 942.021 8	9 942.340 2	27
28	9 935.181 8	9 936.354 9	9 936.673 0	28
29	9 929.508 8	9 930.661 3	9 930.969 4	29
30	9 923.749 7	9 924.891 6	9 925.209 4	30
31	9 917.914 5	9 919.026 0	9 919.353 5	31
32	9 911.953 8	9 913.054 7	9 913.382 1	32
33	9 905.828 2	9 906.928 5	9 907.265 5	33
34	9 899.498 4	9 900.607 8	9 900.964 5	34
35	9 892.915 1	9 894.053 6	9 894.429 9	35
36	9 886.039 5	9 887.206 9	9 887.612 6	36
37	9 878.812 8	9 880.028 8	9 880.454 0	37
38	9 871.166 5	9 872.450 8	9 872.895 4	38
39	9 863.022 7	9 864.404 7	9 864.868 8	39
40	9 854.303 6	9 855.793 1	9 856.286 3	40
41	9 844.902 5	9 846.538 4	9 847.051 0	41
42	9 834.703 0	9 836.524 5	9 837.066 1	42
43	9 823.599 4	9 825.635 4	9 826.206 0	43
44	9 811.447 3	9 813.746 3	9 814.335 9	44
45	9 798.083 7	9 800.693 9	9 801.312 3	45
46	9 783.337 1	9 786.316 2	9 786.953 4	46
47	9 766.998 3	9 770.423 1	9 771.078 9	47
48	9 748.860 3	9 752.787 4	9 753.471 4	48
49	9 728.649 9	9 733.193 8	9 733.886 5	49
50	9 706.097 7	9 711.352 4	9 712.072 8	50
51	9 680.899 0	9 686.966 9	9 687.714 9	51
52	9 652.696 5	9 659.707 5	9 660.502 1	52
53	9 621.100 6	9 629.211 5	9 630.052 2	53
54	9 585.691 6	9 595.056 3	9 595.971 5	54
55	9 545.992 9	9 556.800 3	9 557.817 9	55
56	9 501.483 9	9 513.937 5	9 515.104 0	56
57	9 451.593 8	9 465.929 3	9 467.290 6	57
58	9 395.697 1	9 412.171 2	9 413.800 4	58
59	9 333.128 4	9 352.016 5	9 354.004 0	59
60	9 263.142 2	9 284.764 1	9 287.216 4	60
61	9 184.968 7	9 209.656 8	9 212.714 3	61
62	9 097.740 5	9 125.881 8	9 129.717 0	62
63	9 000.588 4	9 032.564 2	9 037.397 3	63
64	8 892.574 1	8 928.817 7	8 934.877 1	64

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
65	8 772.735 9	8 813.688 1	8 821.261 2	65
66	8 640.048 1	8 686.201 6	8 695.619 9	66
67	8 493.518 7	8 545.353 2	8 557.011 8	67
68	8 332.139 6	8 390.161 1	8 404.491 6	68
69	8 154.931 8	8 219.639 0	8 237.132 9	69
70	7 960.977 6	8 032.860 6	8 054.054 4	70
71	7 749.465 9	7 828.968 6	7 854.450 8	71
72	7 519.702 7	7 607.240 0	7 637.620 8	72
73	7 271.146 1	7 367.082 8	7 403.008 4	73
74	7 003.521 6	7 108.105 2	7 150.240 1	74
75	6 716.823 1	6 830.184 4	6 879.167 3	75
76	6 411.345 9	6 533.500 8	6 589.925 8	76
77	6 087.808 4	6 218.575 9	6 282.980 3	77
78	5 747.362 4	5 886.362 8	5 959.168 0	78
79	5 391.640 0	5 538.279 1	5 619.757 7	79
80	5 022.793 1	5 176.222 4	5 266.460 4	80
81	4 643.512 9	4 802.629 0	4 901.478 9	81
82	4 257.005 6	4 420.452 5	4 527.496 0	82
83	3 866.988 4	4 033.146 7	4 147.670 8	83
84	3 477.592 9	3 644.632 7	3 765.599 8	84
85	3 093.286 3	3 259.186 2	3 385.247 9	85
86	2 718.712 8	2 881.346 7	3 010.839 5	86
87	2 358.529 9	2 515.731 0	2 646.741 6	87
88	2 017.229 8	2 166.880 5	2 297.297 6	88
89	1 698.908 9	1 839.045 8	1 966.649 9	89
90	1 407.055 0	1 535.980 1	1 658.554 5	90
91		1 260.735 4	1 376.190 6	91
92			1 121.988 9	92
93			897.502 5	93
94			703.324 2	94
95			539.064 3	95
96			403.402 3	96
97			294.206 1	97
98			208.706 0	98
99			143.712 0	99
100			95.847 6	100
101			61.773 3	101
102			38.379 6	102
103			22.928 4	103
104			13.135 9	104
105			7.196 8	105
106			3.759 6	106
107			1.866 9	107
108			0.878 4	108
109			0.390 3	109
110			0.163 2	110
111			0.064 0	111
112			0.023 4	112
113			0.008 0	113
114			0.002 5	114
115			0.000 7	115
116			0.000 2	116
117			0.000 0	117
118			0.000 0	118
119			0.000 0	119
120			0.000 0	120

AM92

x	$d_{[x]}$	$d_{[x-1]+1}$	d_x	x
17	4.269 1		6.000 0	17
18	4.256 5	5.476 5	5.936 4	18
19	4.244 1	5.433 3	5.863 0	19
20	4.241 6	5.400 1	5.809 6	20
21	4.239 2	5.367 1	5.756 4	21
22	4.256 7	5.334 1	5.703 2	22
23	4.274 2	5.321 1	5.670 0	23
24	4.291 7	5.308 1	5.646 9	24
25	4.329 1	5.305 1	5.633 7	25
26	4.376 4	5.322 0	5.640 5	26
27	4.443 5	5.348 8	5.667 1	27
28	4.520 5	5.385 5	5.703 7	28
29	4.617 2	5.451 9	5.760 0	29
30	4.723 7	5.538 1	5.855 9	30
31	4.859 8	5.643 9	5.971 5	31
32	5.025 4	5.789 2	6.116 6	32
33	5.220 4	5.964 0	6.301 0	33
34	5.444 7	6.178 0	6.534 6	34
35	5.708 2	6.441 0	6.817 3	35
36	6.010 7	6.753 0	7.158 6	36
37	6.362 0	7.133 4	7.558 5	37
38	6.761 7	7.582 0	8.026 7	38
39	7.229 6	8.118 4	8.582 4	39
40	7.765 2	8.742 1	9.235 3	40
41	8.378 0	9.472 4	9.984 9	41
42	9.067 6	10.318 5	10.860 1	42
43	9.853 1	11.299 5	11.870 1	43
44	10.753 3	12.434 0	13.023 6	44
45	11.767 5	13.740 6	14.358 9	45
46	12.914 0	15.237 3	15.874 4	46
47	14.211 0	16.951 7	17.607 5	47
48	15.666 4	18.900 9	19.585 0	48
49	17.297 5	21.121 0	21.813 6	49
50	19.130 7	23.637 4	24.357 9	50
51	21.191 5	26.464 8	27.212 8	51
52	23.485 0	29.655 3	30.449 9	52
53	26.044 3	33.240 0	34.080 8	53
54	28.891 3	37.238 4	38.153 6	54
55	32.055 4	41.696 3	42.713 9	55
56	35.554 6	46.646 8	47.813 4	56
57	39.422 6	52.128 9	53.490 2	57
58	43.680 6	58.167 2	59.796 5	58
59	48.364 3	64.800 1	66.787 6	59
60	53.485 4	72.049 8	74.502 0	60
61	59.086 9	79.939 8	82.997 3	61
62	65.176 2	88.484 6	92.319 7	62
63	71.770 7	97.687 2	102.520 2	63
64	78.886 0	107.556 5	113.615 9	64

AM92

x	$d_{[x]}$	$d_{[x-1]+1}$	d_x	x
65	86.534 3	118.068 2	125.641 2	65
66	94.694 9	129.189 9	138.608 2	66
67	103.357 6	140.861 6	152.520 2	67
68	112.500 5	153.028 1	167.358 6	68
69	122.071 2	165.584 6	183.078 5	69
70	132.008 9	178.409 8	199.603 6	70
71	142.225 9	191.347 8	216.830 0	71
72	152.619 9	204.231 6	234.612 4	72
73	163.040 9	216.842 7	252.768 3	73
74	173.337 2	228.937 9	271.072 8	74
75	183.322 3	240.258 6	289.241 5	75
76	192.769 9	250.520 6	306.945 6	76
77	201.445 6	259.407 9	323.812 2	77
78	209.083 3	266.605 1	339.410 4	78
79	215.417 6	271.818 7	353.297 3	79
80	220.164 1	274.743 5	364.981 5	80
81	223.060 4	275.133 0	373.982 8	81
82	223.858 9	272.781 7	379.825 2	82
83	222.355 7	267.546 8	382.071 0	83
84	218.406 7	259.384 9	380.351 9	84
85	211.939 6	248.346 7	374.408 4	85
86	202.981 8	234.605 0	364.097 8	86
87	191.649 4	218.433 4	349.444 0	87
88	178.183 9	200.230 6	330.647 8	88
89	162.928 8	180.491 3	308.095 4	89
90	146.319 7	159.789 5	282.363 9	90
91		138.746 4	254.201 7	91
92			224.486 4	92
93			194.178 3	93
94			164.260 0	94
95			135.662 0	95
96			109.196 2	96
97			85.500 1	97
98			64.994 0	98
99			47.864 4	99
100			34.074 3	100
101			23.393 7	101
102			15.451 2	102
103			9.792 5	103
104			5.939 1	104
105			3.437 3	105
106			1.892 7	106
107			.988 5	107
108			.488 1	108
109			.227 1	109
110			.099 2	110
111			.040 5	111
112			.015 4	112
113			.005 5	113
114			.001 8	114
115			.000 5	115
116			.000 1	116
117			.000 0	117
118			.000 0	118
119			.000 0	119
120			.000 0	120

AM92

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x
17	.000 427		.000 600	17
18	.000 426	.000 548	.000 594	18
19	.000 425	.000 544	.000 587	19
20	.000 425	.000 541	.000 582	20
21	.000 425	.000 538	.000 577	21
22	.000 427	.000 535	.000 572	22
23	.000 429	.000 534	.000 569	23
24	.000 431	.000 533	.000 567	24
25	.000 435	.000 533	.000 566	25
26	.000 440	.000 535	.000 567	26
27	.000 447	.000 538	.000 570	27
28	.000 455	.000 542	.000 574	28
29	.000 465	.000 549	.000 580	29
30	.000 476	.000 558	.000 590	30
31	.000 490	.000 569	.000 602	31
32	.000 507	.000 584	.000 617	32
33	.000 527	.000 602	.000 636	33
34	.000 550	.000 624	.000 660	34
35	.000 577	.000 651	.000 689	35
36	.000 608	.000 683	.000 724	36
37	.000 644	.000 722	.000 765	37
38	.000 685	.000 768	.000 813	38
39	.000 733	.000 823	.000 870	39
40	.000 788	.000 887	.000 937	40
41	.000 851	.000 962	.001 014	41
42	.000 922	.001 049	.001 104	42
43	.001 003	.001 150	.001 208	43
44	.001 096	.001 267	.001 327	44
45	.001 201	.001 402	.001 465	45
46	.001 320	.001 557	.001 622	46
47	.001 455	.001 735	.001 802	47
48	.001 607	.001 938	.002 008	48
49	.001 778	.002 170	.002 241	49
50	.001 971	.002 434	.002 508	50
51	.002 189	.002 732	.002 809	51
52	.002 433	.003 070	.003 152	52
53	.002 707	.003 452	.003 539	53
54	.003 014	.003 881	.003 976	54
55	.003 358	.004 363	.004 469	55
56	.003 742	.004 903	.005 025	56
57	.004 171	.005 507	.005 650	57
58	.004 649	.006 180	.006 352	58
59	.005 182	.006 929	.007 140	59
60	.005 774	.007 760	.008 022	60
61	.006 433	.008 680	.009 009	61
62	.007 164	.009 696	.010 112	62
63	.007 974	.010 815	.011 344	63
64	.008 871	.012 046	.012 716	64

AM92

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x
65	.009 864	.013 396	.014 243	65
66	.010 960	.014 873	.015 940	66
67	.012 169	.016 484	.017 824	67
68	.013 502	.018 239	.019 913	68
69	.014 969	.020 145	.022 226	69
70	.016 582	.022 210	.024 783	70
71	.018 353	.024 441	.027 606	71
72	.020 296	.026 847	.030 718	72
73	.022 423	.029 434	.034 144	73
74	.024 750	.032 208	.037 911	74
75	.027 293	.035 176	.042 046	75
76	.030 067	.038 344	.046 578	76
77	.033 090	.041 715	.051 538	77
78	.036 379	.045 292	.056 956	78
79	.039 954	.049 080	.062 867	79
80	.043 833	.053 078	.069 303	80
81	.048 037	.057 288	.076 300	81
82	.052 586	.061 709	.083 893	82
83	.057 501	.066 337	.092 117	83
84	.062 804	.071 169	.101 007	84
85	.068 516	.076 199	.110 600	85
86	.074 661	.081 422	.120 929	86
87	.081 258	.086 827	.132 028	87
88	.088 331	.092 405	.143 929	88
89	.095 902	.098 144	.156 660	89
90	.103 990	.104 031	.170 247	90
91		.110 052	.184 714	91
92			.200 079	92
93			.216 354	93
94			.233 548	94
95			.251 662	95
96			.270 688	96
97			.290 613	97
98			.311 414	98
99			.333 058	99
100			.355 505	100
101			.378 702	101
102			.402 588	102
103			.427 090	103
104			.452 127	104
105			.477 608	105
106			.503 432	106
107			.529 493	107
108			.555 674	108
109			.581 857	109
110			.607 918	110
111			.633 731	111
112			.659 171	112
113			.684 114	113
114			.708 442	114
115			.732 042	115
116			.754 809	116
117			.776 648	117
118			.797 477	118
119			.817 225	119
120			1.000 000	120

AM92

x	$\mu_{[x]}$	$\mu_{[x-1]+1}$	μ_x	x
17	0.000 367		0.000 603	17
18	0.000 367	0.000 488	0.000 597	18
19	0.000 367	0.000 485	0.000 591	19
20	0.000 369	0.000 483	0.000 585	20
21	0.000 370	0.000 482	0.000 580	21
22	0.000 374	0.000 480	0.000 574	22
23	0.000 377	0.000 481	0.000 570	23
24	0.000 380	0.000 481	0.000 568	24
25	0.000 385	0.000 482	0.000 566	25
26	0.000 391	0.000 485	0.000 566	26
27	0.000 400	0.000 489	0.000 568	27
28	0.000 408	0.000 495	0.000 572	28
29	0.000 419	0.000 502	0.000 577	29
30	0.000 430	0.000 512	0.000 585	30
31	0.000 443	0.000 523	0.000 596	31
32	0.000 460	0.000 537	0.000 609	32
33	0.000 479	0.000 555	0.000 626	33
34	0.000 500	0.000 576	0.000 647	34
35	0.000 524	0.000 601	0.000 674	35
36	0.000 551	0.000 630	0.000 706	36
37	0.000 582	0.000 665	0.000 744	37
38	0.000 616	0.000 706	0.000 788	38
39	0.000 656	0.000 754	0.000 840	39
40	0.000 701	0.000 810	0.000 902	40
41	0.000 752	0.000 875	0.000 974	41
42	0.000 808	0.000 950	0.001 057	42
43	0.000 871	0.001 037	0.001 154	43
44	0.000 943	0.001 136	0.001 265	44
45	0.001 023	0.001 250	0.001 394	45
46	0.001 113	0.001 380	0.001 541	46
47	0.001 214	0.001 529	0.001 709	47
48	0.001 326	0.001 698	0.001 902	48
49	0.001 451	0.001 890	0.002 122	49
50	0.001 592	0.002 108	0.002 372	50
51	0.001 750	0.002 354	0.002 656	51
52	0.001 925	0.002 633	0.002 978	52
53	0.002 122	0.002 947	0.003 343	53
54	0.002 342	0.003 300	0.003 756	54
55	0.002 588	0.003 696	0.004 221	55
56	0.002 862	0.004 139	0.004 747	56
57	0.003 170	0.004 636	0.005 340	57
58	0.003 513	0.005 189	0.006 005	58
59	0.003 898	0.005 806	0.006 754	59
60	0.004 327	0.006 493	0.007 593	60
61	0.004 809	0.007 254	0.008 533	61
62	0.005 348	0.008 099	0.009 586	62
63	0.005 949	0.009 032	0.010 763	63
64	0.006 623	0.010 063	0.012 078	64

AM92

x	$\mu_{[x]}$	$\mu_{[x-1]+1}$	μ_x	x
65	0.007 377	0.011 199	0.013 544	65
66	0.008 220	0.012 449	0.015 176	66
67	0.009 162	0.013 821	0.016 993	67
68	0.010 216	0.015 326	0.019 012	68
69	0.011 393	0.016 972	0.021 255	69
70	0.012 709	0.018 771	0.023 741	70
71	0.014 178	0.020 733	0.026 496	71
72	0.015 819	0.022 869	0.029 543	72
73	0.017 648	0.025 190	0.032 912	73
74	0.019 687	0.027 708	0.036 631	74
75	0.021 959	0.030 436	0.040 732	75
76	0.024 487	0.033 385	0.045 251	76
77	0.027 300	0.036 569	0.050 223	77
78	0.030 423	0.040 000	0.055 689	78
79	0.033 892	0.043 691	0.061 689	79
80	0.037 737	0.047 656	0.068 271	80
81	0.041 996	0.051 909	0.075 481	81
82	0.046 709	0.056 462	0.083 372	82
83	0.051 916	0.061 329	0.091 999	83
84	0.057 665	0.066 524	0.101 417	84
85	0.064 000	0.072 061	0.111 691	85
86	0.070 978	0.077 952	0.122 884	86
87	0.078 646	0.084 213	0.135 066	87
88	0.087 067	0.090 853	0.148 309	88
89	0.096 302	0.097 889	0.162 691	89
90	0.106 409	0.105 333	0.178 289	90
91		0.113 198	0.195 190	91
92			0.213 482	92
93			0.233 257	93
94			0.254 610	94
95			0.277 645	95
96			0.302 462	96
97			0.329 170	97
98			0.357 882	98
99			0.388 711	99
100			0.421 777	100
101			0.457 202	101
102			0.495 111	102
103			0.535 631	103
104			0.578 890	104
105			0.625 023	105
106			0.674 162	106
107			0.726 443	107
108			0.782 002	108
109			0.840 973	109
110			0.903 494	110
111			0.969 700	111
112			1.039 723	112
113			1.113 695	113
114			1.191 744	114
115			1.274 000	115
116			1.360 581	116
117			1.451 603	117
118			1.547 178	118
119			1.647 417	119
120			2.000 000	120

AM92

x	$e_{[x]}$	$e_{[x-1]+1}$	e_x	x
17	61.353		61.339	17
18	60.389	60.379	60.376	18
19	59.424	59.414	59.412	19
20	58.458	58.449	58.447	20
21	57.492	57.483	57.481	21
22	56.524	56.516	56.514	22
23	55.556	55.548	55.546	23
24	54.587	54.580	54.578	24
25	53.618	53.611	53.609	25
26	52.648	52.641	52.639	26
27	51.677	51.671	51.669	27
28	50.706	50.700	50.699	28
29	49.735	49.729	49.728	29
30	48.764	48.758	48.757	30
31	47.792	47.787	47.785	31
32	46.821	46.816	46.814	32
33	45.850	45.845	45.843	33
34	44.879	44.874	44.872	34
35	43.909	43.904	43.902	35
36	42.939	42.934	42.932	36
37	41.970	41.965	41.963	37
38	41.003	40.997	40.995	38
39	40.036	40.031	40.029	39
40	39.071	39.066	39.064	40
41	38.108	38.102	38.100	41
42	37.148	37.141	37.139	42
43	36.189	36.182	36.180	43
44	35.234	35.226	35.224	44
45	34.282	34.273	34.271	45
46	33.333	33.323	33.321	46
47	32.388	32.377	32.375	47
48	31.448	31.436	31.433	48
49	30.513	30.499	30.497	49
50	29.583	29.567	29.565	50
51	28.660	28.642	28.639	51
52	27.742	27.722	27.720	52
53	26.833	26.810	26.808	53
54	25.931	25.905	25.903	54
55	25.037	25.009	25.006	55
56	24.153	24.122	24.119	56
57	23.279	23.244	23.240	57
58	22.415	22.376	22.373	58
59	21.563	21.520	21.516	59
60	20.724	20.676	20.670	60
61	19.897	19.844	19.837	61
62	19.084	19.026	19.018	62
63	18.286	18.222	18.212	63
64	17.503	17.433	17.421	64

AM92

x	$e_{[x]}$	$e_{[x-1]+1}$	e_x	x
65	16.736	16.660	16.645	65
66	15.987	15.903	15.886	66
67	15.255	15.164	15.143	67
68	14.541	14.443	14.418	68
69	13.847	13.740	13.711	69
70	13.172	13.057	13.023	70
71	12.517	12.394	12.354	71
72	11.883	11.751	11.704	72
73	11.270	11.129	11.075	73
74	10.679	10.529	10.467	74
75	10.110	9.950	9.879	75
76	9.562	9.393	9.313	76
77	9.037	8.859	8.768	77
78	8.534	8.346	8.244	78
79	8.053	7.856	7.742	79
80	7.594	7.388	7.261	80
81	7.157	6.942	6.802	81
82	6.741	6.518	6.364	82
83	6.347	6.116	5.947	83
84	5.974	5.734	5.550	84
85	5.620	5.374	5.174	85
86	5.287	5.034	4.817	86
87	4.972	4.713	4.480	87
88	4.676	4.412	4.161	88
89	4.397	4.129	3.861	89
90	4.136	3.864	3.578	90
91		3.616	3.312	91
92			3.063	92
93			2.829	93
94			2.610	94
95			2.405	95
96			2.214	96
97			2.035	97
98			1.869	98
99			1.715	99
100			1.571	100
101			1.437	101
102			1.314	102
103			1.199	103
104			1.093	104
105			0.994	105
106			0.904	106
107			0.820	107
108			0.743	108
109			0.672	109
110			0.606	110
111			0.546	111
112			0.491	112
113			0.440	113
114			0.394	114
115			0.352	115
116			0.313	116
117			0.277	117
118			0.240	118
119			0.183	119
120			0.000	120

AM92

4%

x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	x
17	5 132.61		5 133.73	17
18	4 932.28	4 933.09	4 933.32	18
19	4 739.80	4 740.55	4 740.76	19
20	4 554.85	4 555.56	4 555.75	20
21	4 377.15	4 377.80	4 377.98	21
22	4 206.41	4 207.01	4 207.16	22
23	4 042.33	4 042.89	4 043.04	23
24	3 884.66	3 885.19	3 885.32	24
25	3 733.16	3 733.64	3 733.77	25
26	3 587.56	3 588.01	3 588.13	26
27	3 447.63	3 448.06	3 448.17	27
28	3 313.16	3 313.55	3 313.66	28
29	3 183.91	3 184.28	3 184.38	29
30	3 059.68	3 060.03	3 060.13	30
31	2 940.27	2 940.60	2 940.69	31
32	2 825.48	2 825.79	2 825.89	32
33	2 715.13	2 715.43	2 715.52	33
34	2 609.03	2 609.33	2 609.42	34
35	2 507.02	2 507.31	2 507.40	35
36	2 408.92	2 409.20	2 409.30	36
37	2 314.57	2 314.86	2 314.96	37
38	2 223.83	2 224.12	2 224.22	38
39	2 136.53	2 136.83	2 136.93	39
40	2 052.54	2 052.85	2 052.96	40
41	1 971.72	1 972.04	1 972.15	41
42	1 893.92	1 894.27	1 894.37	42
43	1 819.02	1 819.40	1 819.50	43
44	1 746.89	1 747.30	1 747.41	44
45	1 677.42	1 677.86	1 677.97	45
46	1 610.47	1 610.96	1 611.07	46
47	1 545.95	1 546.49	1 546.59	47
48	1 483.73	1 484.32	1 484.43	48
49	1 423.70	1 424.37	1 424.47	49
50	1 365.77	1 366.51	1 366.61	50
51	1 309.83	1 310.65	1 310.75	51
52	1 255.78	1 256.70	1 256.80	52
53	1 203.53	1 204.55	1 204.65	53
54	1 152.98	1 154.11	1 154.22	54
55	1 104.05	1 105.30	1 105.41	55
56	1 056.63	1 058.02	1 058.15	56
57	1 010.66	1 012.19	1 012.34	57
58	966.04	967.73	967.90	58
59	922.70	924.57	924.76	59
60	880.56	882.61	882.85	60
61	839.55	841.80	842.08	61
62	799.59	802.06	802.40	62
63	760.62	763.33	763.74	63
64	722.59	725.54	726.03	64

AM92

4%

x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	x
65	685.44	688.64	689.23	65
66	649.11	652.57	653.28	66
67	613.56	617.30	618.14	67
68	578.75	582.78	583.77	68
69	544.65	548.97	550.14	69
70	511.25	515.87	517.23	70
71	478.53	483.43	485.01	71
72	446.48	451.68	453.48	72
73	415.12	420.59	422.64	73
74	384.46	390.20	392.51	74
75	354.54	360.52	363.11	75
76	325.40	331.60	334.46	76
77	297.09	303.48	306.62	77
78	269.69	276.21	279.63	78
79	243.27	249.89	253.56	79
80	217.91	224.57	228.48	80
81	193.71	200.35	204.47	81
82	170.75	177.31	181.60	82
83	149.14	155.55	159.97	83
84	128.97	135.16	139.65	84
85	110.30	116.22	120.71	85
86	93.22	98.79	103.23	86
87	77.76	82.94	87.26	87
88	63.95	68.69	72.83	88
89	51.78	56.06	59.95	89
90	41.24	45.02	48.61	90
91		35.53	38.78	91
92			30.40	92
93			23.38	93
94			17.62	94
95			12.99	95
96			9.34	96
97			6.55	97
98			4.47	98
99			2.96	99
100			1.90	100
101			1.18	101
102			.70	102
103			.40	103
104			.22	104
105			.12	105
106			.06	106
107			.03	107
108			.01	108
109			.01	109
110			.00	110

AM92

4%

x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	x
17	119 958.58		119 959.94	17
18	114 824.96	114 825.98	114 826.20	18
19	109 891.73	109 892.68	109 892.88	19
20	105 151.06	105 151.94	105 152.13	20
21	100 595.40	100 596.21	100 596.38	21
22	96 217.50	96 218.25	96 218.40	22
23	92 010.40	92 011.10	92 011.24	23
24	87 967.43	87 968.07	87 968.21	24
25	84 082.16	84 082.76	84 082.88	25
26	80 348.43	80 349.00	80 349.12	26
27	76 760.35	76 760.88	76 760.99	27
28	73 312.22	73 312.71	73 312.82	28
29	69 998.60	69 999.06	69 999.16	29
30	66 814.23	66 814.68	66 814.78	30
31	63 754.13	63 754.56	63 754.65	31
32	60 813.46	60 813.87	60 813.96	32
33	57 987.58	57 987.98	57 988.07	33
34	55 272.07	55 272.45	55 272.55	34
35	52 662.65	52 663.03	52 663.13	35
36	50 155.24	50 155.63	50 155.73	36
37	47 745.94	47 746.33	47 746.43	37
38	45 430.98	45 431.37	45 431.47	38
39	43 206.74	43 207.15	43 207.25	39
40	41 069.80	41 070.21	41 070.31	40
41	39 016.82	39 017.25	39 017.36	41
42	37 044.65	37 045.10	37 045.21	42
43	35 150.25	35 150.73	35 150.84	43
44	33 330.72	33 331.23	33 331.34	44
45	31 583.27	31 583.82	31 583.93	45
46	29 905.26	29 905.86	29 905.96	46
47	28 294.14	28 294.79	28 294.89	47
48	26 747.50	26 748.20	26 748.30	48
49	25 263.01	25 263.77	25 263.87	49
50	23 838.46	23 839.30	23 839.41	50
51	22 471.77	22 472.69	22 472.79	51
52	21 160.92	21 161.94	21 162.04	52
53	19 904.01	19 905.14	19 905.24	53
54	18 699.23	18 700.48	18 700.59	54
55	17 544.87	17 546.25	17 546.37	55
56	16 439.29	16 440.82	16 440.95	56
57	15 380.96	15 382.66	15 382.81	57
58	14 368.41	14 370.30	14 370.47	58
59	13 400.27	13 402.37	13 402.57	59
60	12 475.24	12 477.57	12 477.80	60
61	11 592.08	11 594.68	11 594.96	61
62	10 749.66	10 752.54	10 752.88	62
63	9 946.87	9 950.07	9 950.48	63
64	9 182.71	9 186.25	9 186.74	64

AM92

4%

x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	x
65	8 456.21	8 460.12	8 460.71	65
66	7 766.46	7 770.77	7 771.48	66
67	7 112.62	7 117.36	7 118.20	67
68	6 493.86	6 499.06	6 500.06	68
69	5 909.43	5 915.12	5 916.29	69
70	5 358.59	5 364.78	5 366.14	70
71	4 840.63	4 847.34	4 848.92	71
72	4 354.86	4 362.10	4 363.91	72
73	3 900.59	3 908.38	3 910.43	73
74	3 477.14	3 485.47	3 487.78	74
75	3 083.84	3 092.69	3 095.27	75
76	2 719.96	2 729.30	2 732.16	76
77	2 384.76	2 394.56	2 397.70	77
78	2 077.47	2 087.67	2 091.08	78
79	1 797.25	1 807.78	1 811.45	79
80	1 543.20	1 553.98	1 557.89	80
81	1 314.35	1 325.29	1 329.41	81
82	1 109.67	1 120.65	1 124.94	82
83	928.03	938.92	943.34	83
84	768.19	778.88	783.37	84
85	628.87	639.22	643.72	85
86	508.67	518.57	523.01	86
87	406.14	415.45	419.77	87
88	319.75	328.38	332.51	88
89	247.93	255.80	259.69	89
90	189.12	196.15	199.74	90
91		147.88	151.13	91
92			112.35	92
93			81.95	93
94			58.56	94
95			40.94	95
96			27.95	96
97			18.61	97
98			12.06	98
99			7.59	99
100			4.63	100
101			2.73	101
102			1.55	102
103			.85	103
104			.45	104
105			.23	105
106			.11	106
107			.05	107
108			.02	108
109			.01	109
110			.00	110

AM92

4%

x	$S_{[x]}$	$S_{[x-1]+1}$	S_x	x
17	2 398 085.62		2 398 087.20	17
18	2 278 125.81	2 278 127.03	2 278 127.26	18
19	2 163 299.72	2 163 300.85	2 163 301.06	19
20	2 053 406.94	2 053 407.99	2 053 408.17	20
21	1 948 254.91	1 948 255.88	1 948 256.05	21
22	1 847 658.63	1 847 659.51	1 847 659.67	22
23	1 751 440.30	1 751 441.12	1 751 441.27	23
24	1 659 429.12	1 659 429.89	1 659 430.03	24
25	1 571 460.98	1 571 461.70	1 571 461.82	25
26	1 487 378.14	1 487 378.82	1 487 378.94	26
27	1 407 029.07	1 407 029.71	1 407 029.82	27
28	1 330 268.14	1 330 268.73	1 330 268.83	28
29	1 256 955.35	1 256 955.92	1 256 956.02	29
30	1 186 956.21	1 186 956.76	1 186 956.85	30
31	1 120 141.46	1 120 141.98	1 120 142.07	31
32	1 056 386.83	1 056 387.32	1 056 387.42	32
33	995 572.87	995 573.36	995 573.46	33
34	937 584.81	937 585.29	937 585.38	34
35	882 312.25	882 312.74	882 312.84	35
36	829 649.12	829 649.61	829 649.71	36
37	779 493.40	779 493.88	779 493.98	37
38	731 746.96	731 747.45	731 747.56	38
39	686 315.48	686 315.99	686 316.09	39
40	643 108.22	643 108.74	643 108.84	40
41	602 037.89	602 038.43	602 038.53	41
42	563 020.51	563 021.07	563 021.17	42
43	525 975.27	525 975.86	525 975.96	43
44	490 824.40	490 825.02	490 825.13	44
45	457 493.03	457 493.69	457 493.79	45
46	425 909.06	425 909.76	425 909.86	46
47	396 003.05	396 003.80	396 003.90	47
48	367 708.11	367 708.91	367 709.01	48
49	340 959.74	340 960.61	340 960.71	49
50	315 695.79	315 696.73	315 696.84	50
51	291 856.30	291 857.33	291 857.43	51
52	269 383.41	269 384.53	269 384.64	52
53	248 221.26	248 222.49	248 222.60	53
54	228 315.88	228 317.24	228 317.35	54
55	209 615.14	209 616.65	209 616.77	55
56	192 068.59	192 070.27	192 070.40	56
57	175 627.43	175 629.30	175 629.44	57
58	160 244.38	160 246.47	160 246.64	58
59	145 873.64	145 875.97	145 876.17	59
60	132 470.75	132 473.37	132 473.60	60
61	119 992.59	119 995.52	119 995.80	61
62	108 397.21	108 400.50	108 400.84	62
63	97 643.87	97 647.55	97 647.96	63
64	87 692.86	87 696.99	87 697.49	64

AM92

4%

x	$S_{[x]}$	$S_{[x-1]+1}$	S_x	x
65	78 505.54	78 510.15	78 510.74	65
66	70 044.17	70 049.32	70 050.03	66
67	62 271.97	62 277.71	62 278.55	67
68	55 152.99	55 159.35	55 160.35	68
69	48 652.08	48 659.12	48 660.29	69
70	42 734.88	42 742.64	42 744.01	70
71	37 367.77	37 376.29	37 377.86	71
72	32 517.84	32 527.14	32 528.95	72
73	28 152.89	28 162.99	28 165.04	73
74	24 241.39	24 252.30	24 254.61	74
75	20 752.53	20 764.24	20 766.83	75
76	17 656.21	17 668.69	17 671.56	76
77	14 923.03	14 936.25	14 939.39	77
78	12 524.40	12 538.27	12 541.69	78
79	10 432.48	10 446.93	10 450.60	79
80	8 620.33	8 635.24	8 639.15	80
81	7 061.91	7 077.14	7 081.26	81
82	5 732.17	5 747.56	5 751.85	82
83	4 607.11	4 622.49	4 626.91	83
84	3 663.90	3 679.09	3 683.57	84
85	2 880.92	2 895.71	2 900.21	85
86	2 237.83	2 252.05	2 256.49	86
87	1 715.71	1 729.16	1 733.48	87
88	1 297.05	1 309.57	1 313.71	88
89	965.85	977.30	981.19	89
90	707.63	717.91	721.51	90
91		518.51	521.76	91
92			370.63	92
93			258.28	93
94			176.34	94
95			117.78	95
96			76.84	96
97			48.88	97
98			30.28	98
99			18.22	99
100			10.63	100
101			6.00	101
102			3.27	102
103			1.72	103
104			.87	104
105			.42	105
106			.19	106
107			.09	107
108			.04	108
109			.01	109
110			.01	110

AM92

4%

x	$C_{[x]}$	$C_{[x-1]+1}$	C_x	x
17	2.11		2.96	17
18	2.02	2.60	2.82	18
19	1.94	2.48	2.68	19
20	1.86	2.37	2.55	20
21	1.79	2.26	2.43	21
22	1.73	2.16	2.31	22
23	1.67	2.08	2.21	23
24	1.61	1.99	2.12	24
25	1.56	1.91	2.03	25
26	1.52	1.85	1.96	26
27	1.48	1.78	1.89	27
28	1.45	1.73	1.83	28
29	1.42	1.68	1.78	29
30	1.40	1.64	1.74	30
31	1.39	1.61	1.70	31
32	1.38	1.59	1.68	32
33	1.38	1.57	1.66	33
34	1.38	1.57	1.66	34
35	1.39	1.57	1.66	35
36	1.41	1.58	1.68	36
37	1.43	1.61	1.70	37
38	1.46	1.64	1.74	38
39	1.51	1.69	1.79	39
40	1.56	1.75	1.85	40
41	1.61	1.82	1.92	41
42	1.68	1.91	2.01	42
43	1.75	2.01	2.11	43
44	1.84	2.13	2.23	44
45	1.94	2.26	2.36	45
46	2.04	2.41	2.51	46
47	2.16	2.58	2.68	47
48	2.29	2.77	2.87	48
49	2.43	2.97	3.07	49
50	2.59	3.20	3.30	50
51	2.76	3.44	3.54	51
52	2.94	3.71	3.81	52
53	3.13	4.00	4.10	53
54	3.34	4.31	4.41	54
55	3.56	4.64	4.75	55
56	3.80	4.99	5.11	56
57	4.05	5.36	5.50	57
58	4.32	5.75	5.91	58
59	4.60	6.16	6.35	59
60	4.89	6.59	6.81	60
61	5.19	7.03	7.29	61
62	5.51	7.48	7.80	62
63	5.83	7.94	8.33	63
64	6.16	8.40	8.88	64

AM92

4%

x	$C_{[x]}$	$C_{[x-1]+1}$	C_x	x
65	6.50	8.87	9.44	65
66	6.84	9.33	10.01	66
67	7.18	9.78	10.59	67
68	7.51	10.22	11.18	68
69	7.84	10.63	11.76	69
70	8.15	11.02	12.33	70
71	8.44	11.36	12.87	71
72	8.71	11.66	13.39	72
73	8.95	11.90	13.88	73
74	9.15	12.08	14.31	74
75	9.30	12.19	14.68	75
76	9.41	12.23	14.98	76
77	9.45	12.17	15.19	77
78	9.43	12.03	15.31	78
79	9.35	11.79	15.33	79
80	9.18	11.46	15.23	80
81	8.95	11.04	15.00	81
82	8.63	10.52	14.65	82
83	8.25	9.92	14.17	83
84	7.79	9.25	13.56	84
85	7.27	8.52	12.84	85
86	6.69	7.73	12.00	86
87	6.08	6.92	11.08	87
88	5.43	6.10	10.08	88
89	4.78	5.29	9.03	89
90	4.12	4.50	7.96	90
91		3.76	6.89	91
92			5.85	92
93			4.86	93
94			3.96	94
95			3.14	95
96			2.43	96
97			1.83	97
98			1.34	98
99			.95	99
100			.65	100
101			.43	101
102			.27	102
103			.17	103
104			.10	104
105			.05	105
106			.03	106
107			.01	107
108			.01	108
109			.00	109
110			.00	110

AM92

4%

x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	x
17	518.82		519.89	17
18	515.93	516.71	516.93	18
19	513.19	513.91	514.11	19
20	510.58	511.25	511.43	20
21	508.09	508.72	508.88	21
22	505.73	506.31	506.46	22
23	503.47	504.01	504.14	23
24	501.30	501.80	501.93	24
25	499.23	499.69	499.81	25
26	497.23	497.67	497.78	26
27	495.31	495.72	495.82	27
28	493.46	493.83	493.93	28
29	491.66	492.01	492.10	29
30	489.90	490.23	490.33	30
31	488.19	488.50	488.59	31
32	486.50	486.80	486.89	32
33	484.84	485.12	485.21	33
34	483.18	483.46	483.55	34
35	481.53	481.80	481.90	35
36	479.87	480.14	480.24	36
37	478.19	478.46	478.56	37
38	476.48	476.76	476.86	38
39	474.74	475.02	475.12	39
40	472.94	473.23	473.33	40
41	471.07	471.38	471.48	41
42	469.12	469.46	469.56	42
43	467.09	467.44	467.55	43
44	464.94	465.33	465.43	44
45	462.68	463.10	463.20	45
46	460.27	460.74	460.84	46
47	457.71	458.23	458.33	47
48	454.98	455.55	455.65	48
49	452.05	452.68	452.78	49
50	448.91	449.61	449.71	50
51	445.53	446.32	446.42	51
52	441.90	442.78	442.88	52
53	437.99	438.96	439.07	53
54	433.78	434.86	434.97	54
55	429.24	430.44	430.55	55
56	424.35	425.68	425.80	56
57	419.08	420.55	420.69	57
58	413.41	415.03	415.19	58
59	407.30	409.09	409.28	59
60	400.74	402.71	402.93	60
61	393.70	395.85	396.12	61
62	386.14	388.50	388.83	62
63	378.05	380.63	381.02	63
64	369.41	372.22	372.69	64

AM92

4%

x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	x
65	360.20	363.25	363.82	65
66	350.40	353.70	354.38	66
67	339.99	343.56	344.37	67
68	328.98	332.81	333.77	68
69	317.37	321.47	322.59	69
70	305.15	309.53	310.84	70
71	292.35	297.00	298.51	71
72	278.98	283.90	285.64	72
73	265.09	270.27	272.24	73
74	250.72	256.14	258.37	74
75	235.93	241.57	244.06	75
76	220.78	226.63	229.38	76
77	205.37	211.38	214.40	77
78	189.79	195.92	199.20	78
79	174.14	180.36	183.89	79
80	158.56	164.80	168.56	80
81	143.16	149.37	153.34	81
82	128.07	134.21	138.34	82
83	113.45	119.44	123.69	83
84	99.42	105.20	109.52	84
85	86.12	91.63	95.96	85
86	73.65	78.85	83.12	86
87	62.14	66.96	71.11	87
88	51.65	56.06	60.04	88
89	42.25	46.22	49.96	89
90	33.97	37.47	40.93	90
91		29.84	32.97	91
92			26.08	92
93			20.23	93
94			15.37	94
95			11.41	95
96			8.27	96
97			5.84	97
98			4.01	98
99			2.67	99
100			1.72	100
101			1.07	101
102			.64	102
103			.37	103
104			.21	104
105			.11	105
106			.05	106
107			.03	107
108			.01	108
109			.01	109
110			.00	110

AM92

4%

x	$R_{[x]}$	$R_{[x-1]+1}$	R_x	x
17	27 724.52		27 725.81	17
18	27 204.73	27 205.71	27 205.92	18
19	26 687.90	26 688.80	26 689.00	19
20	26 173.87	26 174.71	26 174.89	20
21	25 662.51	25 663.29	25 663.45	21
22	25 153.71	25 154.42	25 154.57	22
23	24 647.32	24 647.98	24 648.11	23
24	24 143.23	24 143.85	24 143.97	24
25	23 641.35	23 641.93	23 642.04	25
26	23 141.58	23 142.12	23 142.23	26
27	22 643.84	22 644.35	22 644.45	27
28	22 148.06	22 148.53	22 148.63	28
29	21 654.16	21 654.60	21 654.70	29
30	21 162.07	21 162.50	21 162.60	30
31	20 671.77	20 672.17	20 672.27	31
32	20 183.20	20 183.59	20 183.68	32
33	19 696.32	19 696.70	19 696.79	33
34	19 211.11	19 211.48	19 211.57	34
35	18 727.56	18 727.93	18 728.02	35
36	18 245.66	18 246.03	18 246.12	36
37	17 765.43	17 765.79	17 765.89	37
38	17 286.86	17 287.23	17 287.33	38
39	16 809.99	16 810.38	16 810.47	39
40	16 334.87	16 335.26	16 335.36	40
41	15 861.52	15 861.93	15 862.03	41
42	15 390.01	15 390.45	15 390.55	42
43	14 920.43	14 920.89	14 920.99	43
44	14 452.85	14 453.35	14 453.45	44
45	13 987.39	13 987.91	13 988.01	45
46	13 524.14	13 524.71	13 524.81	46
47	13 063.26	13 063.87	13 063.97	47
48	12 604.88	12 605.55	12 605.65	48
49	12 149.17	12 149.90	12 150.00	49
50	11 696.32	11 697.12	11 697.22	50
51	11 246.53	11 247.41	11 247.51	51
52	10 800.02	10 800.99	10 801.09	52
53	10 357.04	10 358.12	10 358.22	53
54	9 917.85	9 919.05	9 919.15	54
55	9 482.75	9 484.07	9 484.19	55
56	9 052.04	9 053.51	9 053.63	56
57	8 626.06	8 627.69	8 627.83	57
58	8 205.17	8 206.98	8 207.14	58
59	7 789.75	7 791.76	7 791.95	59
60	7 380.21	7 382.44	7 382.67	60
61	6 976.98	6 979.47	6 979.73	61
62	6 580.53	6 583.29	6 583.61	62
63	6 191.34	6 194.39	6 194.79	63
64	5 809.91	5 813.29	5 813.76	64

AM92

4%

x	$R_{[x]}$	$R_{[x-1]+1}$	R_x	x
65	5 436.77	5 440.50	5 441.07	65
66	5 072.46	5 076.57	5 077.25	66
67	4 717.54	4 722.06	4 722.87	67
68	4 372.60	4 377.55	4 378.51	68
69	4 038.20	4 043.61	4 044.74	69
70	3 714.94	3 720.83	3 722.14	70
71	3 403.41	3 409.79	3 411.31	71
72	3 104.17	3 111.06	3 112.79	72
73	2 817.78	2 825.19	2 827.16	73
74	2 544.78	2 552.69	2 554.91	74
75	2 285.66	2 294.06	2 296.55	75
76	2 040.87	2 049.74	2 052.49	76
77	1 810.80	1 820.09	1 823.11	77
78	1 595.76	1 605.43	1 608.71	78
79	1 396.00	1 405.97	1 409.51	79
80	1 211.64	1 221.85	1 225.62	80
81	1 042.74	1 053.09	1 057.05	81
82	889.21	899.59	903.72	82
83	750.83	761.13	765.38	83
84	627.27	637.38	641.69	84
85	518.06	527.85	532.17	85
86	422.60	431.95	436.22	86
87	340.15	348.95	353.10	87
88	269.86	278.01	281.99	88
89	210.79	218.21	221.95	89
90	161.90	168.54	171.99	90
91		127.94	131.06	91
92			98.09	92
93			72.01	93
94			51.78	94
95			36.41	95
96			25.00	96
97			16.73	97
98			10.89	98
99			6.89	99
100			4.22	100
101			2.50	101
102			1.43	102
103			.79	103
104			.41	104
105			.21	105
106			.10	106
107			.05	107
108			.02	108
109			.01	109
110			.00	110

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	$\ddot{a}_x$	A_x	${}^2A_x$	x
17	23.372	0.101 08	0.016 96	23.367	0.101 27	0.017 16	17
18	23.280	0.104 60	0.017 78	23.276	0.104 78	0.017 97	18
19	23.185	0.108 27	0.018 67	23.180	0.108 44	0.018 85	19
20	23.086	0.112 10	0.019 64	23.081	0.112 26	0.019 82	20
21	22.982	0.116 08	0.020 70	22.978	0.116 24	0.020 86	21
22	22.874	0.120 23	0.021 84	22.870	0.120 38	0.022 00	22
23	22.762	0.124 55	0.023 08	22.758	0.124 69	0.023 24	23
24	22.645	0.129 05	0.024 43	22.641	0.129 19	0.024 58	24
25	22.523	0.133 73	0.025 89	22.520	0.133 86	0.026 03	25
26	22.396	0.138 60	0.027 47	22.393	0.138 73	0.027 61	26
27	22.265	0.143 67	0.029 17	22.261	0.143 79	0.029 31	27
28	22.128	0.148 94	0.031 02	22.124	0.149 06	0.031 15	28
29	21.985	0.154 42	0.033 01	21.982	0.154 54	0.033 14	29
30	21.837	0.160 11	0.035 15	21.834	0.160 23	0.035 28	30
31	21.683	0.166 03	0.037 47	21.680	0.166 15	0.037 59	31
32	21.523	0.172 18	0.039 96	21.520	0.172 30	0.040 08	32
33	21.357	0.178 57	0.042 64	21.354	0.178 68	0.042 76	33
34	21.185	0.185 20	0.045 52	21.182	0.185 31	0.045 65	34
35	21.006	0.192 07	0.048 61	21.003	0.192 19	0.048 74	35
36	20.821	0.199 21	0.051 93	20.818	0.199 33	0.052 07	36
37	20.628	0.206 60	0.055 49	20.625	0.206 72	0.055 63	37
38	20.429	0.214 26	0.059 30	20.426	0.214 39	0.059 45	38
39	20.223	0.222 20	0.063 38	20.219	0.222 34	0.063 54	39
40	20.009	0.230 41	0.067 75	20.005	0.230 56	0.067 92	40
41	19.788	0.238 91	0.072 41	19.784	0.239 07	0.072 59	41
42	19.560	0.247 70	0.077 38	19.555	0.247 87	0.077 58	42
43	19.324	0.256 78	0.082 67	19.319	0.256 96	0.082 89	43
44	19.080	0.266 15	0.088 32	19.075	0.266 36	0.088 56	44
45	18.829	0.275 83	0.094 31	18.823	0.276 05	0.094 58	45
46	18.569	0.285 80	0.100 68	18.563	0.286 05	0.100 98	46
47	18.302	0.296 07	0.107 44	18.295	0.296 35	0.107 78	47
48	18.027	0.306 64	0.114 60	18.019	0.306 95	0.114 98	48
49	17.745	0.317 52	0.122 17	17.736	0.317 86	0.122 60	49
50	17.454	0.328 68	0.130 17	17.444	0.329 07	0.130 65	50
51	17.156	0.340 14	0.138 61	17.145	0.340 58	0.139 15	51
52	16.851	0.351 89	0.147 49	16.838	0.352 38	0.148 11	52
53	16.538	0.363 92	0.156 84	16.524	0.364 48	0.157 55	53
54	16.218	0.376 23	0.166 65	16.202	0.376 85	0.167 45	54
55	15.891	0.388 79	0.176 93	15.873	0.389 50	0.177 85	55
56	15.558	0.401 61	0.187 69	15.537	0.402 40	0.188 74	56
57	15.219	0.414 66	0.198 93	15.195	0.415 56	0.200 12	57
58	14.874	0.427 94	0.210 64	14.847	0.428 96	0.212 00	58
59	14.523	0.441 43	0.222 82	14.493	0.442 58	0.224 37	59
60	14.167	0.455 10	0.235 47	14.134	0.456 40	0.237 23	60
61	13.808	0.468 94	0.248 57	13.769	0.470 41	0.250 58	61
62	13.444	0.482 92	0.262 11	13.401	0.484 58	0.264 40	62
63	13.077	0.497 03	0.276 08	13.029	0.498 90	0.278 68	63
64	12.708	0.511 23	0.290 46	12.653	0.513 33	0.293 40	64

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	$\ddot{a}_x$	A_x	${}^2A_x$	x
65	12.337	0.525 50	0.305 22	12.276	0.527 86	0.308 55	65
66	11.965	0.539 81	0.320 33	11.896	0.542 46	0.324 10	66
67	11.592	0.554 14	0.335 78	11.515	0.557 10	0.340 03	67
68	11.221	0.568 44	0.351 51	11.135	0.571 75	0.356 30	68
69	10.850	0.582 70	0.367 51	10.754	0.586 38	0.372 89	69
70	10.481	0.596 87	0.383 72	10.375	0.600 97	0.389 75	70
71	10.116	0.610 93	0.400 12	9.998	0.615 48	0.406 86	71
72	9.754	0.624 85	0.416 65	9.623	0.629 88	0.424 16	72
73	9.396	0.638 60	0.433 27	9.252	0.644 14	0.441 62	73
74	9.044	0.652 14	0.449 93	8.886	0.658 24	0.459 19	74
75	8.698	0.665 45	0.466 59	8.524	0.672 14	0.476 83	75
76	8.359	0.678 51	0.483 20	8.169	0.685 81	0.494 48	76
77	8.027	0.691 27	0.499 71	7.820	0.699 24	0.512 10	77
78	7.703	0.703 73	0.516 09	7.478	0.712 38	0.529 65	78
79	7.388	0.715 85	0.532 27	7.144	0.725 23	0.547 07	79
80	7.082	0.727 62	0.548 22	6.818	0.737 75	0.564 32	80
81	6.785	0.739 03	0.563 90	6.502	0.749 93	0.581 36	81
82	6.499	0.750 05	0.579 27	6.194	0.761 75	0.598 14	82
83	6.222	0.760 68	0.594 30	5.897	0.773 19	0.614 61	83
84	5.957	0.770 90	0.608 95	5.610	0.784 25	0.630 75	84
85	5.701	0.780 72	0.623 20	5.333	0.794 90	0.646 52	85
86	5.457	0.790 12	0.637 01	5.066	0.805 14	0.661 88	86
87	5.223	0.799 11	0.650 38	4.811	0.814 98	0.676 80	87
88	5.000	0.807 69	0.663 29	4.566	0.824 39	0.691 27	88
89	4.788	0.815 85	0.675 73	4.332	0.833 38	0.705 25	89
90	4.586	0.823 62	0.687 68	4.109	0.841 96	0.718 74	90
91				3.897	0.850 12	0.731 72	91
92				3.695	0.857 87	0.744 17	92
93				3.504	0.865 22	0.756 09	93
94				3.323	0.872 18	0.767 48	94
95				3.153	0.878 75	0.778 34	95
96				2.992	0.884 94	0.788 67	96
97				2.840	0.890 77	0.798 47	97
98				2.698	0.896 25	0.807 76	98
99				2.564	0.901 39	0.816 54	99
100				2.439	0.906 21	0.824 83	100
101				2.321	0.910 71	0.832 63	101
102				2.212	0.914 92	0.839 97	102
103				2.110	0.918 85	0.846 86	103
104				2.015	0.922 51	0.853 31	104
105				1.926	0.925 91	0.859 34	105
106				1.844	0.929 07	0.864 98	106
107				1.768	0.932 01	0.870 23	107
108				1.697	0.934 72	0.875 12	108
109				1.632	0.937 24	0.879 66	109
110				1.571	0.939 56	0.883 87	110
111				1.516	0.941 70	0.887 77	111
112				1.464	0.943 67	0.891 37	112
113				1.417	0.945 49	0.894 69	113
114				1.374	0.947 15	0.897 75	114
115				1.334	0.948 68	0.900 56	115
116				1.298	0.950 08	0.903 15	116
117				1.264	0.951 39	0.905 57	117
118				1.229	0.952 73	0.908 04	118
119				1.176	0.954 78	0.911 81	119
120				1.000	0.961 54	0.924 56	120

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

AM92

4%

x	$(I\ddot{a})_{[x]}$	$(IA)_{[x]}$	$(I\ddot{a})_x$	$(IA)_x$	x
17	467.226	5.401 64	467.124	5.400 71	17
18	461.881	5.515 65	461.784	5.514 73	18
19	456.412	5.630 60	456.320	5.629 69	19
20	450.817	5.746 37	450.729	5.745 47	20
21	445.097	5.862 84	445.013	5.861 95	21
22	439.249	5.979 86	439.170	5.978 99	22
23	433.275	6.097 30	433.200	6.096 44	23
24	427.174	6.215 01	427.102	6.214 15	24
25	420.947	6.332 80	420.878	6.331 95	25
26	414.593	6.450 51	414.528	6.449 67	26
27	408.114	6.567 94	408.051	6.567 10	27
28	401.510	6.684 88	401.450	6.684 05	28
29	394.783	6.801 12	394.726	6.800 29	29
30	387.935	6.916 44	387.878	6.915 59	30
31	380.966	7.030 57	380.911	7.029 72	31
32	373.879	7.143 28	373.825	7.142 42	32
33	366.676	7.254 28	366.623	7.253 40	33
34	359.361	7.363 31	359.308	7.362 39	34
35	351.937	7.470 05	351.883	7.469 09	35
36	344.407	7.574 21	344.353	7.573 20	36
37	336.776	7.675 46	336.720	7.674 38	37
38	329.048	7.773 46	328.991	7.772 31	38
39	321.228	7.867 88	321.169	7.866 63	39
40	313.323	7.958 35	313.260	7.956 99	40
41	305.337	8.044 52	305.271	8.043 03	41
42	297.278	8.126 02	297.207	8.124 35	42
43	289.153	8.202 46	289.077	8.200 60	43
44	280.970	8.273 47	280.888	8.271 37	44
45	272.737	8.338 65	272.647	8.336 28	45
46	264.462	8.397 62	264.365	8.394 93	46
47	256.156	8.450 01	256.049	8.446 95	47
48	247.828	8.495 42	247.711	8.491 93	48
49	239.488	8.533 51	239.360	8.529 50	49
50	231.149	8.563 90	231.007	8.559 29	50
51	222.820	8.586 24	222.664	8.580 95	51
52	214.514	8.600 22	214.342	8.594 12	52
53	206.244	8.605 54	206.053	8.598 51	53
54	198.022	8.601 90	197.811	8.593 81	54
55	189.861	8.589 08	189.627	8.579 76	55
56	181.774	8.566 87	181.516	8.556 11	56
57	173.775	8.535 08	173.489	8.522 68	57
58	165.878	8.493 60	165.561	8.479 31	58
59	158.094	8.442 34	157.744	8.425 88	59
60	150.440	8.381 28	150.053	8.362 34	60
61	142.926	8.310 44	142.499	8.288 67	61
62	135.566	8.229 90	135.096	8.204 91	62
63	128.373	8.139 81	127.856	8.111 17	63
64	121.359	8.040 36	120.790	8.007 60	64

AM92

4%

x	$(I\ddot{a})_{[x]}$	$(IA)_{[x]}$	$(I\ddot{a})_x$	$(IA)_x$	x
65	114.533	7.931 82	113.911	7.894 42	65
66	107.909	7.814 53	107.228	7.771 92	66
67	101.494	7.688 86	100.751	7.640 43	67
68	95.297	7.555 27	94.489	7.500 35	68
69	89.327	7.414 26	88.450	7.352 15	69
70	83.589	7.266 40	82.641	7.196 35	70
71	78.089	7.112 29	77.067	7.033 51	71
72	72.832	6.952 57	71.732	6.864 24	72
73	67.819	6.787 95	66.640	6.689 22	73
74	63.053	6.619 14	61.793	6.509 13	74
75	58.534	6.446 87	57.192	6.324 70	75
76	54.260	6.271 92	52.836	6.136 69	76
77	50.230	6.095 04	48.723	5.945 86	77
78	46.440	5.916 97	44.851	5.752 98	78
79	42.885	5.738 48	41.215	5.558 83	79
80	39.559	5.560 29	37.811	5.364 17	80
81	36.457	5.383 08	34.633	5.169 76	81
82	33.570	5.207 53	31.673	4.976 31	82
83	30.890	5.034 26	28.924	4.784 53	83
84	28.410	4.863 82	26.378	4.595 08	84
85	26.118	4.696 75	24.025	4.408 56	85
86	24.007	4.533 50	21.858	4.225 55	86
87	22.065	4.374 48	19.866	4.046 57	87
88	20.283	4.220 03	18.039	3.872 08	88
89	18.651	4.070 43	16.368	3.702 50	89
90	17.159	3.925 89	14.843	3.538 17	90
91			13.453	3.379 39	91
92			12.191	3.226 40	92
93			11.045	3.079 39	93
94			10.007	2.938 48	94
95			9.070	2.803 78	95
96			8.223	2.675 30	96
97			7.460	2.553 06	97
98			6.774	2.437 01	98
99			6.156	2.327 08	99
100			5.602	2.223 16	100
101			5.104	2.125 12	101
102			4.659	2.032 81	102
103			4.259	1.946 07	103
104			3.902	1.864 71	104
105			3.582	1.788 53	105
106			3.295	1.717 34	106
107			3.039	1.650 92	107
108			2.811	1.589 07	108
109			2.606	1.531 58	109
110			2.424	1.478 23	110
111			2.261	1.428 82	111
112			2.115	1.383 15	112
113			1.985	1.341 02	113
114			1.869	1.302 22	114
115			1.765	1.266 54	115
116			1.672	1.233 70	116
117			1.584	1.202 99	117
118			1.492	1.171 57	118
119			1.351	1.123 76	119
120			1.000	0.961 54	120

AM92

4%

x	$\ddot{a}_{[x]:\overline{n}\vert}$	$A_{[x]:\overline{n}\vert}$	$n = 60 - x$	$\ddot{a}_{x:\overline{n}\vert}$	$A_{x:\overline{n}\vert}$	x
17	20.941	0.194 59	43	20.936	0.194 75	17
18	20.750	0.201 90	42	20.746	0.202 06	18
19	20.552	0.209 53	41	20.548	0.209 68	19
20	20.346	0.217 46	40	20.342	0.217 60	20
21	20.131	0.225 72	39	20.128	0.225 86	21
22	19.908	0.234 32	38	19.904	0.234 45	22
23	19.675	0.243 27	37	19.672	0.243 40	23
24	19.433	0.252 59	36	19.430	0.252 71	24
25	19.181	0.262 28	35	19.178	0.262 40	25
26	18.918	0.272 37	34	18.916	0.272 48	26
27	18.645	0.282 87	33	18.643	0.282 97	27
28	18.361	0.293 79	32	18.359	0.293 89	28
29	18.066	0.305 15	31	18.064	0.305 25	29
30	17.759	0.316 97	30	17.756	0.317 06	30
31	17.439	0.329 26	29	17.437	0.329 35	31
32	17.107	0.342 04	28	17.105	0.342 12	32
33	16.762	0.355 33	27	16.759	0.355 41	33
34	16.402	0.369 14	26	16.400	0.369 23	34
35	16.029	0.383 50	25	16.027	0.383 59	35
36	15.641	0.398 43	24	15.639	0.398 52	36
37	15.237	0.413 95	23	15.235	0.414 03	37
38	14.818	0.430 07	22	14.816	0.430 16	38
39	14.383	0.446 82	21	14.380	0.446 92	39
40	13.930	0.464 23	20	13.927	0.464 33	40
41	13.460	0.482 31	19	13.457	0.482 42	41
42	12.971	0.501 10	18	12.969	0.501 21	42
43	12.464	0.520 61	17	12.461	0.520 73	43
44	11.937	0.540 88	16	11.934	0.541 00	44
45	11.390	0.561 93	15	11.386	0.562 06	45
46	10.821	0.583 80	14	10.818	0.583 93	46
47	10.231	0.606 51	13	10.227	0.606 65	47
48	9.617	0.630 10	12	9.613	0.630 25	48
49	8.980	0.654 61	11	8.976	0.654 77	49
50	8.318	0.680 07	10	8.314	0.680 24	50
51	7.630	0.706 54	9	7.625	0.706 72	51
52	6.914	0.734 06	8	6.910	0.734 24	52
53	6.170	0.762 68	7	6.166	0.762 86	53
54	5.396	0.792 46	6	5.391	0.792 64	54
55	4.590	0.823 48	5	4.585	0.823 65	55
56	3.749	0.855 80	4	3.745	0.855 95	56
57	2.873	0.889 52	3	2.870	0.889 63	57
58	1.957	0.924 73	2	1.955	0.924 79	58
59	1.000	0.961 54	1	1.000	0.961 54	59

AM92

4%

x	$\ddot{a}_{[x]:\overline{n}\rceil}$	$A_{[x]:\overline{n}\rceil}$	$n = 65 - x$	$\ddot{a}_{x:\overline{n}\rceil}$	$A_{x:\overline{n}\rceil}$	x
17	21.723	0.164 48	48	21.719	0.164 66	17
18	21.565	0.170 58	47	21.561	0.170 74	18
19	21.400	0.176 93	46	21.396	0.177 09	19
20	21.228	0.183 54	45	21.224	0.183 69	20
21	21.049	0.190 42	44	21.045	0.190 57	21
22	20.863	0.197 59	43	20.859	0.197 73	22
23	20.669	0.205 05	42	20.665	0.205 18	23
24	20.467	0.212 81	41	20.464	0.212 94	24
25	20.257	0.220 90	40	20.254	0.221 02	25
26	20.038	0.229 31	39	20.035	0.229 42	26
27	19.811	0.238 05	38	19.808	0.238 17	27
28	19.574	0.247 16	37	19.571	0.247 26	28
29	19.328	0.256 62	36	19.325	0.256 73	29
30	19.072	0.266 47	35	19.069	0.266 57	30
31	18.806	0.276 71	34	18.803	0.276 81	31
32	18.529	0.287 35	33	18.526	0.287 45	32
33	18.241	0.298 42	32	18.239	0.298 52	33
34	17.942	0.309 92	31	17.940	0.310 02	34
35	17.631	0.321 87	30	17.629	0.321 97	35
36	17.308	0.334 29	29	17.306	0.334 39	36
37	16.973	0.347 19	28	16.970	0.347 29	37
38	16.625	0.360 59	27	16.622	0.360 70	38
39	16.263	0.374 51	26	16.260	0.374 62	39
40	15.887	0.388 96	25	15.884	0.389 07	40
41	15.497	0.403 95	24	15.494	0.404 07	41
42	15.092	0.419 52	23	15.089	0.419 65	42
43	14.672	0.435 67	22	14.669	0.435 81	43
44	14.237	0.452 43	21	14.233	0.452 58	44
45	13.785	0.469 82	20	13.780	0.469 98	45
46	13.316	0.487 86	19	13.311	0.488 03	46
47	12.829	0.506 56	18	12.824	0.506 75	47
48	12.325	0.525 96	17	12.320	0.526 17	48
49	11.802	0.546 08	16	11.796	0.546 30	49
50	11.259	0.566 95	15	11.253	0.567 19	50
51	10.697	0.588 58	14	10.690	0.588 84	51
52	10.113	0.611 02	13	10.106	0.611 30	52
53	9.508	0.634 30	12	9.500	0.634 60	53
54	8.880	0.658 46	11	8.872	0.658 78	54
55	8.228	0.683 54	10	8.219	0.683 88	55
56	7.551	0.709 58	9	7.542	0.709 93	56
57	6.847	0.736 64	8	6.838	0.737 01	57
58	6.115	0.764 79	7	6.106	0.765 16	58
59	5.353	0.794 10	6	5.344	0.794 46	59
60	4.559	0.824 65	5	4.550	0.824 99	60
61	3.730	0.856 54	4	3.722	0.856 85	61
62	2.863	0.889 90	3	2.857	0.890 13	62
63	1.954	0.924 85	2	1.951	0.924 98	63
64	1.000	0.961 54	1	1.000	0.961 54	64

AM92

6%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	$^2A_{[x]}$	$\ddot{a}_x$	A_x	^2A_x	x
17	16.977	0.039 02	0.006 11	16.974	0.039 21	0.006 30	17
18	16.946	0.040 80	0.006 30	16.943	0.040 99	0.006 48	18
19	16.912	0.042 70	0.006 52	16.909	0.042 88	0.006 69	19
20	16.877	0.044 72	0.006 77	16.874	0.044 89	0.006 93	20
21	16.839	0.046 86	0.007 05	16.836	0.047 03	0.007 21	21
22	16.798	0.049 14	0.007 38	16.796	0.049 30	0.007 53	22
23	16.756	0.051 57	0.007 75	16.753	0.051 72	0.007 90	23
24	16.710	0.054 14	0.008 16	16.708	0.054 28	0.008 31	24
25	16.662	0.056 86	0.008 63	16.660	0.057 01	0.008 77	25
26	16.611	0.059 76	0.009 16	16.609	0.059 90	0.009 30	26
27	16.557	0.062 82	0.009 75	16.554	0.062 96	0.009 88	27
28	16.499	0.066 07	0.010 41	16.497	0.066 20	0.010 54	28
29	16.439	0.069 51	0.011 15	16.436	0.069 64	0.011 28	29
30	16.374	0.073 16	0.011 97	16.372	0.073 28	0.012 10	30
31	16.306	0.077 01	0.012 89	16.304	0.077 14	0.013 01	31
32	16.234	0.081 09	0.013 90	16.232	0.081 21	0.014 03	32
33	16.158	0.085 40	0.015 03	16.156	0.085 52	0.015 15	33
34	16.078	0.089 95	0.016 27	16.075	0.090 07	0.016 40	34
35	15.993	0.094 75	0.017 65	15.990	0.094 88	0.017 78	35
36	15.903	0.099 82	0.019 16	15.901	0.099 95	0.019 30	36
37	15.809	0.105 16	0.020 84	15.806	0.105 30	0.020 98	37
38	15.709	0.110 79	0.022 67	15.707	0.110 94	0.022 82	38
39	15.605	0.116 72	0.024 69	15.602	0.116 88	0.024 85	39
40	15.494	0.122 96	0.026 90	15.491	0.123 13	0.027 07	40
41	15.378	0.129 52	0.029 33	15.375	0.129 70	0.029 51	41
42	15.257	0.136 41	0.031 98	15.253	0.136 60	0.032 18	42
43	15.129	0.143 65	0.034 87	15.125	0.143 85	0.035 09	43
44	14.995	0.151 23	0.038 02	14.991	0.151 46	0.038 26	44
45	14.855	0.159 18	0.041 45	14.850	0.159 43	0.041 72	45
46	14.708	0.167 50	0.045 17	14.703	0.167 78	0.045 48	46
47	14.554	0.176 19	0.049 21	14.548	0.176 51	0.049 56	47
48	14.393	0.185 28	0.053 59	14.387	0.185 63	0.053 98	48
49	14.226	0.194 76	0.058 32	14.219	0.195 16	0.058 76	49
50	14.051	0.204 63	0.063 42	14.044	0.205 08	0.063 92	50
51	13.870	0.214 91	0.068 92	13.861	0.215 42	0.069 49	51
52	13.681	0.225 60	0.074 83	13.671	0.226 17	0.075 48	52
53	13.485	0.236 69	0.081 18	13.474	0.237 34	0.081 92	53
54	13.282	0.248 18	0.087 97	13.269	0.248 92	0.088 82	54
55	13.072	0.260 08	0.095 24	13.057	0.260 92	0.096 21	55
56	12.855	0.272 37	0.102 98	12.838	0.273 33	0.104 09	56
57	12.631	0.285 06	0.111 23	12.612	0.286 14	0.112 50	57
58	12.400	0.298 12	0.119 98	12.378	0.299 35	0.121 44	58
59	12.163	0.311 55	0.129 26	12.138	0.312 94	0.130 93	59
60	11.919	0.325 33	0.139 07	11.891	0.326 92	0.140 98	60
61	11.670	0.339 45	0.149 41	11.638	0.341 25	0.151 60	61
62	11.415	0.353 88	0.160 29	11.379	0.355 92	0.162 80	62
63	11.155	0.368 61	0.171 71	11.114	0.370 91	0.174 57	63
64	10.890	0.383 60	0.183 66	10.844	0.386 20	0.186 92	64

Note. $^2A_{[x]} = A_{[x]}$ at 12.36% and $^2A_x = A_x$ at 12.36%.

AM92

6%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	$^2A_{[x]}$	$\ddot{a}_x$	A_x	^2A_x	x
65	10.621	0.398 83	0.196 14	10.569	0.401 77	0.199 85	65
66	10.348	0.414 27	0.209 13	10.289	0.417 58	0.213 35	66
67	10.072	0.429 88	0.222 62	10.006	0.433 61	0.227 40	67
68	9.794	0.445 64	0.236 58	9.720	0.449 82	0.242 00	68
69	9.513	0.461 50	0.251 00	9.431	0.466 17	0.257 12	69
70	9.232	0.477 43	0.265 83	9.140	0.482 65	0.272 74	70
71	8.950	0.493 38	0.281 06	8.848	0.499 19	0.288 82	71
72	8.669	0.509 33	0.296 64	8.555	0.515 78	0.305 34	72
73	8.388	0.525 21	0.312 54	8.262	0.532 36	0.322 26	73
74	8.109	0.541 01	0.328 70	7.969	0.548 90	0.339 55	74
75	7.832	0.556 67	0.345 09	7.679	0.565 35	0.357 14	75
76	7.559	0.572 15	0.361 64	7.390	0.581 69	0.375 01	76
77	7.289	0.587 42	0.378 33	7.105	0.597 86	0.393 09	77
78	7.024	0.602 44	0.395 08	6.822	0.613 83	0.411 33	78
79	6.763	0.617 17	0.411 86	6.544	0.629 56	0.429 69	79
80	6.509	0.631 59	0.428 60	6.271	0.645 01	0.448 11	80
81	6.260	0.645 66	0.445 25	6.004	0.660 16	0.466 52	81
82	6.018	0.659 35	0.461 77	5.742	0.674 97	0.484 88	82
83	5.783	0.672 65	0.478 11	5.487	0.689 42	0.503 13	83
84	5.556	0.685 53	0.494 22	5.239	0.703 46	0.521 21	84
85	5.336	0.697 97	0.510 05	4.998	0.717 10	0.539 07	85
86	5.124	0.709 97	0.525 57	4.765	0.730 29	0.556 67	86
87	4.920	0.721 50	0.540 75	4.540	0.743 04	0.573 96	87
88	4.724	0.732 58	0.555 55	4.323	0.755 31	0.590 88	88
89	4.537	0.743 18	0.569 94	4.114	0.767 11	0.607 41	89
90	4.358	0.753 32	0.583 90	3.914	0.778 43	0.623 50	90
91				3.723	0.789 25	0.639 13	91
92				3.541	0.799 59	0.654 26	92
93				3.367	0.809 44	0.668 88	93
94				3.201	0.818 80	0.682 96	94
95				3.044	0.827 69	0.696 49	95
96				2.896	0.836 10	0.709 46	96
97				2.755	0.844 06	0.721 87	97
98				2.622	0.851 56	0.733 70	98
99				2.498	0.858 63	0.744 96	99
100				2.380	0.865 27	0.755 65	100
101				2.270	0.871 51	0.765 79	101
102				2.167	0.877 36	0.775 37	102
103				2.070	0.882 83	0.784 42	103
104				1.980	0.887 94	0.792 93	104
105				1.895	0.892 71	0.800 94	105
106				1.817	0.897 15	0.808 45	106
107				1.744	0.901 28	0.815 48	107
108				1.676	0.905 11	0.822 05	108
109				1.614	0.908 66	0.828 17	109
110				1.556	0.911 95	0.833 87	110
111				1.502	0.914 99	0.839 17	111
112				1.452	0.917 79	0.844 08	112
113				1.407	0.920 37	0.848 61	113
114				1.365	0.922 75	0.852 80	114
115				1.326	0.924 92	0.856 66	115
116				1.291	0.926 93	0.860 22	116
117				1.258	0.928 80	0.863 55	117
118				1.224	0.930 72	0.866 94	118
119				1.172	0.933 64	0.872 10	119
120				1.000	0.943 40	0.890 00	120

Note. $^2A_{[x]} = A_{[x]}$ at 12.36% and $^2A_x = A_x$ at 12.36%.

AM92

6%

x	$(I\ddot{a})_{[x]}$	$(IA)_{[x]}$	$(I\ddot{a})_x$	$(IA)_x$	x
17	268.142	1.799 55	268.083	1.799 40	17
18	266.392	1.867 08	266.336	1.866 92	18
19	264.567	1.936 81	264.514	1.936 64	19
20	262.666	2.008 74	262.615	2.008 56	20
21	260.687	2.082 89	260.638	2.082 70	21
22	258.626	2.159 25	258.579	2.159 06	22
23	256.482	2.237 82	256.437	2.237 62	23
24	254.253	2.318 58	254.210	2.318 37	24
25	251.936	2.401 51	251.896	2.401 29	25
26	249.531	2.486 57	249.491	2.486 35	26
27	247.034	2.573 73	246.996	2.573 50	27
28	244.444	2.662 93	244.407	2.662 70	28
29	241.759	2.754 10	241.724	2.753 86	29
30	238.978	2.847 18	238.943	2.846 92	30
31	236.099	2.942 06	236.065	2.941 80	31
32	233.120	3.038 64	233.087	3.038 37	32
33	230.041	3.136 81	230.008	3.136 53	33
34	226.861	3.236 43	226.827	3.236 13	34
35	223.579	3.337 35	223.545	3.337 02	35
36	220.194	3.439 40	220.159	3.439 04	36
37	216.706	3.542 39	216.671	3.542 00	37
38	213.116	3.646 13	213.079	3.645 69	38
39	209.424	3.750 37	209.385	3.749 89	39
40	205.630	3.854 89	205.589	3.854 35	40
41	201.736	3.959 42	201.692	3.958 80	41
42	197.744	4.063 68	197.696	4.062 97	42
43	193.654	4.167 36	193.603	4.166 55	43
44	189.471	4.270 14	189.416	4.269 22	44
45	185.197	4.371 70	185.136	4.370 62	45
46	180.834	4.471 66	180.768	4.470 41	46
47	176.388	4.569 65	176.315	4.568 20	47
48	171.863	4.665 29	171.783	4.663 59	48
49	167.264	4.758 18	167.175	4.756 18	49
50	162.597	4.847 89	162.497	4.845 55	50
51	157.867	4.934 00	157.757	4.931 26	51
52	153.082	5.016 09	152.959	5.012 87	52
53	148.249	5.093 72	148.113	5.089 94	53
54	143.376	5.166 47	143.224	5.162 03	54
55	138.472	5.233 89	138.302	5.228 68	55
56	133.545	5.295 58	133.356	5.289 47	56
57	128.605	5.351 13	128.394	5.343 97	57
58	123.662	5.400 16	123.427	5.391 76	58
59	118.726	5.442 29	118.464	5.432 47	59
60	113.808	5.477 20	113.516	5.465 72	60
61	108.918	5.504 57	108.594	5.491 18	61
62	104.067	5.524 16	103.707	5.508 56	62
63	99.267	5.535 74	98.868	5.517 59	63
64	94.528	5.539 13	94.087	5.518 08	64

AM92

6%

x	$(I\ddot{a})_{[x]}$	$(IA)_{[x]}$	$(I\ddot{a})_x$	$(IA)_x$	x
65	89.861	5.534 21	89.374	5.509 85	65
66	85.277	5.520 93	84.740	5.492 80	66
67	80.785	5.499 28	80.196	5.466 88	67
68	76.397	5.469 31	75.752	5.432 09	68
69	72.121	5.431 14	71.416	5.388 51	69
70	67.965	5.384 97	67.198	5.336 28	70
71	63.939	5.331 01	63.105	5.275 60	71
72	60.048	5.269 59	59.146	5.206 73	72
73	56.300	5.201 07	55.326	5.129 99	73
74	52.700	5.125 86	51.652	5.045 77	74
75	49.251	5.044 44	48.128	4.954 52	75
76	45.958	4.957 31	44.758	4.856 72	76
77	42.822	4.865 04	41.545	4.752 91	77
78	39.846	4.768 19	38.491	4.643 69	78
79	37.028	4.667 37	35.596	4.529 64	79
80	34.369	4.563 20	32.860	4.411 42	80
81	31.866	4.456 30	30.283	4.289 68	81
82	29.517	4.347 29	27.861	4.165 09	82
83	27.320	4.236 78	25.594	4.038 31	83
84	25.268	4.125 36	23.475	3.910 00	84
85	23.359	4.013 61	21.503	3.780 82	85
86	21.586	3.902 05	19.671	3.651 39	86
87	19.944	3.791 19	17.974	3.522 31	87
88	18.426	3.681 49	16.406	3.394 16	88
89	17.026	3.573 36	14.962	3.267 46	89
90	15.738	3.467 16	13.634	3.142 70	90
91			12.417	3.020 33	91
92			11.303	2.900 75	92
93			10.287	2.784 31	93
94			9.361	2.671 32	94
95			8.518	2.562 02	95
96			7.754	2.456 63	96
97			7.061	2.355 32	97
98			6.435	2.258 21	98
99			5.869	2.165 37	99
100			5.358	2.076 86	100
101			4.898	1.992 70	101
102			4.483	1.912 86	102
103			4.111	1.837 31	103
104			3.776	1.765 98	104
105			3.475	1.698 78	105
106			3.205	1.635 63	106
107			2.963	1.576 39	107
108			2.746	1.520 96	108
109			2.551	1.469 20	109
110			2.377	1.420 96	110
111			2.221	1.376 11	111
112			2.081	1.334 50	112
113			1.956	1.295 98	113
114			1.845	1.260 40	114
115			1.744	1.227 60	115
116			1.654	1.197 34	116
117			1.570	1.169 04	117
118			1.481	1.140 18	118
119			1.345	1.096 31	119
120			1.000	0.943 40	120

AM92

6%

x	$\ddot{a}_{[x]:\overline{n}\vert}$	$A_{[x]:\overline{n}\vert}$	$n = 60 - x$	$\ddot{a}_{x:\overline{n}\vert}$	$A_{x:\overline{n}\vert}$	x
17	16.076	0.090 05	43	16.072	0.090 24	17
18	15.990	0.094 93	42	15.986	0.095 11	18
19	15.898	0.100 11	41	15.895	0.100 28	19
20	15.801	0.105 61	40	15.798	0.105 77	20
21	15.698	0.111 45	39	15.695	0.111 60	21
22	15.588	0.117 64	38	15.586	0.117 79	22
23	15.472	0.124 22	37	15.470	0.124 36	23
24	15.349	0.131 19	36	15.347	0.131 33	24
25	15.218	0.138 59	35	15.216	0.138 72	25
26	15.080	0.146 43	34	15.078	0.146 56	26
27	14.933	0.154 75	33	14.931	0.154 87	27
28	14.777	0.163 57	32	14.775	0.163 69	28
29	14.612	0.172 92	31	14.610	0.173 03	29
30	14.437	0.182 83	30	14.435	0.182 94	30
31	14.251	0.193 33	29	14.249	0.193 44	31
32	14.054	0.204 46	28	14.053	0.204 57	32
33	13.846	0.216 26	27	13.844	0.216 36	33
34	13.625	0.228 75	26	13.624	0.228 85	34
35	13.392	0.241 98	25	13.390	0.242 08	35
36	13.144	0.255 99	24	13.142	0.256 09	36
37	12.882	0.270 82	23	12.880	0.270 93	37
38	12.605	0.286 53	22	12.603	0.286 64	38
39	12.311	0.303 16	21	12.309	0.303 27	39
40	12.000	0.320 76	20	11.998	0.320 88	40
41	11.671	0.339 38	19	11.669	0.339 51	41
42	11.323	0.359 10	18	11.320	0.359 23	42
43	10.954	0.379 96	17	10.952	0.380 10	43
44	10.564	0.402 03	16	10.561	0.402 19	44
45	10.151	0.425 39	15	10.149	0.425 56	45
46	9.715	0.450 11	14	9.712	0.450 28	46
47	9.253	0.476 26	13	9.249	0.476 45	47
48	8.764	0.503 94	12	8.760	0.504 15	48
49	8.246	0.533 24	11	8.242	0.533 46	49
50	7.698	0.564 26	10	7.694	0.564 49	50
51	7.118	0.597 11	9	7.114	0.597 35	51
52	6.503	0.631 91	8	6.499	0.632 16	52
53	5.851	0.668 79	7	5.847	0.669 04	53
54	5.160	0.707 91	6	5.156	0.708 15	54
55	4.427	0.749 41	5	4.423	0.749 65	55
56	3.648	0.793 50	4	3.645	0.793 70	56
57	2.820	0.840 36	3	2.817	0.840 52	57
58	1.939	0.890 24	2	1.937	0.890 34	58
59	1.000	0.943 40	1	1.000	0.943 40	59

AM92

6%

x	$\ddot{a}_{[x]:\overline{n}|}$	$A_{[x]:\overline{n}|}$	$n = 65 - x$	$\ddot{a}_{x:\overline{n}|}$	$A_{x:\overline{n}|}$	x
17	16.409	0.071 21	48	16.405	0.071 40	17
18	16.343	0.074 95	47	16.339	0.075 13	18
19	16.272	0.078 92	46	16.269	0.079 09	19
20	16.198	0.083 13	45	16.195	0.083 30	20
21	16.119	0.087 61	44	16.116	0.087 77	21
22	16.035	0.092 36	43	16.032	0.092 51	22
23	15.946	0.097 40	42	15.943	0.097 54	23
24	15.852	0.102 74	41	15.849	0.102 88	24
25	15.751	0.108 42	40	15.749	0.108 55	25
26	15.645	0.114 43	39	15.643	0.114 56	26
27	15.532	0.120 81	38	15.530	0.120 94	27
28	15.413	0.127 58	37	15.411	0.127 70	28
29	15.286	0.134 75	36	15.284	0.134 86	29
30	15.152	0.142 34	35	15.150	0.142 46	30
31	15.010	0.150 39	34	15.008	0.150 50	31
32	14.859	0.158 92	33	14.857	0.159 03	32
33	14.700	0.167 95	32	14.698	0.168 06	33
34	14.531	0.177 51	31	14.529	0.177 62	34
35	14.352	0.187 63	30	14.350	0.187 74	35
36	14.163	0.198 33	29	14.161	0.198 45	36
37	13.963	0.209 67	28	13.960	0.209 79	37
38	13.751	0.221 65	27	13.749	0.221 78	38
39	13.527	0.234 33	26	13.525	0.234 46	39
40	13.290	0.247 74	25	13.288	0.247 87	40
41	13.040	0.261 91	24	13.037	0.262 06	41
42	12.775	0.276 89	23	12.772	0.277 05	42
43	12.495	0.292 72	22	12.492	0.292 89	43
44	12.200	0.309 44	21	12.197	0.309 63	44
45	11.888	0.327 11	20	11.884	0.327 31	45
46	11.558	0.345 78	19	11.554	0.345 99	46
47	11.210	0.365 49	18	11.206	0.365 72	47
48	10.842	0.386 30	17	10.837	0.386 56	48
49	10.454	0.408 28	16	10.449	0.408 57	49
50	10.044	0.431 50	15	10.038	0.431 81	50
51	9.610	0.456 02	14	9.604	0.456 35	51
52	9.153	0.481 91	13	9.146	0.482 28	52
53	8.669	0.509 27	12	8.662	0.509 67	53
54	8.159	0.538 19	11	8.151	0.538 62	54
55	7.618	0.568 77	10	7.610	0.569 22	55
56	7.047	0.601 12	9	7.038	0.601 60	56
57	6.442	0.635 36	8	6.433	0.635 86	57
58	5.801	0.671 65	7	5.792	0.672 16	58
59	5.121	0.710 15	6	5.112	0.710 66	59
60	4.398	0.751 04	5	4.390	0.751 52	60
61	3.630	0.794 54	4	3.622	0.794 97	61
62	2.811	0.840 90	3	2.805	0.841 23	62
63	1.936	0.890 42	2	1.933	0.890 60	63
64	1.000	0.943 40	1	1.000	0.943 40	64

PENSIONER MORTALITY TABLES

PMA92 and PFA92 (Base tables)
and
PMA92C20 and PFA92C20 (Projected tables)

The Base tables are based on the mortality of pensioners insured by UK life offices during the years 1991, 1992, 1993, and 1994. Mortality is measured by amounts of annuities held.

The projected tables are projected to the calendar year 2020.

Full details are given in *C.M.I.R.* **16** and **17**.

PROJECTION FORMULAE

The projected mortality rate applicable in a particular calendar year is calculated using the formula:

$$q_x^{Year}\ (projected) = q_x^{Base} \times RF(x,t) \quad \text{where} \quad t = Year - 1992$$

The reduction factor is calculated as: $RF(x,t) = \alpha + (1-\alpha)(1-f)^{t/20}$

The parameters used are:

Age range	α	f
$x < 60$	0.13	0.55
$60 \le x \le 110$	$1 - 0.87\left(\frac{110-x}{50}\right)$	$0.55\left(\frac{110-x}{50}\right) + 0.29\left(\frac{x-60}{50}\right)$
$x > 110$	1	0.29

PMA92Base

x	q_x
50	0.001 315
51	0.001 519
52	0.001 761
53	0.002 045
54	0.002 379
55	0.002 771
56	0.003 228
57	0.003 759
58	0.004 376
59	0.005 090
60	0.005 914
61	0.006 861
62	0.007 947
63	0.009 189
64	0.010 604
65	0.012 211
66	0.014 032
67	0.016 088
68	0.018 402
69	0.020 998
70	0.023 901
71	0.027 137
72	0.030 732
73	0.034 713
74	0.039 105
75	0.043 935
76	0.049 227
77	0.055 006
78	0.061 292
79	0.068 106
80	0.075 464
81	0.083 379
82	0.091 862
83	0.100 917
84	0.110 544
85	0.120 739
86	0.131 492
87	0.142 786
88	0.154 599
89	0.166 903
90	0.179 664
91	0.192 841
92	0.206 389
93	0.220 257
94	0.234 389
95	0.248 727
96	0.263 206
97	0.277 762
98	0.292 327
99	0.306 832
100	0.321 209
101	0.335 389
102	0.349 305
103	0.362 893
104	0.376 091
105	0.388 838

PFA92base

x	q_x
50	0.001 271
51	0.001 456
52	0.001 670
53	0.001 917
54	0.002 200
55	0.002 524
56	0.002 894
57	0.003 317
58	0.003 799
59	0.004 345
60	0.004 965
61	0.005 667
62	0.006 458
63	0.007 350
64	0.008 352
65	0.009 476
66	0.010 734
67	0.012 138
68	0.013 703
69	0.015 442
70	0.017 371
71	0.019 505
72	0.021 861
73	0.024 455
74	0.027 306
75	0.030 432
76	0.033 849
77	0.037 577
78	0.041 632
79	0.046 035
80	0.050 800
81	0.055 946
82	0.061 488
83	0.067 441
84	0.073 817
85	0.080 629
86	0.087 885
87	0.095 594
88	0.103 761
89	0.112 386
90	0.121 470
91	0.131 009
92	0.140 996
93	0.151 420
94	0.162 267
95	0.173 519
96	0.185 155
97	0.197 150
98	0.209 477
99	0.222 103
100	0.234 995
101	0.248 115
102	0.261 424
103	0.274 879
104	0.288 437
105	0.302 054

PMA92C20

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
50	9 941.923	5.418	0.000 545	0.000 507	34.10	50
51	9 936.504	6.260	0.000 630	0.000 585	33.12	51
52	9 930.244	7.249	0.000 730	0.000 677	32.14	52
53	9 922.995	8.415	0.000 848	0.000 786	31.17	53
54	9 914.580	9.776	0.000 986	0.000 914	30.19	54
55	9 904.805	11.371	0.001 148	0.001 063	29.22	55
56	9 893.434	13.237	0.001 338	0.001 239	28.25	56
57	9 880.196	15.393	0.001 558	0.001 444	27.29	57
58	9 864.803	17.895	0.001 814	0.001 681	26.33	58
59	9 846.908	20.777	0.002 110	0.001 957	25.38	59
60	9 826.131	24.084	0.002 451	0.002 266	24.43	60
61	9 802.048	28.965	0.002 955	0.002 685	23.49	61
62	9 773.083	34.694	0.003 550	0.003 241	22.56	62
63	9 738.388	41.398	0.004 251	0.003 889	21.64	63
64	9 696.990	49.193	0.005 073	0.004 651	20.73	64
65	9 647.797	58.195	0.006 032	0.005 543	19.83	65
66	9 589.602	68.537	0.007 147	0.006 583	18.95	66
67	9 521.065	80.348	0.008 439	0.007 792	18.08	67
68	9 440.717	93.746	0.009 930	0.009 191	17.23	68
69	9 346.970	108.836	0.011 644	0.010 806	16.40	69
70	9 238.134	125.685	0.013 605	0.012 661	15.59	70
71	9 112.449	144.350	0.015 841	0.014 783	14.79	71
72	8 968.099	164.834	0.018 380	0.017 204	14.02	72
73	8 803.265	187.096	0.021 253	0.019 956	13.28	73
74	8 616.170	211.010	0.024 490	0.023 072	12.55	74
75	8 405.160	236.362	0.028 121	0.026 587	11.86	75
76	8 168.798	262.864	0.032 179	0.030 537	11.18	76
77	7 905.934	290.116	0.036 696	0.034 962	10.54	77
78	7 615.818	317.595	0.041 702	0.039 899	9.92	78
79	7 298.223	344.688	0.047 229	0.045 390	9.33	79
80	6 953.536	370.644	0.053 303	0.051 473	8.77	80
81	6 582.891	394.658	0.059 952	0.058 188	8.23	81
82	6 188.234	415.856	0.067 201	0.065 576	7.73	82
83	5 772.378	433.321	0.075 068	0.073 676	7.25	83
84	5 339.057	446.180	0.083 569	0.082 522	6.80	84
85	4 892.878	453.648	0.092 716	0.092 149	6.37	85
86	4 439.230	455.092	0.102 516	0.102 590	5.97	86
87	3 984.138	450.084	0.112 969	0.113 873	5.59	87
88	3 534.054	438.463	0.124 068	0.126 023	5.24	88
89	3 095.591	420.387	0.135 802	0.139 060	4.91	89
90	2 675.203	396.334	0.148 151	0.152 998	4.61	90
91	2 278.869	367.099	0.161 088	0.167 846	4.32	91
92	1 911.771	333.759	0.174 581	0.183 606	4.06	92
93	1 578.012	297.596	0.188 589	0.200 273	3.81	93
94	1 280.416	260.008	0.203 065	0.217 836	3.59	94
95	1 020.409	222.405	0.217 957	0.236 273	3.38	95
96	798.003	186.098	0.233 205	0.255 556	3.18	96
97	611.905	152.209	0.248 746	0.275 647	3.00	97
98	459.696	121.595	0.264 511	0.296 499	2.84	98
99	338.101	94.813	0.280 429	0.318 054	2.68	99
100	243.288	72.117	0.296 425	0.340 247	2.54	100
101	171.171	53.478	0.312 423	0.363 002	2.41	101
102	117.693	38.644	0.328 344	0.386 232	2.29	102
103	79.050	27.202	0.344 113	0.409 842	2.18	103
104	51.848	18.647	0.359 653	0.433 729	2.08	104
105	33.200	12.446	0.374 887	0.457 778	1.99	105

PFA92C20

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
50	9 952.697	5.245	0.000 527	0.000 492	37.08	50
51	9 947.452	5.998	0.000 603	0.000 563	36.10	51
52	9 941.454	6.879	0.000 692	0.000 645	35.12	52
53	9 934.574	7.898	0.000 795	0.000 741	34.15	53
54	9 926.676	9.053	0.000 912	0.000 851	33.17	54
55	9 917.623	10.374	0.001 046	0.000 976	32.20	55
56	9 907.249	11.879	0.001 199	0.001 120	31.24	56
57	9 895.370	13.606	0.001 375	0.001 284	30.27	57
58	9 881.764	15.564	0.001 575	0.001 472	29.31	58
59	9 866.200	17.769	0.001 801	0.001 685	28.36	59
60	9 848.431	20.268	0.002 058	0.001 918	27.41	60
61	9 828.163	23.991	0.002 441	0.002 236	26.46	61
62	9 804.173	28.285	0.002 885	0.002 655	25.53	62
63	9 775.888	33.248	0.003 401	0.003 135	24.60	63
64	9 742.640	38.932	0.003 996	0.003 691	23.68	64
65	9 703.708	45.423	0.004 681	0.004 332	22.78	65
66	9 658.285	52.802	0.005 467	0.005 069	21.88	66
67	9 605.483	61.158	0.006 367	0.005 914	21.00	67
68	9 544.325	70.580	0.007 395	0.006 882	20.13	68
69	9 473.745	81.124	0.008 563	0.007 986	19.28	69
70	9 392.621	92.874	0.009 888	0.009 240	18.44	70
71	9 299.747	105.887	0.011 386	0.010 663	17.62	71
72	9 193.860	120.210	0.013 075	0.012 272	16.81	72
73	9 073.650	135.860	0.014 973	0.014 086	16.03	73
74	8 937.791	152.836	0.017 100	0.016 126	15.27	74
75	8 784.955	171.113	0.019 478	0.018 414	14.52	75
76	8 613.841	190.598	0.022 127	0.020 974	13.80	76
77	8 423.243	211.162	0.025 069	0.023 829	13.10	77
78	8 212.080	232.615	0.028 326	0.027 004	12.42	78
79	7 979.465	254.729	0.031 923	0.030 527	11.77	79
80	7 724.737	277.179	0.035 882	0.034 425	11.14	80
81	7 447.558	299.593	0.040 227	0.038 728	10.54	81
82	7 147.965	321.523	0.044 981	0.043 464	9.96	82
83	6 826.442	342.455	0.050 166	0.048 664	9.41	83
84	6 483.987	361.832	0.055 804	0.054 357	8.88	84
85	6 122.154	379.053	0.061 915	0.060 576	8.37	85
86	5 743.101	393.506	0.068 518	0.067 349	7.89	86
87	5 349.595	404.595	0.075 631	0.074 708	7.43	87
88	4 945.000	411.770	0.083 270	0.082 686	7.00	88
89	4 533.230	414.537	0.091 444	0.091 308	6.59	89
90	4 118.693	412.545	0.100 164	0.100 604	6.20	90
91	3 706.149	405.590	0.109 437	0.110 601	5.84	91
92	3 300.559	393.644	0.119 266	0.121 325	5.49	92
93	2 906.914	376.882	0.129 650	0.132 801	5.17	93
94	2 530.033	355.677	0.140 582	0.145 048	4.87	94
95	2 174.356	330.617	0.152 053	0.158 084	4.58	95
96	1 843.738	302.467	0.164 051	0.171 926	4.32	96
97	1 541.271	272.119	0.176 555	0.186 586	4.07	97
98	1 269.152	240.562	0.189 545	0.202 071	3.84	98
99	1 028.591	208.795	0.202 991	0.218 386	3.62	99
100	819.796	177.783	0.216 863	0.235 531	3.41	100
101	642.013	148.385	0.231 125	0.253 502	3.22	101
102	493.627	121.303	0.245 737	0.272 288	3.05	102
103	372.325	97.048	0.260 654	0.291 872	2.89	103
104	275.277	75.930	0.275 830	0.312 234	2.73	104
105	199.347	58.053	0.291 217	0.333 348	2.59	105

4%

PMA92C20

x	$\ddot{a}_x$	^2A_x
50	18.843	0.088 02
51	18.567	0.094 71
52	18.281	0.101 87
53	17.985	0.109 54
54	17.680	0.117 73
55	17.364	0.126 47
56	17.038	0.135 80
57	16.702	0.145 74
58	16.356	0.156 32
59	15.999	0.167 56
60	15.632	0.179 50
61	15.254	0.192 17
62	14.868	0.205 50
63	14.475	0.219 50
64	14.073	0.234 16
65	13.666	0.249 46
66	13.252	0.265 38
67	12.834	0.281 90
68	12.412	0.298 99
69	11.988	0.316 60
70	11.562	0.334 69
71	11.136	0.353 20
72	10.711	0.372 08
73	10.288	0.391 25
74	9.870	0.410 65
75	9.456	0.430 21
76	9.049	0.449 84
77	8.649	0.469 47
78	8.258	0.489 03
79	7.877	0.508 44
80	7.506	0.527 62
81	7.148	0.546 50
82	6.801	0.565 01
83	6.468	0.583 10
84	6.148	0.600 71
85	5.842	0.617 79
86	5.551	0.634 29
87	5.273	0.650 19
88	5.010	0.665 45
89	4.762	0.680 06
90	4.527	0.693 99
91	4.306	0.707 25
92	4.098	0.719 83
93	3.903	0.731 74
94	3.721	0.742 97
95	3.551	0.753 56
96	3.393	0.763 50
97	3.245	0.772 82
98	3.109	0.781 55
99	2.982	0.789 69
100	2.864	0.797 28
101	2.755	0.804 34
102	2.655	0.810 89
103	2.562	0.816 96
104	2.477	0.822 57
105	2.399	0.827 74

PFA92C20

x	$\ddot{a}_x$	^2A_x
50	19.539	0.074 21
51	19.291	0.079 78
52	19.034	0.085 74
53	18.768	0.092 11
54	18.494	0.098 91
55	18.210	0.106 16
56	17.917	0.113 90
57	17.615	0.122 14
58	17.303	0.130 91
59	16.982	0.140 24
60	16.652	0.150 15
61	16.311	0.160 68
62	15.963	0.171 77
63	15.606	0.183 43
64	15.242	0.195 66
65	14.871	0.208 47
66	14.494	0.221 83
67	14.111	0.235 76
68	13.723	0.250 22
69	13.330	0.265 21
70	12.934	0.280 69
71	12.535	0.296 64
72	12.135	0.313 02
73	11.734	0.329 80
74	11.333	0.346 93
75	10.933	0.364 37
76	10.536	0.382 07
77	10.142	0.399 97
78	9.752	0.418 02
79	9.367	0.436 16
80	8.989	0.454 33
81	8.618	0.472 47
82	8.254	0.490 53
83	7.900	0.508 45
84	7.555	0.526 16
85	7.220	0.543 63
86	6.896	0.560 80
87	6.582	0.577 62
88	6.281	0.594 05
89	5.991	0.610 06
90	5.713	0.625 60
91	5.447	0.640 66
92	5.193	0.655 20
93	4.951	0.669 21
94	4.722	0.682 68
95	4.504	0.695 59
96	4.297	0.707 94
97	4.102	0.719 73
98	3.918	0.730 97
99	3.744	0.741 64
100	3.581	0.751 77
101	3.428	0.761 36
102	3.284	0.770 43
103	3.149	0.778 99
104	3.023	0.787 05
105	2.905	0.794 63

Note. $^2A_x = A_x$ at 8.16%.

PMA92C20 and PFA92C20

4%

$\ddot{a}_{xy}$ for male (x) and female (y)

Age difference d $(= y - x)$

x	*d* −20	−10	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+10	+20 *d*	*x*
50	18.746	18.493	18.192	18.110	18.019	17.918	17.808	17.688	17.556	17.413	17.258	17.090	16.909	15.801	12.638	50
51	18.467	18.206	17.894	17.809	17.715	17.612	17.498	17.374	17.238	17.091	16.931	16.758	16.572	15.433	12.232	51
52	18.179	17.908	17.586	17.499	17.402	17.295	17.178	17.050	16.910	16.758	16.594	16.416	16.225	15.057	11.823	52
53	17.881	17.601	17.269	17.178	17.078	16.968	16.848	16.716	16.572	16.415	16.246	16.064	15.867	14.672	11.413	53
54	17.573	17.283	16.941	16.847	16.744	16.631	16.507	16.371	16.223	16.062	15.888	15.701	15.499	14.279	11.004	54
55	17.255	16.955	16.602	16.506	16.400	16.284	16.156	16.016	15.864	15.699	15.521	15.328	15.121	13.880	10.595	55
56	16.926	16.617	16.253	16.155	16.046	15.926	15.795	15.651	15.495	15.326	15.143	14.945	14.733	13.473	10.189	56
57	16.587	16.269	15.894	15.793	15.681	15.558	15.423	15.276	15.116	14.942	14.755	14.553	14.337	13.061	9.786	57
58	16.238	15.910	15.525	15.421	15.306	15.180	15.041	14.891	14.727	14.549	14.357	14.151	13.932	12.644	9.387	58
59	15.879	15.541	15.146	15.039	14.921	14.791	14.650	14.495	14.327	14.145	13.950	13.742	13.520	12.222	8.993	59
60	15.509	15.161	14.756	14.646	14.526	14.393	14.248	14.090	13.918	13.734	13.536	13.325	13.101	11.796	8.605	60
61	15.129	14.772	14.356	14.244	14.121	13.985	13.837	13.675	13.501	13.314	13.114	12.901	12.675	11.368	8.224	61
62	14.740	14.374	13.949	13.834	13.708	13.569	13.418	13.254	13.078	12.888	12.686	12.472	12.245	10.939	7.851	62
63	14.343	13.968	13.533	13.416	13.287	13.145	12.992	12.826	12.648	12.458	12.255	12.039	11.812	10.511	7.487	63
64	13.939	13.555	13.111	12.991	12.859	12.716	12.561	12.394	12.215	12.023	11.819	11.604	11.376	10.085	7.133	64
65	13.529	13.136	12.682	12.560	12.427	12.282	12.126	11.958	11.778	11.586	11.382	11.167	10.940	9.662	6.790	65
66	13.112	12.711	12.248	12.125	11.991	11.845	11.688	11.520	11.339	11.147	10.944	10.729	10.504	9.243	6.457	66
67	12.692	12.282	11.811	11.687	11.552	11.406	11.248	11.080	10.900	10.708	10.506	10.293	10.070	8.830	6.137	67
68	12.267	11.849	11.372	11.247	11.112	10.966	10.808	10.640	10.460	10.270	10.070	9.859	9.639	8.423	5.829	68
69	11.840	11.414	10.933	10.807	10.672	10.526	10.369	10.201	10.023	9.835	9.637	9.429	9.213	8.025	5.533	69
70	11.412	10.978	10.494	10.368	10.233	10.088	9.932	9.766	9.590	9.404	9.209	9.005	8.792	7.636	5.250	70
75	9.295	8.833	8.357	8.238	8.110	7.975	7.831	7.679	7.520	7.355	7.182	7.005	6.822	5.860	4.027	75
80	7.335	6.876	6.441	6.336	6.224	6.107	5.985	5.857	5.725	5.588	5.449	5.306	5.161	4.422	3.108	80
85	5.660	5.235	4.864	4.777	4.687	4.593	4.496	4.396	4.294	4.189	4.084	3.977	3.870	3.340	2.449	85
90	4.339	3.963	3.664	3.597	3.528	3.456	3.384	3.310	3.235	3.160	3.084	3.008	2.933	2.571	1.998	90
95	3.361	3.039	2.808	2.757	2.706	2.654	2.602	2.549	2.496	2.444	2.391	2.339	2.288	2.049	1.708	95
100	2.670	2.400	2.223	2.186	2.149	2.112	2.075	2.038	2.001	1.965	1.930	1.895	1.861	1.708	1.000	100

INTERNATIONAL ACTUARIAL NOTATION

Reproduced from *Bulletin of the Permanent Committee of the International Congress of Actuaries,* **46**, 207 (1949), *Journal of the Institute of Actuaries,* **75**, 121 (1949) and *Transactions of the Faculty of Actuaries,* **19**, 89 (1949–50).

The existing international actuarial notation was founded on the "Key to the Notation" given in the *Institute of Actuaries Text Book, Part II, Life Contingencies* by George King (1887), and was adopted by the Second International Actuarial Congress, London, 1898 (*Transactions*, pp. 618–640) with minor revisions approved by the Third International Congress, Paris, 1900 (*Transactions*, pp. 622–651). Further revisions were discussed during 1937–1939, and were introduced by the Institute and the Faculty in 1949 (*J.I.A.*, **75,** 121 and *T.F.A.*, **19,** 89). These revisions were finally adopted internationally at the Fourteenth International Actuarial Congress, Madrid, 1954 (*Bulletin of the Permanent Committee of the International Congress of Actuaries* (1949), **46,** pp. 207–217).

The general principles on which the system is based are as follows:

To each fundamental symbolic letter are attached signs and letters each having its own signification.

The lower space to the left is reserved for signs indicating the conditions relative to the duration of the operations and to their position with regard to time.

The lower space to the right is reserved for signs indicating the conditions relative to ages and the order of succession of the events.

The upper space to the right is reserved for signs indicating the periodicity of events.

The upper space to the left is free, and in it can be placed signs corresponding to other notions.

In what follows these two conventions are used:

A letter enclosed in brackets, thus (x), denotes "a person aged x".

A letter or number enclosed in a right angle, thus $\overline{n|}$ or $\overline{15|}$, denotes a term-certain of years.

1 FUNDAMENTAL SYMBOLIC LETTERS

1.1 INTEREST

i = the effective rate of interest, namely, the total interest earned on 1 in a year on the assumption that the actual interest (if receivable otherwise than yearly) is invested forthwith as it becomes due on the same terms as the original principal.

$v = (1+i)^{-1} =$ the present value of 1 due one year hence.

$d = 1 - v$ = the discount on 1 due one year hence.

$\delta = \log_e(1+i) = -\log_e(1-d)$ = the force of interest or the force of discount.

1.2 MORTALITY TABLES

l = number living.
d = number dying.
p = probability of living.
q = probability of dying.
μ = force of mortality.
m = central death rate.
a = present value of an annuity.
s = amount of an annuity.
e = expectation of life.
A = present value of an assurance.
E = present value of an endowment.
P = premium per annum. } P generally refers to net premiums, π to
π = premium per annum. } special premiums.
V = policy value.
W = paid-up policy.

The methods of using the foregoing principal letters and their precise meaning when added to by suffixes, etc., follow.

1.3 INTEREST

$i^{(m)} = m\{(1+i)^{1/m} - 1\} =$ the nominal rate of interest, convertible m times a year.

$a_{\overline{n|}} = v + v^2 + ... + v^n =$ the value of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.

$\ddot{a}_{\overline{n|}} = 1 + v + v^2 + ... + v^{n-1} =$ the value of a similar annuity, the payments being made at the beginning of each year.

$s_{\overline{n|}} = 1 + (1+i) + (1+i)^2 + ... + (1+i)^{n-1} =$ the amount of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.

$\ddot{s}_{\overline{n|}} = (1+i) + (1+i)^2 + ... + (1+i)^n =$ the amount of a similar annuity, the payments being made at the beginning of each year.

The diaeresis or trema (¨) above the letters *a* and *s* is used as a symbol of acceleration of payments.

1.4 MORTALITY TABLES

The ages of the lives involved are denoted by letters placed as suffixes in the lower space to the right. Thus:

$l_x =$ the number of persons who attain age x according to the mortality table.

$d_x = l_x - l_{x+1} =$ the number of persons who die between ages x and $x + 1$ according to the mortality table.

$p_x =$ the probability that (x) will live 1 year.

$q_x =$ the probability that (x) will die within 1 year.

$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} =$ the force of mortality at age x.

$m_x =$ the central death-rate for the year of age x to $x + 1$
$= d_x \Big/ \int_0^1 l_{x+t} dt.$

$e_x =$ the curtate "expectation of life" (or average after-lifetime) of (x).

In the following it is always to be understood (unless otherwise expressed) that the annual payment of an annuity is 1, that the sum assured in any case is 1, and that the symbols indicate the present values:

$a_x =$ an annuity, first payment at the end of a year, to continue during the life of (x).

$\ddot{a}_x = 1 + a_x =$ an "annuity-due" to continue during the life of (x), the first payment to be made at once.

$A_x =$ an assurance payable at the end of the year of death of (x).

Note. $e_x = a_x$ at rate of interest $i = 0$.

A letter or number at the lower left corner of the principal symbol denotes the number of years involved in the probability or benefit in question. Thus:

${}_n p_x =$ the probability that (x) will live n years.
${}_n q_x =$ the probability that (x) will die within n years.

Note. When $n = 1$ it is customary to omit it (as shown above) provided no ambiguity is introduced.

${}_n E_x = v^n {}_n p_x =$ the value of an endowment on (x) payable at the end of n years if (x) be then alive.

If the letter or number comes before a perpendicular bar it shows that a period of deferment is meant. Thus:

${}_{n|}q_x =$ the probability that (x) will die in a year, deferred n years; that is, that he will die in the $(n + 1)$th year.

${}_{n|}a_x =$ an annuity on (x) deferred n years; that is, that the first payment is to be made at the end of $(n + 1)$ years.

${}_{n|t}a_x =$ an intercepted, or deferred, temporary annuity on (x) deferred n years and, after that, to run for t years.

A letter or number in brackets at the upper right corner of the principal symbol shows the number of intervals into which the year is to be divided. Thus:

$a_x^{(m)} =$ an annuity of (x) payable by m instalments of $1/m$ each throughout the year, the first payment being one of $1/m$ at the end of the first $1/m$th of a year.

$\ddot{a}_x^{(m)} =$ a similar annuity but the first payment of $1/m$ is to be made at once, so that $\ddot{a}_x^{(m)} = 1/m + a_x^{(m)}$.

$A_x^{(m)} =$ an assurance payable at the end of that fraction $1/m$ of a year in which (x) dies.

If $m \to \infty$ then instead of writing (∞) a bar is placed over the principal symbol. Thus:

$\bar{a} =$ a continuous or momently annuity.
$\bar{A} =$ an assurance payable at the moment of death.

A small circle placed over the principal symbol shows that the benefit is to be complete. Thus:

$\overset{\circ}{a} =$ a complete annuity.
$\overset{\circ}{e} =$ the complete expectation of life.

Note. Some consider that $\overline{e}$ would be as appropriate as $\overset{\circ}{e}$. As $e_x = a_x$ at rate of interest $i = 0$, so also the complete expectation of life $= \overline{a}_x$ at rate of interest $i = 0$.

When more than one life is involved the following rules are observed:

If there are two or more letters or numbers in a suffix without any distinguishing mark, joint lives are intended. Thus:

$$l_{xy} = l_x \times l_y, \quad d_{xy} = l_{xy} - l_{x+1:y+1}.$$

Note. When, for the sake of distinctness, it is desired to separate letters or numbers in a suffix, a colon is placed between them. A colon is used instead of a point or comma to avoid confusion with decimals when numbers are involved.

$a_{xyz} =$ an annuity, first payment at the end of a year, to continue during the joint lives of (x), (y) and (z).

$A_{xyz} =$ an assurance payable at the end of the year of the failure of the joint lives (x), (y) and (z).

In place of a life a term-certain may be involved. Thus:

$a_{x:\overline{n}|} =$ an annuity to continue during the joint duration of the life of (x) and a term of n years certain; that is, a temporary annuity for n years on the life of (x).

$A_{x:\overline{n}|} =$ an assurance payable at the end of the year of death of (x) if he dies within n years, or at the end of n years if (x) be then alive; that is, an endowment assurance for n years.

If a perpendicular bar separates the letters in the suffix, then the status after the bar is to follow the status before the bar. Thus:

$a_{y|x}$ = a reversionary annuity, that is, an annuity on the life of (x) after the death of (y).

$A_{z|xy}$ = an assurance payable on the failure of the joint lives (x) and (y) provided both these lives survive (z).

If a horizontal bar appears above the suffix then survivors of the lives, and not joint lives, are intended. The number of survivors can be denoted by a letter or number over the right end of the bar. If that letter, say r, is not distinguished by any mark, then the meaning is *at least r* survivors; but if it is enclosed in square brackets, [r], then the meaning is *exactly r* survivors. If no letter or number appears over the bar, then unity is supposed and the meaning is *at least one* survivor. Thus:

$a_{\overline{xyz}}$ = an annuity payable so long as at least one of the three lives (x), (y) and (z) is alive.

$a_{\overline{xyz}}^{\;2}$ = an annuity payable so long as at least two of the three lives (x), (y) and (z) are alive.

$p_{\overline{xyz}}^{[2]}$ = probability that exactly two of the three lives (x), (y) and (z) will survive a year.

${}_nq_{\overline{xy}}$ = probability that the survivor of the two lives (x) and (y) will die within n years = ${}_nq_x \times {}_nq_y$.

${}_nA_{\overline{xy}}$ = an assurance payable at the end of the year of death of the survivor of the lives (x) and (y) provided the death occurs within n years.

When numerals are placed above or below the letters of the suffix, they designate the order in which the lives are to fail. The numeral placed *over* the suffix points out the life whose failure will finally determine the event; and the numerals placed *under* the suffix indicate the order in which the other lives involved are to fail. Thus:

A^{1}_{xy} = an assurance payable at the end of the year of death of (x) if he dies first of the two lives (x) and (y).

A^{2}_{xyz} = an assurance payable at the end of the year of death of (x) if he dies second of the three lives (x), (y) and (z).

$A^{2}_{x\underset{1}{y}z}$ = an assurance payable at the end of the year of death of (x) if he dies second of the three lives, (y) having died first.

$A_{xy:\underset{3}{z}}$ = an assurance payable at the end of the year of death of the survivor of (x) and (y) if he dies before (z).

$A^{1}_{x:\overline{n}|}$ = an assurance payable at the end of the year of death of (x) if he dies within a term of n years.

$$\left.\begin{array}{l} a_{\underset{1}{\overline{yz}}|x} \\ \text{or} \\ a^{2}_{\overline{yz}|x} \end{array}\right\} = \begin{array}{l} \text{an annuity to } (x) \text{ after the failure of the survivor of } (y) \text{ and } (z), \\ \text{provided } (z) \text{ fails before } (y). \end{array}$$

Note. Sometimes to make quite clear that a joint-life status is involved a symbol $\sqcap$ is placed above the lives included. Thus $A^{\overset{1}{\sqcap}}_{xy:\overline{n}|}$ = a joint-life temporary assurance on (x) and (y).

In the case of reversionary annuities, distinction has sometimes to be made between those where the times of year at which payments are to take place are determined at the outset and those where the times depend on the failure of the preceding status. Thus:

$a_{y|x}$ = annuity to (x), first payment at the end of the year of death of (y) or, on the average, about 6 months after his death.

$\hat{a}_{y|x}$ = annuity to (x), first payment 1 year after the death of (y).

$\hat{\mathring{a}}_{y|x}$ = complete annuity to (x), first payment 1 year after the death of (y).

2 ANNUAL PREMIUMS

The symbol P with the appropriate suffix or suffixes is used in simple cases, where no misunderstanding can occur, to denote the annual premium for a benefit. Thus:

$P_x =$ the annual premium for an assurance payable at the end of the year of death of (x).

$P_{x:\overline{n|}} =$ the annual premium for an endowment assurance on (x) payable after n years or at the end of the year of death of (x) if he dies within n years.

$P_{xy}^{1} =$ the annual premium for a contingent assurance payable at the end of the year of death of (x) if he dies before (y).

In all cases it is optional to use the symbol P in conjunction with the principal symbol denoting the benefit. Thus instead of $P_{x:\overline{n|}}$ we may write $P(A_{x:\overline{n|}})$. In the more complicated cases it is necessary to use the two symbols in this way. Suffixes, etc., showing the conditions of the benefit are to be attached to the principal letter, and those showing the condition of payment of the premium are to be attached to the subsidiary symbol P. Thus:

${}_nP(\bar{A}_x) =$ the annual premium payable for n years only for an assurance payable at the moment of death of (x).

$P_{xy}(A_x) =$ the annual premium payable during the joint lives of (x) and (y) for an assurance payable at the end of the year of death of (x).

${}_nP({}_{n|}a_x) =$ the annual premium payable for n years only for an annuity on (x) deferred n years.

${}_tP^{(m)}(A_{x:\overline{n|}}) =$ the annual premium payable for t years only, by m instalments throughout the year, for an endowment assurance for n years on (x) (see below as to $P^{(m)}$).

Notes. (1) As a general rule the symbol P could be used without the principal symbol in the case of assurances where the sum assured is payable at the end of the year of death, but if it is payable at other times, or if the benefit is an annuity, then the principal symbol should be used.

(2) $P_x^{(m)}$. A point which was not brought out when the international system was adopted is that there are two kinds of premiums payable m times a year, namely those which cease on payment of the instalment immediately preceding death and those which continue to be payable to the end of the year of death. To distinguish the latter, the m is sometimes enclosed in square brackets, thus $P^{[m]}$.

3 POLICY VALUES AND PAID-UP POLICIES

${}_tV_x =$ the value of an ordinary whole-life assurance on (x) which has been t years in force, the premium then just due being unpaid.

${}_tW_x =$ the paid-up policy the present value of which is ${}_tV_x$.

The symbols V and W may, in simple cases, be used alone, but in the more complicated cases it is necessary to insert the full symbol for the benefit thus:

$${}_tV^{(m)}(\bar{A}_{x:\overline{n|}}) \text{ (corresponding to } P^{(m)}(\bar{A}_{x:\overline{n|}})\text{)}, \quad {}_tV({}_{n|}a_x).$$

Note. As a general rule V or W can be used as the main symbol if the sum assured is payable at the end of the year of death and the premium is payable periodically throughout the duration of the assurance. If the premium is payable for a limited number of years, say n, the policy value after t years could be written ${}_tV[{}_nP(A)]$, or, if desired, ${}_t^nV(A)$.

In investigations where modified premiums and policy values are in question such modification may be denoted by adding accents to the symbols. Thus, when a premium other than the net premium (a valuation premium) is used in a valuation it may be denoted by P' and the corresponding policy value by V'. Similarly, the office (or commercial) premium may be denoted by P'' and the corresponding paid-up policy by W''.

4 COMPOUND SYMBOLS

$$\left.\begin{array}{l}(Ia) = \text{an annuity}\\(IA) = \text{an assurance}\end{array}\right\}\text{commencing at 1 and increasing 1 per annum.}$$

If the whole benefit is to be temporary the symbol of limitation is placed outside the brackets. Thus:

$(Ia)_{x:\overline{n}|} =$ a temporary increasing annuity.

$(IA)^{1}_{x:\overline{n}|} =$ a temporary increasing assurance.

If only the increase is to be temporary but the benefit is to continue thereafter, then the symbol of limitation is placed immediately after the symbol *I*. Thus:

$$\left.\begin{array}{l}(I_{\overline{n}|}a)_x = \text{a whole-life annuity}\\(I_{\overline{n}|}A)_x = \text{a whole-life assurance}\end{array}\right\}\begin{array}{l}\text{increasing for } n \text{ years and thereafter}\\\text{stationary.}\end{array}$$

If the benefit is a decreasing one, the corresponding symbol is *D*. From the nature of the case this decrease must have a limit, as otherwise negative values might be implied. Thus:

$(D_{\overline{n}|}A)^{1}_{x:\overline{n}|} =$ a temporary assurance commencing at *n* and decreasing by 1 in each successive year.

If the benefit is a varying one the corresponding symbol is *v*. Thus:

$(va) =$ a varying annuity.

5 COMMUTATION COLUMNS

5.1 SINGLE LIVES

$D_x = v^x l_x,$

$N_x = D_x + D_{x+1} + D_{x+2} + \text{etc.},$

$S_x = N_x + N_{x+1} + N_{x+2} + \text{etc.},$

$C_x = v^{x+1} d_x,$

$M_x = C_x + C_{x+1} + C_{x+2} + \text{etc.},$

$R_x = M_x + M_{x+1} + M_{x+2} + \text{etc.}$

When it is desired to construct the assurance columns so as to give directly assurances payable at the moment of death, the symbols are distinguished by a bar placed over them. Thus:

$\bar{C}_x = v^{x+½} d_x$ which is an approximation to $\int_0^1 v^{x+t} \mu_{x+t} l_{x+t} dt.$

$\bar{M}_x = \bar{C}_x + \bar{C}_{x+1} + \bar{C}_{x+2} + \text{etc.}$

$\bar{R}_x = \bar{M}_x + \bar{M}_{x+1} + \bar{M}_{x+2} + \text{etc.}$

5.2 JOINT LIVES

$D_{xy} = v^{½(x+y)} l_{xy},$

$N_{xy} = D_{xy} + D_{x+1:y+1} + D_{x+2:y+2} + \text{etc.}$

$C_{xy} = v^{½(x+y)+1} d_{xy},$

$M_{xy} = C_{xy} + C_{x+1:y+1} + C_{x+2:y+2} + \text{etc.}$

$C^1_{xy} = v^{½(x+y)+1} d_x l_{y+½},$

$M^1_{xy} = C^1_{xy} + C^{\,1}_{x+1:y+1} + C^{\,1}_{x+2:y+2} + \text{etc.}$

6 SELECTION

If the suffix to a symbol which denotes the age is enclosed in a square bracket it indicates the age at which the life was selected. To this may be added, outside the bracket, the number of years which have elapsed since selection, so that the total suffix denotes the present age. Thus:

$l_{[x]+t} =$ the number in the select life table who were selected at age x and have attained age $x + t$.

$d_{[x]+t} = l_{[x]+t} - l_{[x]+t+1}$.

$a_{[x]} =$ value of an annuity on a life now aged x and now select.

$a_{[x-n]+n} =$ value of an annuity on a life now aged x and select n years ago at age $x - n$.

$N_{[x]} = D_{[x]} + D_{[x]+1} + D_{[x]+2} + \cdots$

$\ddot{a}_{[x]} = N_{[x]} \div D_{[x]} = 1 + a_{[x]},$

and similarly for other functions.

When Dr Sprague presented his statement [in 1900] he mentioned that an objection had been raised that the notation in some cases offers the choice of two symbols for the same benefit. For instance, a temporary annuity may be denoted either by ${}_n a_x$ or by $a_{x:\overline{n}|}$. This is, he says, a necessary consequence of the principles underlying the system, and neither of the alternative forms could have been suppressed without injury to the symmetry of the system.

SICKNESS TABLE
(MANCHESTER UNITY METHODOLOGY)

S(MU)

This table was produced using the methodology underlying that of the Manchester Unity Sickness Experience 1893–97. The underlying rates of sickness have, however, been updated to reflect more modern experience, and have been combined with the mortality of English Life Tables No. 15 (Males).

S(MU)

Central rates of sickness (weeks per annum)

Duration of sickness in weeks

Age	0–13	13–26	26–52	52–104	≥104	All	Age
16	0.315 0	0.004 8	0.001 2	0.000 0	0.000 0	0.321 0	16
17	0.332 3	0.008 0	0.004 4	0.002 0	0.000 0	0.346 7	17
18	0.348 2	0.008 8	0.005 0	0.003 9	0.001 1	0.367 0	18
19	0.357 6	0.009 7	0.005 6	0.004 4	0.003 0	0.380 3	19
20	0.366 5	0.010 6	0.006 3	0.005 1	0.004 8	0.393 3	20
21	0.374 9	0.011 6	0.007 0	0.005 8	0.006 8	0.406 1	21
22	0.383 0	0.012 7	0.007 8	0.006 6	0.008 9	0.419 0	22
23	0.390 5	0.013 9	0.008 7	0.007 4	0.011 3	0.431 8	23
24	0.397 7	0.015 1	0.009 7	0.008 4	0.014 0	0.444 9	24
25	0.402 6	0.016 4	0.010 8	0.009 5	0.017 0	0.456 3	25
26	0.410 9	0.017 8	0.011 9	0.010 7	0.020 3	0.471 6	26
27	0.417 1	0.019 3	0.013 2	0.012 0	0.024 1	0.485 7	27
28	0.423 0	0.020 9	0.014 6	0.013 5	0.028 4	0.500 4	28
29	0.428 7	0.022 5	0.016 1	0.015 1	0.033 2	0.515 6	29
30	0.434 4	0.024 3	0.017 7	0.016 9	0.038 6	0.531 9	30
31	0.439 8	0.026 2	0.019 5	0.018 9	0.044 8	0.549 2	31
32	0.445 4	0.028 3	0.021 5	0.021 1	0.051 8	0.568 1	32
33	0.451 0	0.030 4	0.023 6	0.023 6	0.059 6	0.588 2	33
34	0.456 7	0.032 8	0.025 9	0.026 3	0.068 6	0.610 3	34
35	0.462 6	0.035 3	0.028 4	0.029 3	0.078 7	0.634 3	35
36	0.468 8	0.037 9	0.031 2	0.032 7	0.090 1	0.660 7	36
37	0.475 2	0.040 8	0.034 2	0.036 4	0.103 1	0.689 7	37
38	0.482 2	0.043 9	0.037 6	0.040 5	0.117 9	0.722 1	38
39	0.489 8	0.047 3	0.041 2	0.045 2	0.134 6	0.758 1	39
40	0.497 9	0.050 9	0.045 3	0.050 3	0.153 6	0.798 0	40
41	0.506 7	0.054 8	0.049 7	0.056 1	0.175 2	0.842 5	41
42	0.516 3	0.059 1	0.054 6	0.062 5	0.199 7	0.892 2	42
43	0.526 9	0.063 8	0.060 1	0.069 7	0.227 7	0.948 2	43
44	0.538 6	0.068 9	0.066 1	0.077 8	0.259 5	1.010 9	44
45	0.551 4	0.074 5	0.072 9	0.086 9	0.295 9	1.081 6	45
46	0.565 6	0.080 6	0.080 4	0.097 2	0.337 4	1.161 2	46
47	0.581 2	0.087 4	0.088 8	0.108 8	0.385 0	1.251 2	47
48	0.598 6	0.094 8	0.098 2	0.122 0	0.439 5	1.353 1	48
49	0.617 8	0.103 1	0.108 8	0.137 0	0.502 0	1.468 7	49
50	0.639 0	0.112 3	0.120 7	0.154 0	0.574 0	1.600 0	50
51	0.662 6	0.122 5	0.134 1	0.173 4	0.656 9	1.749 5	51
52	0.688 8	0.133 9	0.149 3	0.195 6	0.752 7	1.920 3	52
53	0.717 8	0.146 6	0.166 6	0.221 0	0.863 6	2.115 6	53
54	0.749 9	0.160 9	0.186 2	0.250 3	0.992 1	2.339 4	54
55	0.785 6	0.176 9	0.208 5	0.283 9	1.141 6	2.596 5	55
56	0.825 1	0.194 9	0.234 0	0.322 8	1.315 8	2.892 6	56
57	0.869 1	0.215 3	0.263 2	0.367 7	1.519 3	3.234 6	57
58	0.917 7	0.238 2	0.296 7	0.419 9	1.757 8	3.630 3	58
59	0.971 7	0.264 2	0.335 1	0.480 4	2.037 8	4.089 2	59
60	1.031 1	0.293 5	0.379 3	0.550 8	2.367 7	4.622 4	60
61	1.096 8	0.326 8	0.430 0	0.632 8	2.757 4	5.243 8	61
62	1.169 0	0.364 3	0.488 4	0.728 5	3.218 9	5.969 1	62
63	1.247 8	0.406 7	0.555 5	0.840 0	3.767 0	6.817 0	63
64	1.333 5	0.454 3	0.632 5	0.970 0	4.419 8	7.810 1	64

S(MU)

Present value of a sickness benefit payable at the rate of 1 per week during sickness of the following durations.

All benefits cease at the earlier of death or attainment of age 65.

4%

	Duration of sickness in weeks						
Age	0–13	13–26	26–52	52–104	≥104	All	Age
16	10.236	1.113	1.171	1.515	5.786	19.821	16
17	10.329	1.153	1.217	1.576	6.021	20.297	17
18	10.412	1.192	1.262	1.639	6.266	20.771	18
19	10.482	1.232	1.309	1.702	6.522	21.246	19
20	10.546	1.272	1.357	1.767	6.785	21.726	20
21	10.603	1.313	1.406	1.834	7.057	22.213	21
22	10.654	1.355	1.456	1.903	7.339	22.707	22
23	10.699	1.398	1.508	1.974	7.630	23.209	23
24	10.739	1.441	1.560	2.047	7.931	23.718	24
25	10.772	1.484	1.614	2.122	8.241	24.235	25
26	10.802	1.528	1.669	2.199	8.561	24.760	26
27	10.825	1.573	1.725	2.278	8.890	25.291	27
28	10.842	1.617	1.783	2.359	9.229	25.830	28
29	10.853	1.662	1.841	2.442	9.578	26.376	29
30	10.860	1.707	1.899	2.527	9.936	26.929	30
31	10.862	1.752	1.959	2.613	10.303	27.489	31
32	10.858	1.797	2.020	2.701	10.680	28.055	32
33	10.849	1.842	2.080	2.790	11.065	28.626	33
34	10.834	1.887	2.142	2.880	11.458	29.201	34
35	10.813	1.931	2.203	2.972	11.859	29.778	35
36	10.787	1.974	2.265	3.064	12.267	30.358	36
37	10.754	2.017	2.327	3.158	12.682	30.939	37
38	10.715	2.059	2.388	3.251	13.103	31.517	38
39	10.668	2.100	2.449	3.345	13.527	32.089	39
40	10.613	2.139	2.509	3.438	13.953	32.653	40
41	10.548	2.176	2.568	3.531	14.380	33.203	41
42	10.473	2.212	2.625	3.622	14.804	33.735	42
43	10.387	2.245	2.680	3.710	15.223	34.245	43
44	10.288	2.274	2.732	3.796	15.634	34.725	44
45	10.176	2.301	2.780	3.878	16.034	35.169	45
46	10.048	2.323	2.825	3.955	16.418	35.569	46
47	9.904	2.341	2.864	4.026	16.781	35.916	47
48	9.740	2.353	2.898	4.090	17.117	36.199	48
49	9.556	2.360	2.925	4.145	17.419	36.405	49
50	9.348	2.359	2.944	4.189	17.678	36.517	50
51	9.114	2.350	2.952	4.219	17.884	36.520	51
52	8.851	2.331	2.949	4.233	18.025	36.390	52
53	8.554	2.302	2.932	4.228	18.085	36.101	53
54	8.219	2.259	2.899	4.200	18.046	35.624	54
55	7.842	2.202	2.846	4.143	17.888	34.921	55
56	7.417	2.127	2.770	4.053	17.584	33.951	56
57	6.938	2.033	2.667	3.922	17.104	32.663	57
58	6.397	1.915	2.532	3.743	16.409	30.995	58
59	5.786	1.769	2.358	3.506	15.455	28.875	59
60	5.096	1.592	2.140	3.199	14.184	26.211	60
61	4.316	1.378	1.867	2.808	12.528	22.897	61
62	3.433	1.120	1.531	2.316	10.401	18.800	62
63	2.431	0.810	1.118	1.702	7.698	13.759	63
64	1.293	0.441	0.613	0.941	4.286	7.574	64

Annuity values, allowing for mortality only, on the basis of ELT15 (Males)

4%

x	$\bar{a}_{x:\overline{65-x}\rvert}$
16	21.231
17	21.072
18	20.911
19	20.746
20	20.573
21	20.394
22	20.208
23	20.015
24	19.813
25	19.604
26	19.385
27	19.157
28	18.920
29	18.674
30	18.418
31	18.152
32	17.875
33	17.588
34	17.289
35	16.979
36	16.658
37	16.326
38	15.982
39	15.626
40	15.256
41	14.873
42	14.476
43	14.064
44	13.638
45	13.197
46	12.740
47	12.268
48	11.779
49	11.274
50	10.752
51	10.212
52	9.653
53	9.075
54	8.475
55	7.854
56	7.210
57	6.541
58	5.846
59	5.123
60	4.368
61	3.580
62	2.754
63	1.886
64	0.970

SICKNESS TABLE
(INCEPTION RATE / DISABILITY ANNUITY METHODOLOGY)

S(ID)

This table was produced using an inception rate/disability annuity method based on results presented in *C.M.I.R.* **12**. The following are tabulated:

- claim inception rates
- present values of current claim sickness annuities
- present values of annuities payable during sickness for lives currently healthy

The annuities cease at the earliest of:

death;
attainment of age 65;
recovery from sickness.

S(ID)

Claim inception rates, $(ia)_{x,d}$, for the given ages x and deferred periods d years.

(These rates are central, and (when $d = 0$) allow for the possibility of falling sick more than once during the year of age from x to $x + 1$. It was assumed in the construction of this table that all lives are healthy at exact age 30.)

	Deferred period in years, d		
Age, x	0	1	2
30	0.322 744		
31	0.318 254	0.000 521	
32	0.313 615	0.000 578	0.000 294
33	0.308 879	0.000 641	0.000 330
34	0.304 097	0.000 709	0.000 371
35	0.299 317	0.000 785	0.000 416
36	0.294 583	0.000 869	0.000 467
37	0.289 937	0.000 961	0.000 524
38	0.285 418	0.001 063	0.000 588
39	0.281 061	0.001 176	0.000 659
40	0.276 901	0.001 301	0.000 739
41	0.272 968	0.001 440	0.000 829
42	0.269 290	0.001 594	0.000 930
43	0.265 896	0.001 767	0.001 044
44	0.262 810	0.001 959	0.001 172
45	0.260 057	0.002 175	0.001 317
46	0.257 659	0.002 416	0.001 482
47	0.255 639	0.002 688	0.001 669
48	0.254 018	0.002 994	0.001 882
49	0.252 816	0.003 340	0.002 125
50	0.252 056	0.003 732	0.002 403
51	0.251 758	0.004 177	0.002 721
52	0.251 943	0.004 682	0.003 086
53	0.252 630	0.005 259	0.003 507
54	0.253 841	0.005 918	0.003 992
55	0.255 594	0.006 674	0.004 554
56	0.257 906	0.007 541	0.005 205
57	0.260 793	0.008 539	0.005 962
58	0.264 262	0.009 690	0.006 843
59	0.268 316	0.011 018	0.007 873
60	0.272 945	0.012 554	0.009 076
61	0.278 123	0.014 332	0.010 487
62	0.283 800	0.016 390	0.012 141
63	0.289 890	0.018 772	0.014 083
64	0.296 263	0.021 524	0.016 362

S(ID)

Present values of sickness benefit payable continuously at the rate of 1 per annum during sickness of the specified duration.

All benefits cease at the earlier of death or attainment of age 65.

6%

CURRENT STATUS = SICK

The table below gives the present value, $\overline{a}^{\overline{SS}}_{x,z}$, of a "current claim" sickness annuity for a sick life now aged x with current duration of sickness z years. (The annuity does not allow for the possibility of future new episodes of sickness.)

	Current duration of sickness, z years		
Age, x	0	1	2
30	0.033 3	3.570 2	5.418 0
31	0.035 0	3.660 4	5.505 1
32	0.036 8	3.751 9	5.591 5
33	0.038 8	3.844 3	5.676 9
34	0.041 0	3.937 5	5.761 0
35	0.043 5	4.031 1	5.843 2
36	0.046 2	4.124 6	5.923 0
37	0.049 2	4.217 8	5.999 7
38	0.052 5	4.309 9	6.072 8
39	0.056 2	4.400 6	6.141 3
40	0.060 3	4.488 9	6.204 4
41	0.064 9	4.574 3	6.261 2
42	0.069 9	4.655 7	6.310 6
43	0.075 4	4.732 1	6.351 2
44	0.081 5	4.802 3	6.381 9
45	0.088 3	4.865 1	6.401 1
46	0.095 7	4.918 9	6.407 1
47	0.103 8	4.961 9	6.398 1
48	0.112 6	4.992 3	6.372 1
49	0.122 1	5.008 0	6.326 9
50	0.132 4	5.006 4	6.259 9
51	0.143 3	4.984 9	6.168 6
52	0.154 9	4.940 5	6.049 8
53	0.167 0	4.869 7	5.900 4
54	0.179 3	4.768 8	5.716 9
55	0.191 7	4.633 7	5.495 2
56	0.203 5	4.459 6	5.231 2
57	0.214 4	4.241 4	4.920 2
58	0.223 4	3.973 3	4.557 1
59	0.229 5	3.649 0	4.136 3
60	0.231 2	3.261 4	3.651 8
61	0.226 7	2.802 9	3.097 0
62	0.213 4	2.264 3	2.464 8
63	0.187 5	1.633 6	1.746 9
64	0.142 9	0.892 5	0.931 5

CURRENT STATUS = HEALTHY

The table below gives the present value, $\overline{a}_x^{HS(d/\text{all})}$, of sickness benefit payable during sickness of duration at least d years for a life aged x who is currently healthy. (The value allows for the possibility of more than one episode of sickness.)

	Deferred period, d years		
Age, x	0	1	2
30	0.330 580	0.142 025	0.111 543
31	0.339 378	0.148 808	0.116 826
32	0.348 311	0.155 754	0.122 226
33	0.357 354	0.162 837	0.127 714
34	0.366 480	0.170 038	0.133 274
35	0.375 647	0.177 324	0.138 875
36	0.384 822	0.184 665	0.144 486
37	0.393 952	0.192 016	0.150 067
38	0.402 981	0.199 327	0.155 573
39	0.411 815	0.206 529	0.160 944
40	0.420 352	0.213 550	0.166 111
41	0.428 479	0.220 304	0.171 001
42	0.436 077	0.226 698	0.175 528
43	0.443 010	0.232 611	0.179 594
44	0.449 125	0.237 925	0.183 090
45	0.454 221	0.242 488	0.185 885
46	0.458 091	0.246 146	0.187 843
47	0.460 523	0.248 719	0.188 814
48	0.461 260	0.250 010	0.188 628
49	0.460 010	0.249 788	0.187 096
50	0.456 447	0.247 810	0.184 025
51	0.450 241	0.243 825	0.179 219
52	0.440 992	0.237 558	0.172 462
53	0.428 296	0.228 736	0.163 569
54	0.411 745	0.217 100	0.152 372
55	0.390 935	0.202 426	0.138 768
56	0.365 518	0.184 575	0.122 748
57	0.335 193	0.163 508	0.104 447
58	0.299 804	0.139 390	0.084 219
59	0.259 410	0.112 669	0.062 755
60	0.214 401	0.084 217	0.041 213
61	0.165 680	0.055 536	0.021 441
62	0.114 894	0.029 046	0.006 275
63	0.064 864	0.008 533	0.000 000
64	0.020 334	0.000 000	0.000 000

Annuity values, allowing for mortality only, on the basis of ELT15 (Males)

6%

x	$\bar{a}_{x:\overline{65-x}\rceil}$
16	15.881
17	15.813
18	15.744
19	15.673
20	15.597
21	15.517
22	15.432
23	15.342
24	15.247
25	15.146
26	15.038
27	14.924
28	14.803
29	14.674
30	14.538
31	14.394
32	14.242
33	14.081
34	13.911
35	13.731
36	13.541
37	13.342
38	13.131
39	12.909
40	12.675
41	12.428
42	12.168
43	11.893
44	11.604
45	11.299
46	10.978
47	10.640
48	10.284
49	9.910
50	9.516
51	9.102
52	8.666
53	8.207
54	7.722
55	7.211
56	6.671
57	6.101
58	5.496
59	4.856
60	4.176
61	3.452
62	2.679
63	1.851
64	0.961

EXAMPLE PENSION SCHEME TABLE

PEN

PEN

Service table and relative salary scale

Age x	l_x	w_x	d_x	i_x	r_x	s_x^*	$s_x = (1.02)^x s_x^*$	z_x	$z_{x+½}$	Age x
16	100 000	10 000	50			1.000	1.373			16
17	89 950	8 995	45			1.177	1.648			17
18	80 910	8 091	41			1.349	1.927			18
19	72 778	7 278	36			1.513	2.204			19
20	65 464	6 546	33			1.672	2.485			20
21	58 885	5 888	24			1.823	2.763			21
22	52 973	5 296	21			1.970	3.045			22
23	47 656	4 763	19			2.108	3.324			23
24	42 874	4 070	17			2.241	3.605			24
25	38 787	3 487	16			2.366	3.882			25
26	35 284	2 994	11			2.483	4.155			26
27	32 279	2 577	10			2.595	4.429			27
28	29 692	2 221	9			2.707	4.713			28
29	27 462	1 916	8			2.810	4.991			29
30	25 538	1 679	8	10		2.914	5.278	4.711	4.852	30
31	23 841	1 472	10	12		3.004	5.551	4.994	5.133	31
32	22 347	1 290	9	13		3.095	5.832	5.273	5.413	32
33	21 035	1 131	8	15		3.181	6.115	5.554	5.693	33
34	19 881	989	8	18		3.259	6.389	5.833	5.972	34
35	18 866	863	9	21		3.328	6.655	6.112	6.249	35
36	17 973	751	11	21		3.392	6.920	6.386	6.520	36
37	17 190	650	12	22		3.448	7.175	6.655	6.786	37
38	16 506	558	12	25		3.491	7.410	6.916	7.042	38
39	15 911	474	13	27		3.522	7.623	7.168	7.285	39
40	15 397	413	14	31		3.539	7.814	7.403	7.509	40
41	14 939	356	13	34		3.543	7.980	7.616	7.711	41
42	14 536	303	14	38		3.539	8.129	7.806	7.890	42
43	14 181	254	16	41		3.522	8.252	7.974	8.047	43
44	13 870	207	17	44		3.504	8.375	8.120	8.186	44
45	13 602	162	18	47		3.487	8.501	8.252	8.314	45
46	13 375	120	19	51		3.470	8.628	8.376	8.439	46
47	13 185	79	22	55		3.457	8.768	8.502	8.567	47
48	13 029	52	26	62		3.440	8.899	8.632	8.699	48
49	12 889	26	28	72		3.422	9.031	8.765	8.832	49
50	12 763		32	82		3.405	9.165	8.899	8.965	50
51	12 649		35	94		3.392	9.313	9.032	9.101	51
52	12 520		39	108		3.375	9.451	9.170	9.240	52
53	12 373		43	125		3.358	9.591	9.310	9.381	53
54	12 205		47	145		3.345	9.745	9.452	9.524	54
55	12 013		51	168		3.328	9.889	9.596	9.669	55
56	11 794		55	193		3.310	10.034	9.742	9.815	56
57	11 546		58	220		3.297	10.195	9.889	9.964	57
58	11 268		63	248		3.280	10.344	10.039	10.115	58
59	10 957		67	278		3.267	10.510	10.191	10.270	59
60	10 612		73	310	3 681	3.250	10.663	10.350	10.428	60
61	6 548		50	219	516	3.233	10.819	10.506	10.585	61
62	5 763		49	223	453	3.220	10.991	10.664	10.744	62
63	5 038		48	224	395	3.203	11.151	10.824	10.906	63
64	4 371		47	225	342	3.190	11.328	10.987	11.072	64
65	3 757				3 757			11.157		65

$$z_x = \frac{1}{3}(s_{x-3} + s_{x-2} + s_{x-1}) \text{ and } z_{x+½} = \frac{1}{2}(z_x + z_{x+1})$$

PEN

Contribution functions 4%

Age *x*	$D_x =$ $v^x l_x$	$\bar{D}_x =$ ½$(D_x + D_{x+1})$	$\bar{N}_x =$ $\Sigma\, \bar{D}_x$	${}^s\bar{D}_x =$ $s_x \bar{D}_x$	${}^s\bar{N}_x =$ $\Sigma\, {}^s\bar{D}_x$	${}^sD_x =$ $s_x D_x$	Age *x*
16	53 391	49 784	413 287	68 343	1 513 322	73 294	16
17	46 178	43 059	363 503	70 948	1 444 979	76 087	17
18	39 939	37 241	320 444	71 761	1 374 031	76 959	18
19	34 544	32 210	283 203	70 993	1 302 270	76 136	19
20	29 877	27 859	250 992	69 232	1 231 277	74 248	20
21	25 841	24 096	223 134	66 590	1 162 045	71 410	21
22	22 352	20 844	199 037	63 476	1 095 455	68 070	22
23	19 335	18 031	178 193	59 929	1 031 979	64 265	23
24	16 726	15 638	160 163	56 376	972 050	60 299	24
25	14 550	13 638	144 525	52 947	915 673	56 486	25
26	12 727	11 961	130 887	49 693	862 726	52 875	26
27	11 195	10 548	118 926	46 719	813 033	49 583	27
28	9 902	9 354	108 378	44 082	766 314	46 664	28
29	8 806	8 340	99 024	41 622	722 232	43 947	29
30	7 874	7 471	90 684	39 431	680 611	41 558	30
31	7 068	6 719	83 213	37 296	641 180	39 232	31
32	6 370	6 068	76 494	35 390	603 884	37 153	32
33	5 766	5 503	70 427	33 647	568 494	35 255	33
34	5 240	5 010	64 924	32 011	534 848	33 477	34
35	4 781	4 580	59 914	30 480	502 836	31 816	35
36	4 379	4 204	55 333	29 087	472 356	30 305	36
37	4 028	3 873	51 130	27 788	443 269	28 897	37
38	3 719	3 583	47 257	26 546	415 480	27 554	38
39	3 447	3 327	43 674	25 361	388 934	26 275	39
40	3 207	3 099	40 347	24 219	363 573	25 059	40
41	2 992	2 896	37 248	23 106	339 354	23 875	41
42	2 799	2 713	34 352	22 052	316 248	22 757	42
43	2 626	2 548	31 640	21 023	294 196	21 668	43
44	2 470	2 399	29 092	20 093	273 173	20 683	44
45	2 329	2 265	26 693	19 256	253 080	19 796	45
46	2 202	2 144	24 428	18 502	233 824	18 997	46
47	2 087	2 035	22 283	17 842	215 322	18 298	47
48	1 983	1 935	20 248	17 215	197 480	17 645	48
49	1 886	1 841	18 314	16 627	180 265	17 034	49
50	1 796	1 754	16 473	16 073	163 638	16 460	50
51	1 711	1 670	14 719	15 554	147 565	15 939	51
52	1 629	1 588	13 049	15 011	132 011	15 394	52
53	1 548	1 508	11 461	14 462	117 000	14 845	53
54	1 468	1 429	9 953	13 923	102 538	14 306	54
55	1 389	1 350	8 524	13 354	88 615	13 739	55
56	1 312	1 273	7 174	12 775	75 261	13 161	56
57	1 235	1 197	5 901	12 199	62 486	12 587	57
58	1 159	1 121	4 704	11 595	50 287	11 984	58
59	1 083	1 046	3 583	10 993	38 692	11 385	59
60	1 009	804	2 537	8 570	27 699	10 757	60
61	599	553	1 733	5 978	19 129	6 475	61
62	507	466	1 181	5 123	13 152	5 567	62
63	426	390	715	4 354	8 028	4 748	63
64	355	324	324	3 674	3 674	4 023	64
65	294						65

PEN

4%

Ill health retirement functions

Age x	$\bar{a}^{i}_{x+½}$	$C^{i}_{x} = v^{x+½}i_{x}$	$M^{i}_{x} = \Sigma C^{i}_{x}$	$\bar{R}^{i}_{x} = \Sigma (M^{i}_{x} - ½C^{i}_{x})$	$C^{ia}_{x} = C^{i}_{x}\bar{a}^{i}_{x+½}$	$M^{ia}_{x} = \Sigma C^{ia}_{x}$	$\bar{R}^{ia}_{x} = \Sigma (M^{ia}_{x} - ½C^{ia}_{x})$	Age x
16			414	15 416		7 023	252 924	16
17			414	15 002		7 023	245 901	17
18			414	14 588		7 023	238 878	18
19			414	14 173		7 023	231 855	19
20			414	13 759		7 023	224 831	20
21			414	13 345		7 023	217 808	21
22			414	12 930		7 023	210 785	22
23			414	12 516		7 023	203 762	23
24			414	12 102		7 023	196 739	24
25			414	11 688		7 023	189 715	25
26			414	11 273		7 023	182 692	26
27			414	10 859		7 023	175 669	27
28			414	10 445		7 023	168 646	28
29			414	10 030		7 023	161 622	29
30	21.852	3	414	9 616	66	7 023	154 599	30
31	21.720	3	411	9 203	76	6 957	147 609	31
32	21.583	4	408	8 794	78	6 881	140 690	32
33	21.441	4	404	8 388	86	6 803	133 848	33
34	21.294	5	400	7 986	99	6 717	127 088	34
35	21.142	5	395	7 588	110	6 617	120 421	35
36	20.985	5	390	7 195	105	6 507	113 859	36
37	20.822	5	385	6 807	105	6 402	107 404	37
38	20.654	6	380	6 425	114	6 297	101 055	38
39	20.481	6	375	6 047	117	6 183	94 815	39
40	20.302	6	369	5 676	129	6 065	88 691	40
41	20.118	7	363	5 310	134	5 937	82 691	41
42	19.929	7	356	4 951	143	5 802	76 821	42
43	19.734	7	349	4 598	147	5 659	71 091	43
44	19.534	8	341	4 253	150	5 512	65 505	44
45	19.330	8	334	3 916	153	5 362	60 068	45
46	19.120	8	326	3 586	157	5 210	54 782	46
47	18.906	9	317	3 265	161	5 052	49 651	47
48	18.669	9	309	2 951	173	4 891	44 679	48
49	18.407	10	300	2 647	190	4 718	39 875	49
50	18.135	11	289	2 353	205	4 528	35 251	50
51	17.853	12	278	2 069	223	4 323	30 826	51
52	17.561	14	266	1 797	242	4 100	26 615	52
53	17.259	15	252	1 538	265	3 858	22 635	53
54	16.948	17	236	1 294	290	3 594	18 909	54
55	16.625	19	219	1 066	317	3 304	15 461	55
56	16.292	21	200	856	343	2 987	12 315	56
57	15.949	23	179	667	368	2 644	9 500	57
58	15.594	25	156	499	390	2 276	7 040	58
59	15.229	27	131	355	410	1 886	4 958	59
60	14.855	29	104	238	429	1 476	3 277	60
61	14.472	20	75	148	284	1 047	2 016	61
62	14.081	19	56	82	271	763	1 111	62
63	13.682	19	36	36	254	492	484	63
64	13.277	18	18	9	238	238	119	64

PEN

Ill health retirement functions **4%**

Age x	${}^s\overline{M}_x^{ia} =$	${}^s\overline{R}_x^{ia} =$	${}^zC_x^{ia} =$	${}^zM_x^{ia} =$	${}^z\overline{R}_x^{ia} =$	Age x
	$s_x(M_x^{ia} - \frac{1}{2}C_x^{ia})$	$\Sigma\, {}^s\overline{M}_x^{ia}$	$z_{x+\frac{1}{2}}C_x^{ia}$	$\Sigma\, {}^zC_x^{ia}$	$\Sigma\, ({}^zM_x^{ia} - \frac{1}{2}\, {}^zC_x^{ia})$	
16	9 641	1 533 946		64 061	2 399 660	16
17	11 572	1 524 304		64 061	2 335 599	17
18	13 533	1 512 732		64 061	2 271 539	18
19	15 480	1 499 199		64 061	2 207 478	19
20	17 454	1 483 720		64 061	2 143 417	20
21	19 409	1 466 266		64 061	2 079 357	21
22	21 388	1 446 858		64 061	2 015 296	22
23	23 343	1 425 470		64 061	1 951 235	23
24	25 320	1 402 126		64 061	1 887 175	24
25	27 266	1 376 807		64 061	1 823 114	25
26	29 179	1 349 541		64 061	1 759 054	26
27	31 106	1 320 361		64 061	1 694 993	27
28	33 099	1 289 255		64 061	1 630 932	28
29	35 051	1 256 156		64 061	1 566 872	29
30	36 894	1 221 105	321	64 061	1 502 811	30
31	38 407	1 184 211	389	63 740	1 438 911	31
32	39 906	1 145 804	425	63 351	1 375 365	32
33	41 334	1 105 898	492	62 927	1 312 226	33
34	42 596	1 064 565	592	62 434	1 249 546	34
35	43 671	1 021 969	689	61 843	1 187 407	35
36	44 664	978 298	687	61 153	1 125 909	36
37	45 554	933 634	714	60 467	1 065 099	37
38	46 234	888 080	803	59 753	1 004 989	38
39	46 683	841 846	856	58 949	945 638	39
40	46 889	795 163	965	58 094	887 117	40
41	46 836	748 274	1 036	57 128	829 506	41
42	46 587	701 438	1 128	56 093	772 895	42
43	46 092	654 850	1 182	54 964	717 367	43
44	45 540	608 759	1 228	53 782	662 994	44
45	44 936	563 219	1 268	52 554	609 826	45
46	44 271	518 283	1 328	51 286	557 906	46
47	43 590	474 013	1 383	49 957	507 285	47
48	42 754	430 422	1 503	48 575	458 019	48
49	41 752	387 668	1 680	47 072	410 195	49
50	40 560	345 917	1 840	45 392	363 963	50
51	39 222	305 356	2 026	43 553	319 490	51
52	37 608	266 134	2 236	41 527	276 951	52
53	35 735	228 526	2 482	39 291	236 542	53
54	33 607	192 792	2 760	36 809	198 492	54
55	31 104	159 184	3 063	34 048	163 063	55
56	28 252	128 080	3 366	30 986	130 546	56
57	25 081	99 828	3 666	27 620	101 243	57
58	21 530	74 747	3 944	23 954	75 455	58
59	17 668	53 218	4 215	20 011	53 473	59
60	13 449	35 550	4 476	15 795	35 570	60
61	9 787	22 100	3 007	11 319	22 013	61
62	6 895	12 313	2 907	8 313	12 197	62
63	4 070	5 418	2 770	5 405	5 338	63
64	1 348	1 348	2 635	2 635	1 318	64

PEN

4%

Age retirement functions

Age x	$\bar{a}^r_{x+½}$ ($\bar{a}^r_{65}$ at 65)	$C^r_x = v^{x+½} r_x$ ($v^{65} r_{65}$ at 65)	$M^r_x = \Sigma\, C^r_x$	$\bar{R}^r_x = \Sigma\,(M^r_x - ½C^r_x)$	$C^{ra}_x = C^r_x \bar{a}^r_{x+½}$ ($v^{65} r^{65} \bar{a}^r_{65}$ at 65)	$M^{ra}_x = \Sigma\, C^{ra}_x$	$\bar{R}^{ra}_x = \Sigma\,(M^{ra}_x - ½C^{ra}_x)$	Age x
16			782	36 449		11 915	553 630	16
17			782	35 667		11 915	541 715	17
18			782	34 885		11 915	529 800	18
19			782	34 103		11 915	517 885	19
20			782	33 321		11 915	505 970	20
21			782	32 539		11 915	494 055	21
22			782	31 757		11 915	482 140	22
23			782	30 975		11 915	470 225	23
24			782	30 193		11 915	458 310	24
25			782	29 411		11 915	446 395	25
26			782	28 629		11 915	434 479	26
27			782	27 847		11 915	422 564	27
28			782	27 065		11 915	410 649	28
29			782	26 284		11 915	398 734	29
30			782	25 502		11 915	386 819	30
31			782	24 720		11 915	374 904	31
32			782	23 938		11 915	362 989	32
33			782	23 156		11 915	351 074	33
34			782	22 374		11 915	339 159	34
35			782	21 592		11 915	327 244	35
36			782	20 810		11 915	315 328	36
37			782	20 028		11 915	303 413	37
38			782	19 246		11 915	291 498	38
39			782	18 464		11 915	279 583	39
40			782	17 682		11 915	267 668	40
41			782	16 900		11 915	255 753	41
42			782	16 118		11 915	243 838	42
43			782	15 336		11 915	231 923	43
44			782	14 554		11 915	220 008	44
45			782	13 773		11 915	208 093	45
46			782	12 991		11 915	196 177	46
47			782	12 209		11 915	184 262	47
48			782	11 427		11 915	172 347	48
49			782	10 645		11 915	160 432	49
50			782	9 863		11 915	148 517	50
51			782	9 081		11 915	136 602	51
52			782	8 299		11 915	124 687	52
53			782	7 517		11 915	112 772	53
54			782	6 735		11 915	100 857	54
55			782	5 953		11 915	88 942	55
56			782	5 171		11 915	77 027	56
57			782	4 389		11 915	65 111	57
58			782	3 607		11 915	53 196	58
59			782	2 825		11 915	41 281	59
60	16.292	343	782	2 043	5 590	11 915	29 366	60
61	15.949	46	439	1 433	738	6 325	20 246	61
62	15.594	39	393	1 017	609	5 587	14 290	62
63	15.229	33	354	644	498	4 979	9 007	63
64	14.855	27	321	307	405	4 480	4 278	64
65	13.883	294	294		4 075	4 075		65

PEN

Age retirement functions 4%

Age x	${}^{s}\bar{M}_x^{ra} =$ $s_x(M_x^{ra} - \tfrac{1}{2}C_x^{ra})$	${}^{s}\bar{R}_x^{ra} =$ $\Sigma\, {}^{s}\bar{M}_x^{ra}$	${}^{z}C_x^{ra} =$ $z_{x+\frac{1}{2}}C_x^{ra}$ ($z_{65}C_{65}^{ra}$ at 65)	${}^{z}M_x^{ra} =$ $\Sigma\, {}^{z}C_x^{ra}$	${}^{z}\bar{R}_x^{ra} =$ $\Sigma\, ({}^{z}M_x^{ra} - \tfrac{1}{2}\, {}^{z}C_x^{ra})$	Age x
16	16 357	3 801 411		128 026	5 956 885	16
17	19 632	3 785 055		128 026	5 828 859	17
18	22 959	3 765 422		128 026	5 700 833	18
19	26 262	3 742 463		128 026	5 572 807	19
20	29 610	3 716 201		128 026	5 444 781	20
21	32 927	3 686 591		128 026	5 316 755	21
22	36 285	3 653 664		128 026	5 188 729	22
23	39 602	3 617 379		128 026	5 060 703	23
24	42 955	3 577 776		128 026	4 932 677	24
25	46 258	3 534 821		128 026	4 804 651	25
26	49 504	3 488 563		128 026	4 676 625	26
27	52 773	3 439 059		128 026	4 548 599	27
28	56 153	3 386 286		128 026	4 420 573	28
29	59 465	3 330 133		128 026	4 292 547	29
30	62 887	3 270 668		128 026	4 164 521	30
31	66 137	3 207 781		128 026	4 036 495	31
32	69 493	3 141 643		128 026	3 908 469	32
33	72 857	3 072 151		128 026	3 780 443	33
34	76 127	2 999 294		128 026	3 652 417	34
35	79 293	2 923 167		128 026	3 524 390	35
36	82 450	2 843 874		128 026	3 396 364	36
37	85 488	2 761 424		128 026	3 268 338	37
38	88 288	2 675 936		128 026	3 140 312	38
39	90 832	2 587 648		128 026	3 012 286	39
40	93 102	2 496 816		128 026	2 884 260	40
41	95 080	2 403 714		128 026	2 756 234	41
42	96 863	2 308 634		128 026	2 628 208	42
43	98 319	2 211 771		128 026	2 500 182	43
44	99 795	2 113 452		128 026	2 372 156	44
45	101 290	2 013 657		128 026	2 244 130	45
46	102 805	1 912 367		128 026	2 116 104	46
47	104 470	1 809 562		128 026	1 988 078	47
48	106 028	1 705 092		128 026	1 860 052	48
49	107 607	1 599 064		128 026	1 732 026	49
50	109 206	1 491 457		128 026	1 604 000	50
51	110 967	1 382 252		128 026	1 475 974	51
52	112 611	1 271 285		128 026	1 347 948	52
53	114 276	1 158 674		128 026	1 219 921	53
54	116 113	1 044 398		128 026	1 091 895	54
55	117 825	928 285		128 026	963 869	55
56	119 559	810 460		128 026	835 843	56
57	121 473	690 902		128 026	707 817	57
58	123 255	569 428		128 026	579 791	58
59	125 224	446 173		128 026	451 765	59
60	97 250	320 949	58 293	128 026	323 739	60
61	64 439	223 699	7 807	69 733	224 859	61
62	58 066	159 260	6 541	61 926	159 030	62
63	52 736	101 194	5 436	55 385	100 374	63
64	48 458	48 458	4 482	49 949	47 708	64
65			45 467	45 467		65

PEN

4%

Functions for return of contributions, accumulated with interest at 2% p.a., on death

Age x	${}^{j}C_x^d =$ $v^{x+½}(1+j)^{x+½}d_x$	${}^{j}M_x^d =$ $\Sigma\,{}^{j}C_x^d$	${}^{j}\overline{R}_x^d =$ $\Sigma\left(\frac{{}^{j}M_x^d - ½\,{}^{j}C_x^d}{(1+j)^{x+½}}\right)$	${}^{sj}\overline{R}_x^d =$ $\Sigma s_x\left(\frac{{}^{j}M_x^d - ½\,{}^{j}C_x^d}{(1+j)^{x+½}}\right)$	Age x
16	36	601	7 617	39 369	16
17	32	565	7 196	38 791	17
18	29	533	6 808	38 152	18
19	25	504	6 449	37 459	19
20	22	480	6 114	36 722	20
21	16	457	5 802	35 946	21
22	14	442	5 508	35 134	22
23	12	428	5 230	34 286	23
24	11	416	4 965	33 405	24
25	10	406	4 712	32 494	25
26	7	396	4 470	31 555	26
27	6	389	4 238	30 590	27
28	5	383	4 014	29 598	28
29	5	378	3 797	28 577	29
30	4	374	3 588	27 531	30
31	5	369	3 385	26 460	31
32	5	364	3 188	25 369	32
33	4	359	2 998	24 262	33
34	4	355	2 815	23 138	34
35	5	351	2 636	22 000	35
36	5	346	2 464	20 852	36
37	6	341	2 297	19 698	37
38	6	335	2 136	18 544	38
39	6	329	1 981	17 396	39
40	6	323	1 832	16 258	40
41	6	317	1 689	15 136	41
42	6	311	1 551	14 035	42
43	7	305	1 418	12 956	43
44	7	298	1 290	11 904	44
45	7	291	1 168	10 882	45
46	8	283	1 052	9 891	46
47	9	276	940	8 930	47
48	10	267	834	8 001	48
49	11	257	734	7 109	49
50	12	246	640	6 256	50
51	13	234	551	5 446	51
52	14	221	469	4 681	52
53	15	207	394	3 965	53
54	16	192	324	3 302	54
55	17	176	262	2 693	55
56	18	158	206	2 142	56
57	19	140	157	1 653	57
58	20	121	116	1 227	58
59	21	101	81	867	59
60	23	80	53	575	60
61	15	57	32	355	61
62	15	42	18	197	62
63	14	27	8	86	63
64	13	13	2	21	64

PEN

Functions for return of contributions, accumulated with interest at 2% p.a., on withdrawal

4%

Age x	${}^{j}C_x^w =$ $v^{x+\frac{1}{2}}(1+j)^{x+\frac{1}{2}}w_x$	${}^{j}M_x^w =$ $\Sigma\, {}^{j}C_x^w$	${}^{j}\bar{R}_x^w =$ $\Sigma\left(\frac{{}^{j}M_x^w - \frac{1}{2}\,{}^{j}C_x^w}{(1+j)^{x+\frac{1}{2}}}\right)$	${}^{sj}\bar{R}_x^w =$ $\Sigma s_x\left(\frac{{}^{j}M_x^w - \frac{1}{2}\,{}^{j}C_x^w}{(1+j)^{x+\frac{1}{2}}}\right)$	Age x
16	7 259	55 286	230 458	622 984	16
17	6 404	48 027	193 200	571 836	17
18	5 649	41 624	161 503	519 609	18
19	4 984	35 974	134 605	467 779	19
20	4 396	30 991	111 848	417 622	20
21	3 878	26 594	92 662	369 943	21
22	3 421	22 716	76 556	325 433	22
23	3 018	19 294	63 103	284 465	23
24	2 529	16 277	51 935	247 347	24
25	2 125	13 747	42 694	214 031	25
26	1 790	11 622	35 038	184 310	26
27	1 511	9 832	28 691	157 939	27
28	1 277	8 322	23 425	134 617	28
29	1 080	7 045	19 056	114 025	29
30	929	5 964	15 429	95 926	30
31	798	5 036	12 423	80 058	31
32	686	4 237	9 938	66 267	32
33	590	3 551	7 892	54 334	33
34	506	2 961	6 215	44 080	34
35	433	2 454	4 848	35 344	35
36	370	2 021	3 740	27 971	36
37	314	1 652	2 849	21 802	37
38	264	1 338	2 137	16 699	38
39	220	1 074	1 575	12 531	39
40	188	853	1 134	9 171	40
41	159	665	794	6 510	41
42	133	506	536	4 454	42
43	109	374	346	2 913	43
44	87	264	212	1 800	44
45	67	177	120	1 034	45
46	49	110	62	538	46
47	31	62	27	242	47
48	20	30	10	85	48
49	10	10	2	17	49

SAMPLE TIME SERIES

This section shows the data values and related summary statistics for various observed time series. These could be used in discussions of time series modelling focusing on the following concepts:

- stationarity
- differencing
- seasonality
- autocorrelation
- choice of model
- ARIMA models
- parameter estimation
- residual analysis
- forecasting

The left-hand side of each table shows an extract of the data values for the time series.

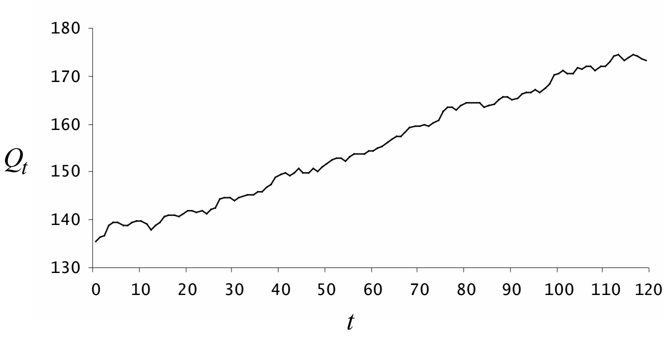
The right-hand side of each table shows summary statistics based on the full range of values for the series over the stated period.

Time Series – RPI

This dataset shows the monthly Retail Prices Index for the 10-year period from January 1992 to December 2001. These figures represent the prices of a representative “basket” of goods purchased in the UK.

		Data values		
Month	t	Q_t	∇Q_t	$\nabla^2 Q_t$
Jan-92	0	135.6		
Feb-92	1	136.3	0.7	
Mar-92	2	136.7	0.4	–0.3
Apr-92	3	138.8	2.1	1.7
May-92	4	139.3	0.5	–1.6
Jun-92	5	139.3	0.0	–0.5
Jul-92	6	138.8	–0.5	–0.5
Aug-92	7	138.9	0.1	0.6
Sep-92	8	139.4	0.5	0.4
Oct-92	9	139.9	0.5	0.0
...	...	...	...	...
Mar-01	110	172.2	0.2	–0.7
Apr-01	111	173.1	0.9	0.7
May-01	112	174.2	1.1	0.2
Jun-01	113	174.4	0.2	–0.9
Jul-01	114	173.3	–1.1	–1.3
Aug-01	115	174.0	0.7	1.8
Sep-01	116	174.6	0.6	–0.1
Oct-01	117	174.3	–0.3	–0.9
Nov-01	118	173.6	–0.7	–0.4
Dec-01	119	173.4	–0.2	0.5

	Summary statistics		
	Q_t	∇Q_t	$\nabla^2 Q_t$
n	120	119	118
mean	155.4	0.3	0.0
s.d.	11.9	0.6	0.8
min	135.6	–1.3	–1.6
max	174.6	2.1	2.2
r_1	0.977	0.083	–0.404
r_2	0.954	–0.101	–0.006
r_3	0.930	–0.285	–0.218
r_4	0.908	–0.047	0.114
r_5	0.887	–0.012	–0.128
r_6	0.866	0.240	0.315
r_{12}	0.729	0.637	0.671
ϕ_1	0.977	0.083	–0.404
ϕ_2	–0.019	–0.109	–0.201
ϕ_3	–0.028	–0.272	–0.376
ϕ_4	0.037	–0.017	–0.235
ϕ_5	0.003	–0.066	–0.391
ϕ_6	–0.011	0.179	–0.028



Time Series – NAEI

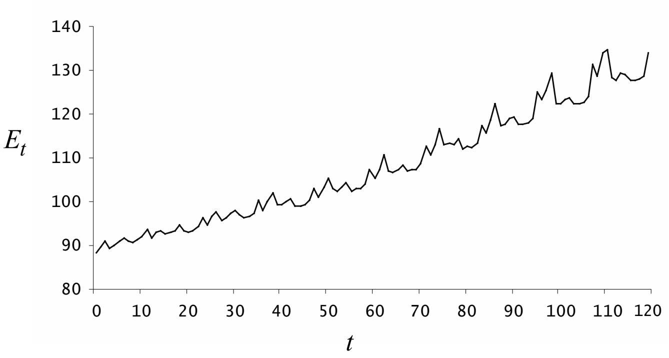
This dataset shows the monthly UK National Average Earnings Index for the 10-year period from January 1992 to December 2001. These figures are NOT seasonally adjusted.

Data values

Month	t	E_t	∇E_t	$\nabla^2 E_t$
Jan-92	0	88.5		
Feb-92	1	89.8	1.3	
Mar-92	2	91.1	1.3	0.0
Apr-92	3	89.5	−1.6	−2.9
May-92	4	90.1	0.6	2.2
Jun-92	5	91.1	1.0	0.4
Jul-92	6	91.6	0.5	−0.5
Aug-92	7	90.9	−0.7	−1.2
Sep-92	8	90.7	−0.2	0.5
Oct-92	9	91.5	0.8	1.0
...	...	...	...	...
Mar-01	110	134.8	0.9	−4.3
Apr-01	111	128.4	−6.4	−7.3
May-01	112	127.7	−0.7	5.7
Jun-01	113	129.3	1.6	2.3
Jul-01	114	128.9	−0.4	−2.0
Aug-01	115	127.8	−1.1	−0.7
Sep-01	116	127.6	−0.2	0.9
Oct-01	117	128.1	0.5	0.7
Nov-01	118	128.6	0.5	0.0
Dec-01	119	134.1	5.5	5.0

Summary statistics

	E_t	∇E_t	$\nabla^2 E_t$
n	120	119	118
mean	108.0	0.4	0.0
s.d.	12.9	2.2	3.4
min	88.5	−6.8	−10.8
max	134.8	7.3	7.8
r_1	0.959	−0.245	−0.511
r_2	0.932	−0.197	−0.163
r_3	0.912	0.252	0.358
r_4	0.883	−0.170	−0.212
r_5	0.859	−0.065	0.056
r_6	0.835	−0.103	−0.042
r_{12}	0.706	0.823	0.801
ϕ_1	0.959	−0.245	−0.511
ϕ_2	0.160	−0.274	−0.573
ϕ_3	0.103	0.141	−0.131
ϕ_4	−0.084	−0.131	−0.153
ϕ_5	0.015	−0.065	0.050
ϕ_6	−0.008	−0.276	−0.140



Time Series – FTSE 100

This dataset shows the monthly FTSE 100 index for the 10-year period from January 1992 to December 2001. The index is based on the average closing prices of the top 100 UK shares on the last day of each month.

		Data values		
Month	t	S_t	∇S_t	$\nabla^2 S_t$
Jan-92	0	2 571.2		
Feb-92	1	2 562.1	–9.1	
Mar-92	2	2 440.1	–122.0	–112.9
Apr-92	3	2 654.1	214.0	336.0
May-92	4	2 707.6	53.5	–160.5
Jun-92	5	2 521.2	–186.4	–239.9
Jul-92	6	2 399.6	–121.6	64.8
Aug-92	7	2312.6	–87.0	34.6
Sep-92	8	2 553.0	240.4	327.4
Oct-92	9	2 658.3	105.3	–135.1
...	...	...	...	...
Mar-01	110	5 633.7	–284.2	95.4
Apr-01	111	5 966.9	333.2	617.4
May-01	112	5 796.1	–170.8	–504.0
Jun-01	113	5 642.5	–153.6	17.2
Jul-01	114	5 529.1	–113.4	40.2
Aug-01	115	5 345.0	–184.1	–70.7
Sep-01	116	4 903.4	–441.6	–257.5
Oct-01	117	5 039.7	136.3	577.9
Nov-01	118	5 203.6	163.9	27.6
Dec-01	119	5 217.4	13.8	–150.1

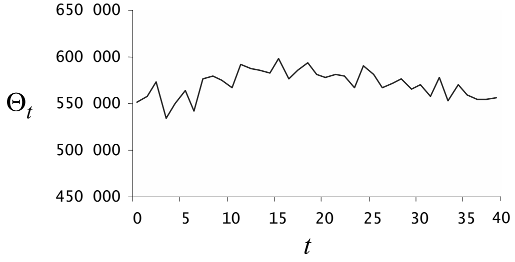
	Summary statistics		
	S_t	∇S_t	$\nabla^2 S_t$
n	120	119	118
mean	4 447.4	22.2	0.2
s.d.	1 394.3	192.3	277.4
min	2 312.6	–661.7	–994.7
max	6 930.2	426.7	625.8
r_1	0.982	–0.031	–0.474
r_2	0.963	–0.085	–0.043
r_3	0.946	–0.049	–0.052
r_4	0.932	0.094	0.127
r_5	0.914	–0.028	–0.030
r_6	0.895	–0.087	–0.020
r_{12}	0.768	0.026	–0.010
ϕ_1	0.982	–0.031	–0.474
ϕ_2	–0.001	–0.087	–0.345
ϕ_3	0.016	–0.055	–0.356
ϕ_4	0.070	0.084	–0.178
ϕ_5	–0.090	–0.031	–0.113
ϕ_6	–0.059	–0.078	–0.083

Time Series – Death Counts

This dataset shows the annual number of deaths recorded in England & Wales for the 39-year period from 1961 to 1999.

		Data values		
Year	t	Θ_t	$\nabla\Theta_t$	$\nabla^2\Theta_t$
1961	0	551 752		
1962	1	557 636	5 884	
1963	2	572 868	15 232	9 348
1964	3	534 737	–38 131	–53 363
1965	4	549 379	14 642	52 773
1966	5	563 624	14 245	–397
1967	6	542 516	–21 108	–35 353
1968	7	576 754	34 238	55 346
1969	8	579 378	2 624	–31 614
1970	9	575 194	–4 184	–6 808
…	…	…	…	…
1990	29	564 846	–12 026	–17 490
1991	30	570 044	5 198	17 224
1992	31	558 313	–11 731	–16 929
1993	32	578 799	20 486	32 217
1994	33	553 194	–25 605	–46 091
1995	34	569 683	16 489	42 094
1996	35	560 135	–9 548	–26 037
1997	36	555 281	–4 854	4 694
1998	37	555 015	–266	4 588
1999	38	556 118	1 103	1 369

	Summary statistics		
	Θ_t	$\nabla\Theta_t$	$\nabla^2\Theta_t$
n	39	38	37
mean	570 980	115	–129
s.d.	14 695	15 067	26 798
min	534 737	–38 131	–53 363
max	598 516	34 238	55 346
r_1	0.452	–0.541	–0.668
r_2	0.470	–0.033	0.100
r_3	0.558	0.204	0.113
r_4	0.356	0.059	0.061
r_5	0.145	–0.278	–0.249
r_6	0.222	0.181	0.143
ϕ_1	0.452	–0.541	–0.668
ϕ_2	0.334	–0.460	–0.624
ϕ_3	0.375	–0.127	–0.578
ϕ_4	–0.026	0.264	–0.163
ϕ_5	–0.353	0.000	–0.082
ϕ_6	–0.089	–0.064	–0.327



Time Series – Bank Base Rates

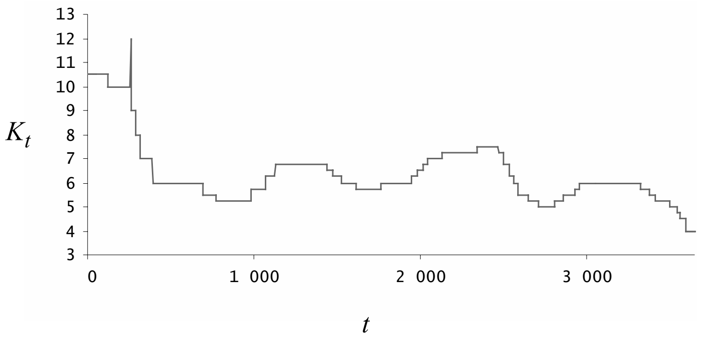
This dataset shows the daily Bank Base Rate for the 10-year period from 1 January 1992 to 31 December 2001. These figures act as a benchmark for interest rates in the UK.

Data values

Date	t	K_t	∇K_t	$\nabla^2 K_t$
01-Jan-92	0	10.50		
02-Jan-92	1	10.50	0.00	
03-Jan-92	2	10.50	0.00	0.00
04-Jan-92	3	10.50	0.00	0.00
05-Jan-92	4	10.50	0.00	0.00
06-Jan-92	5	10.50	0.00	0.00
07-Jan-92	6	10.50	0.00	0.00
08-Jan-92	7	10.50	0.00	0.00
09-Jan-92	8	10.50	0.00	0.00
10-Jan-92	9	10.50	0.00	0.00
22-Dec-01	3 643	4.00	0.00	0.00
23-Dec-01	3 644	4.00	0.00	0.00
24-Dec-01	3 645	4.00	0.00	0.00
25-Dec-01	3 646	4.00	0.00	0.00
26-Dec-01	3 647	4.00	0.00	0.00
27-Dec-01	3 648	4.00	0.00	0.00
28-Dec-01	3 649	4.00	0.00	0.00
29-Dec-01	3 650	4.00	0.00	0.00
30-Dec-01	3 651	4.00	0.00	0.00
31-Dec-01	3 652	4.00	0.00	0.00

Summary statistics

	K_t	∇K_t	$\nabla^2 K_t$
n	3 653	3 652	3 651
mean	6.39	0.00	0.00
s.d.	1.33	0.07	0.09
min	4.00	–2.00	–2.00
max	12.00	2.00	2.00
r_1	0.997	–0.001	–0.374
r_2	0.994	–0.253	–0.252
r_3	0.992	–0.001	0.063
r_4	0.989	0.125	0.126
r_5	0.987	–0.001	0.000
r_6	0.984	–0.127	–0.126
r_{365}	–0.064	–0.004	0.006
ϕ_1	0.997	–0.001	–0.374
ϕ_2	–0.002	–0.253	–0.456
ϕ_3	0.103	–0.001	–0.359
ϕ_4	–0.001	0.066	–0.215
ϕ_5	–0.043	–0.001	–0.107
ϕ_6	–0.001	–0.086	–0.174

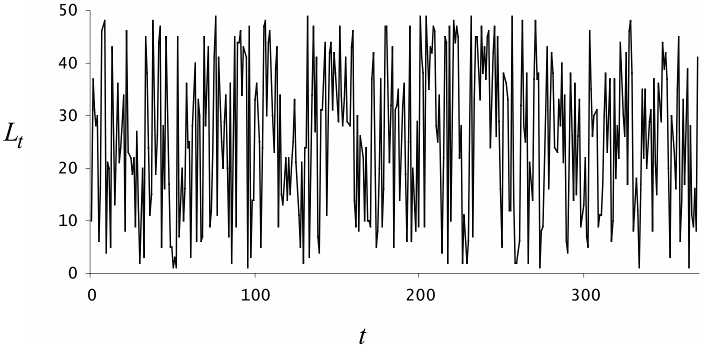


Time Series – National Lottery

This dataset shows the bonus ball number drawn in the UK National Lottery* (Saturdays only) up to 29 December 2001.

		Data values		
Date	t	L_t	∇L_t	$\nabla^2 L_t$
19-Nov-94	0	10		
26-Nov-94	1	37	27.0	
03-Dec-94	2	31	–6.0	–33.0
10-Dec-94	3	28	–3.0	3.0
17-Dec-94	4	30	2.0	5.0
24-Dec-94	5	6	–24.0	–26.0
31-Dec-94	6	16	10.0	34.0
07-Jan-95	7	46	30.0	20.0
14-Jan-95	8	48	2.0	–28.0
21-Jan-95	9	4	–44.0	–46.0
27-Oct-01	360	33	19.0	11.0
03-Nov-01	361	17	–16.0	–35.0
10-Nov-01	362	39	22.0	38.0
17-Nov-01	363	1	–38.0	–60.0
24-Nov-01	364	28	27.0	65.0
01-Dec-01	365	11	–17.0	–44.0
08-Dec-01	366	9	–2.0	15.0
15-Dec-01	367	16	7.0	9.0
22-Dec-01	368	8	–8.0	–15.0
29-Dec-01	369	41	33.0	41.0

	Summary statistics		
	L_t	∇L_t	$\nabla^2 L_t$
n	370	369	368
mean	25.87	0.08	0.02
s.d.	14.40	19.32	33.10
min	1.00	–46.00	–90.00
max	49.00	46.00	87.00
r_1	0.100	–0.471	–0.648
r_2	0.056	–0.027	0.139
r_3	0.059	0.005	0.003
r_4	0.054	0.030	0.036
r_5	–0.005	–0.042	-0.031
r_6	0.003	–0.038	–0.049
r_{52}	0.010	0.048	0.052
ϕ_1	0.100	–0.471	–0.648
ϕ_2	0.047	–0.320	–0.484
ϕ_3	0.049	–0.233	–0.400
ϕ_4	0.041	–0.135	–0.266
ϕ_5	–0.019	–0.139	–0.165
ϕ_6	–0.002	–0.192	–0.251



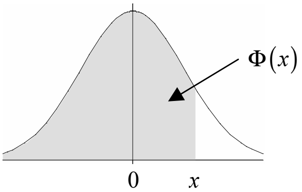
* *Note.* The UK National Lottery draws seven balls (without replacement) from 49 balls numbered from 1 to 49. The bonus ball is the seventh ball drawn.

STATISTICAL TABLES

Probabilities for the Standard Normal distribution

The distribution function is denoted by Φ(*x*), and the probability density function is denoted by ϕ(*x*).

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$



x	Φ(*x*)	***x***	Φ(*x*)	***x***	Φ(*x*)	***x***	Φ(*x*)	***x***	Φ(*x*)
0.00	0.50000	**0.40**	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520
0.01	0.50399	**0.41**	0.65910	**0.81**	0.79103	**1.21**	0.88686	**1.61**	0.94630
0.02	0.50798	**0.42**	0.66276	**0.82**	0.79389	**1.22**	0.88877	**1.62**	0.94738
0.03	0.51197	**0.43**	0.66640	**0.83**	0.79673	**1.23**	0.89065	**1.63**	0.94845
0.04	0.51595	**0.44**	0.67003	**0.84**	0.79955	**1.24**	0.89251	**1.64**	0.94950
0.05	0.51994	**0.45**	0.67364	**0.85**	0.80234	**1.25**	0.89435	**1.65**	0.95053
0.06	0.52392	**0.46**	0.67724	**0.86**	0.80511	**1.26**	0.89617	**1.66**	0.95154
0.07	0.52790	**0.47**	0.68082	**0.87**	0.80785	**1.27**	0.89796	**1.67**	0.95254
0.08	0.53188	**0.48**	0.68439	**0.88**	0.81057	**1.28**	0.89973	**1.68**	0.95352
0.09	0.53586	**0.49**	0.68793	**0.89**	0.81327	**1.29**	0.90147	**1.69**	0.95449
0.10	0.53983	**0.50**	0.69146	**0.90**	0.81594	**1.30**	0.90320	**1.70**	0.95543
0.11	0.54380	**0.51**	0.69497	**0.91**	0.81859	**1.31**	0.90490	**1.71**	0.95637
0.12	0.54776	**0.52**	0.69847	**0.92**	0.82121	**1.32**	0.90658	**1.72**	0.95728
0.13	0.55172	**0.53**	0.70194	**0.93**	0.82381	**1.33**	0.90824	**1.73**	0.95818
0.14	0.55567	**0.54**	0.70540	**0.94**	0.82639	**1.34**	0.90988	**1.74**	0.95907
0.15	0.55962	**0.55**	0.70884	**0.95**	0.82894	**1.35**	0.91149	**1.75**	0.95994
0.16	0.56356	**0.56**	0.71226	**0.96**	0.83147	**1.36**	0.91309	**1.76**	0.96080
0.17	0.56749	**0.57**	0.71566	**0.97**	0.83398	**1.37**	0.91466	**1.77**	0.96164
0.18	0.57142	**0.58**	0.71904	**0.98**	0.83646	**1.38**	0.91621	**1.78**	0.96246
0.19	0.57535	**0.59**	0.72240	**0.99**	0.83891	**1.39**	0.91774	**1.79**	0.96327
0.20	0.57926	**0.60**	0.72575	**1.00**	0.84134	**1.40**	0.91924	**1.80**	0.96407
0.21	0.58317	**0.61**	0.72907	**1.01**	0.84375	**1.41**	0.92073	**1.81**	0.96485
0.22	0.58706	**0.62**	0.73237	**1.02**	0.84614	**1.42**	0.92220	**1.82**	0.96562
0.23	0.59095	**0.63**	0.73565	**1.03**	0.84849	**1.43**	0.92364	**1.83**	0.96638
0.24	0.59483	**0.64**	0.73891	**1.04**	0.85083	**1.44**	0.92507	**1.84**	0.96712
0.25	0.59871	**0.65**	0.74215	**1.05**	0.85314	**1.45**	0.92647	**1.85**	0.96784
0.26	0.60257	**0.66**	0.74537	**1.06**	0.85543	**1.46**	0.92785	**1.86**	0.96856
0.27	0.60642	**0.67**	0.74857	**1.07**	0.85769	**1.47**	0.92922	**1.87**	0.96926
0.28	0.61026	**0.68**	0.75175	**1.08**	0.85993	**1.48**	0.93056	**1.88**	0.96995
0.29	0.61409	**0.69**	0.75490	**1.09**	0.86214	**1.49**	0.93189	**1.89**	0.97062
0.30	0.61791	**0.70**	0.75804	**1.10**	0.86433	**1.50**	0.93319	**1.90**	0.97128
0.31	0.62172	**0.71**	0.76115	**1.11**	0.86650	**1.51**	0.93448	**1.91**	0.97193
0.32	0.62552	**0.72**	0.76424	**1.12**	0.86864	**1.52**	0.93574	**1.92**	0.97257
0.33	0.62930	**0.73**	0.76730	**1.13**	0.87076	**1.53**	0.93699	**1.93**	0.97320
0.34	0.63307	**0.74**	0.77035	**1.14**	0.87286	**1.54**	0.93822	**1.94**	0.97381
0.35	0.63683	**0.75**	0.77337	**1.15**	0.87493	**1.55**	0.93943	**1.95**	0.97441
0.36	0.64058	**0.76**	0.77637	**1.16**	0.87698	**1.56**	0.94062	**1.96**	0.97500
0.37	0.64431	**0.77**	0.77935	**1.17**	0.87900	**1.57**	0.94179	**1.97**	0.97558
0.38	0.64803	**0.78**	0.78230	**1.18**	0.88100	**1.58**	0.94295	**1.98**	0.97615
0.39	0.65173	**0.79**	0.78524	**1.19**	0.88298	**1.59**	0.94408	**1.99**	0.97670
0.40	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520	**2.00**	0.97725

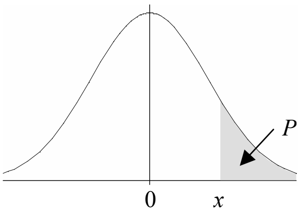
Probabilities for the Standard Normal distribution

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2.00	0.97725	**2.40**	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997
2.01	0.97778	**2.41**	0.99202	**2.81**	0.99752	**3.21**	0.99934	**3.61**	0.99985	**4.01**	0.99997
2.02	0.97831	**2.42**	0.99224	**2.82**	0.99760	**3.22**	0.99936	**3.62**	0.99985	**4.02**	0.99997
2.03	0.97882	**2.43**	0.99245	**2.83**	0.99767	**3.23**	0.99938	**3.63**	0.99986	**4.03**	0.99997
2.04	0.97932	**2.44**	0.99266	**2.84**	0.99774	**3.24**	0.99940	**3.64**	0.99986	**4.04**	0.99997
2.05	0.97982	**2.45**	0.99286	**2.85**	0.99781	**3.25**	0.99942	**3.65**	0.99987	**4.05**	0.99997
2.06	0.98030	**2.46**	0.99305	**2.86**	0.99788	**3.26**	0.99944	**3.66**	0.99987	**4.06**	0.99998
2.07	0.98077	**2.47**	0.99324	**2.87**	0.99795	**3.27**	0.99946	**3.67**	0.99988	**4.07**	0.99998
2.08	0.98124	**2.48**	0.99343	**2.88**	0.99801	**3.28**	0.99948	**3.68**	0.99988	**4.08**	0.99998
2.09	0.98169	**2.49**	0.99361	**2.89**	0.99807	**3.29**	0.99950	**3.69**	0.99989	**4.09**	0.99998
2.10	0.98214	**2.50**	0.99379	**2.90**	0.99813	**3.30**	0.99952	**3.70**	0.99989	**4.10**	0.99998
2.11	0.98257	**2.51**	0.99396	**2.91**	0.99819	**3.31**	0.99953	**3.71**	0.99990	**4.11**	0.99998
2.12	0.98300	**2.52**	0.99413	**2.92**	0.99825	**3.32**	0.99955	**3.72**	0.99990	**4.12**	0.99998
2.13	0.98341	**2.53**	0.99430	**2.93**	0.99831	**3.33**	0.99957	**3.73**	0.99990	**4.13**	0.99998
2.14	0.98382	**2.54**	0.99446	**2.94**	0.99836	**3.34**	0.99958	**3.74**	0.99991	**4.14**	0.99998
2.15	0.98422	**2.55**	0.99461	**2.95**	0.99841	**3.35**	0.99960	**3.75**	0.99991	**4.15**	0.99998
2.16	0.98461	**2.56**	0.99477	**2.96**	0.99846	**3.36**	0.99961	**3.76**	0.99992	**4.16**	0.99998
2.17	0.98500	**2.57**	0.99492	**2.97**	0.99851	**3.37**	0.99962	**3.77**	0.99992	**4.17**	0.99998
2.18	0.98537	**2.58**	0.99506	**2.98**	0.99856	**3.38**	0.99964	**3.78**	0.99992	**4.18**	0.99999
2.19	0.98574	**2.59**	0.99520	**2.99**	0.99861	**3.39**	0.99965	**3.79**	0.99992	**4.19**	0.99999
2.20	0.98610	**2.60**	0.99534	**3.00**	0.99865	**3.40**	0.99966	**3.80**	0.99993	**4.20**	0.99999
2.21	0.98645	**2.61**	0.99547	**3.01**	0.99869	**3.41**	0.99968	**3.81**	0.99993	**4.21**	0.99999
2.22	0.98679	**2.62**	0.99560	**3.02**	0.99874	**3.42**	0.99969	**3.82**	0.99993	**4.22**	0.99999
2.23	0.98713	**2.63**	0.99573	**3.03**	0.99878	**3.43**	0.99970	**3.83**	0.99994	**4.23**	0.99999
2.24	0.98745	**2.64**	0.99585	**3.04**	0.99882	**3.44**	0.99971	**3.84**	0.99994	**4.24**	0.99999
2.25	0.98778	**2.65**	0.99598	**3.05**	0.99886	**3.45**	0.99972	**3.85**	0.99994	**4.25**	0.99999
2.26	0.98809	**2.66**	0.99609	**3.06**	0.99889	**3.46**	0.99973	**3.86**	0.99994	**4.26**	0.99999
2.27	0.98840	**2.67**	0.99621	**3.07**	0.99893	**3.47**	0.99974	**3.87**	0.99995	**4.27**	0.99999
2.28	0.98870	**2.68**	0.99632	**3.08**	0.99896	**3.48**	0.99975	**3.88**	0.99995	**4.28**	0.99999
2.29	0.98899	**2.69**	0.99643	**3.09**	0.99900	**3.49**	0.99976	**3.89**	0.99995	**4.29**	0.99999
2.30	0.98928	**2.70**	0.99653	**3.10**	0.99903	**3.50**	0.99977	**3.90**	0.99995	**4.30**	0.99999
2.31	0.98956	**2.71**	0.99664	**3.11**	0.99906	**3.51**	0.99978	**3.91**	0.99995	**4.31**	0.99999
2.32	0.98983	**2.72**	0.99674	**3.12**	0.99910	**3.52**	0.99978	**3.92**	0.99996	**4.32**	0.99999
2.33	0.99010	**2.73**	0.99683	**3.13**	0.99913	**3.53**	0.99979	**3.93**	0.99996	**4.33**	0.99999
2.34	0.99036	**2.74**	0.99693	**3.14**	0.99916	**3.54**	0.99980	**3.94**	0.99996	**4.34**	0.99999
2.35	0.99061	**2.75**	0.99702	**3.15**	0.99918	**3.55**	0.99981	**3.95**	0.99996	**4.35**	0.99999
2.36	0.99086	**2.76**	0.99711	**3.16**	0.99921	**3.56**	0.99981	**3.96**	0.99996	**4.36**	0.99999
2.37	0.99111	**2.77**	0.99720	**3.17**	0.99924	**3.57**	0.99982	**3.97**	0.99996	**4.37**	0.99999
2.38	0.99134	**2.78**	0.99728	**3.18**	0.99926	**3.58**	0.99983	**3.98**	0.99997	**4.38**	0.99999
2.39	0.99158	**2.79**	0.99736	**3.19**	0.99929	**3.59**	0.99983	**3.99**	0.99997	**4.39**	0.99999
2.40	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997	**4.40**	0.99999

Percentage Points for the Standard Normal distribution

The table gives percentage points *x* defined by the equation

$$P = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2}\,dt$$

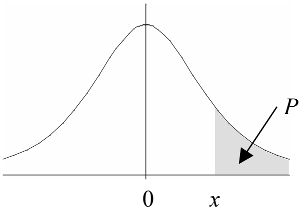


P	x	**P**	x	**P**	x	**P**	x	**P**	x	**P**	x
50%	0.0000	**5.0%**	1.6449	**3.0%**	1.8808	**2.0%**	2.0537	**1.0%**	2.3263	**0.10%**	3.0902
45%	0.1257	**4.8%**	1.6646	**2.9%**	1.8957	**1.9%**	2.0749	**0.9%**	2.3656	**0.09%**	3.1214
40%	0.2533	**4.6%**	1.6849	**2.8%**	1.9110	**1.8%**	2.0969	**0.8%**	2.4089	**0.08%**	3.1559
35%	0.3853	**4.4%**	1.7060	**2.7%**	1.9268	**1.7%**	2.1201	**0.7%**	2.4573	**0.07%**	3.1947
30%	0.5244	**4.2%**	1.7279	**2.6%**	1.9431	**1.6%**	2.1444	**0.6%**	2.5121	**0.06%**	3.2389
25%	0.6745	**4.0%**	1.7507	**2.5%**	1.9600	**1.5%**	2.1701	**0.5%**	2.5758	**0.05%**	3.2905
20%	0.8416	**3.8%**	1.7744	**2.4%**	1.9774	**1.4%**	2.1973	**0.4%**	2.6521	**0.01%**	3.7190
15%	1.0364	**3.6%**	1.7991	**2.3%**	1.9954	**1.3%**	2.2262	**0.3%**	2.7478	**0.005%**	3.8906
10%	1.2816	**3.4%**	1.8250	**2.2%**	2.0141	**1.2%**	2.2571	**0.2%**	2.8782	**0.001%**	4.2649
5%	1.6449	**3.2%**	1.8522	**2.1%**	2.0335	**1.1%**	2.2904	**0.1%**	3.0902	**0.0005%**	4.4172

Percentage Points for the *t* distribution

This table gives percentage points *x* defined by the equation

$$P = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma\left(\tfrac{1}{2}\nu + \tfrac{1}{2}\right)}{\Gamma\left(\tfrac{1}{2}\nu\right)} \int_x^\infty \frac{dt}{\left(1 + t^2/\nu\right)^{\frac{1}{2}(\nu+1)}}$$



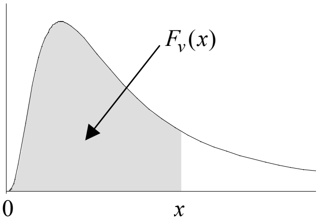
The limiting distribution of *t* as *v* tends to infinity is the standard normal distribution. When *v* is large, interpolation in *v* should be harmonic.

P **=**	**40%**	**30%**	**25%**	**20%**	**15%**	**10%**	**5%**	**2.5%**	**1%**	**0.5%**	**0.1%**	**0.05%**
v												
1	0.3249	0.7265	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Probabilities for the χ^2 distribution

The function tabulated is:

$$F_\nu(x) = \frac{1}{2^{\frac{1}{2}\nu}\Gamma\left(\frac{1}{2}\nu\right)}\int_0^x t^{\frac{1}{2}\nu - 1} e^{-\frac{1}{2}t}\,dt$$



(The above shape applies for $\nu \geq 3$ only. When $\nu < 3$ the mode is at the origin.)

ν =	1		1		2		2		3		3
x		***x***		***x***		***x***		***x***		***x***	
0.0	0.0000	**4.0**	0.9545	**0.0**	0.0000	**4.0**	0.8647	**0.0**	0.0000	**4.0**	0.7385
0.1	0.2482	**4.1**	0.9571	**0.1**	0.0488	**4.1**	0.8713	**0.1**	0.0082	**4.2**	0.7593
0.2	0.3453	**4.2**	0.9596	**0.2**	0.0952	**4.2**	0.8775	**0.2**	0.0224	**4.4**	0.7786
0.3	0.4161	**4.3**	0.9619	**0.3**	0.1393	**4.3**	0.8835	**0.3**	0.0400	**4.6**	0.7965
0.4	0.4729	**4.4**	0.9641	**0.4**	0.1813	**4.4**	0.8892	**0.4**	0.0598	**4.8**	0.8130
0.5	0.5205	**4.5**	0.9661	**0.5**	0.2212	**4.5**	0.8946	**0.5**	0.0811	**5.0**	0.8282
0.6	0.5614	**4.6**	0.9680	**0.6**	0.2592	**4.6**	0.8997	**0.6**	0.1036	**5.2**	0.8423
0.7	0.5972	**4.7**	0.9698	**0.7**	0.2953	**4.7**	0.9046	**0.7**	0.1268	**5.4**	0.8553
0.8	0.6289	**4.8**	0.9715	**0.8**	0.3297	**4.8**	0.9093	**0.8**	0.1505	**5.6**	0.8672
0.9	0.6572	**4.9**	0.9731	**0.9**	0.3624	**4.9**	0.9137	**0.9**	0.1746	**5.8**	0.8782
1.0	0.6827	**5.0**	0.9747	**1.0**	0.3935	**5.0**	0.9179	**1.0**	0.1987	**6.0**	0.8884
1.1	0.7057	**5.1**	0.9761	**1.1**	0.4231	**5.1**	0.9219	**1.1**	0.2229	**6.2**	0.8977
1.2	0.7267	**5.2**	0.9774	**1.2**	0.4512	**5.2**	0.9257	**1.2**	0.2470	**6.4**	0.9063
1.3	0.7458	**5.3**	0.9787	**1.3**	0.4780	**5.3**	0.9293	**1.3**	0.2709	**6.6**	0.9142
1.4	0.7633	**5.4**	0.9799	**1.4**	0.5034	**5.4**	0.9328	**1.4**	0.2945	**6.8**	0.9214
1.5	0.7793	**5.5**	0.9810	**1.5**	0.5276	**5.5**	0.9361	**1.5**	0.3177	**7.0**	0.9281
1.6	0.7941	**5.6**	0.9820	**1.6**	0.5507	**5.6**	0.9392	**1.6**	0.3406	**7.2**	0.9342
1.7	0.8077	**5.7**	0.9830	**1.7**	0.5726	**5.7**	0.9422	**1.7**	0.3631	**7.4**	0.9398
1.8	0.8203	**5.8**	0.9840	**1.8**	0.5934	**5.8**	0.9450	**1.8**	0.3851	**7.6**	0.9450
1.9	0.8319	**5.9**	0.9849	**1.9**	0.6133	**5.9**	0.9477	**1.9**	0.4066	**7.8**	0.9497
2.0	0.8427	**6.0**	0.9857	**2.0**	0.6321	**6.0**	0.9502	**2.0**	0.4276	**8.0**	0.9540
2.1	0.8527	**6.1**	0.9865	**2.1**	0.6501	**6.2**	0.9550	**2.1**	0.4481	**8.2**	0.9579
2.2	0.8620	**6.2**	0.9872	**2.2**	0.6671	**6.4**	0.9592	**2.2**	0.4681	**8.4**	0.9616
2.3	0.8706	**6.3**	0.9879	**2.3**	0.6834	**6.6**	0.9631	**2.3**	0.4875	**8.6**	0.9649
2.4	0.8787	**6.4**	0.9886	**2.4**	0.6988	**6.8**	0.9666	**2.4**	0.5064	**8.8**	0.9679
2.5	0.8862	**6.5**	0.9892	**2.5**	0.7135	**7.0**	0.9698	**2.5**	0.5247	**9.0**	0.9707
2.6	0.8931	**6.6**	0.9898	**2.6**	0.7275	**7.2**	0.9727	**2.6**	0.5425	**9.2**	0.9733
2.7	0.8997	**6.7**	0.9904	**2.7**	0.7408	**7.4**	0.9753	**2.7**	0.5598	**9.4**	0.9756
2.8	0.9057	**6.8**	0.9909	**2.8**	0.7534	**7.6**	0.9776	**2.8**	0.5765	**9.6**	0.9777
2.9	0.9114	**6.9**	0.9914	**2.9**	0.7654	**7.8**	0.9798	**2.9**	0.5927	**9.8**	0.9797
3.0	0.9167	**7.0**	0.9918	**3.0**	0.7769	**8.0**	0.9817	**3.0**	0.6084	**10.0**	0.9814
3.1	0.9217	**7.1**	0.9923	**3.1**	0.7878	**8.2**	0.9834	**3.1**	0.6235	**10.2**	0.9831
3.2	0.9264	**7.2**	0.9927	**3.2**	0.7981	**8.4**	0.9850	**3.2**	0.6382	**10.4**	0.9845
3.3	0.9307	**7.3**	0.9931	**3.3**	0.8080	**8.6**	0.9864	**3.3**	0.6524	**10.6**	0.9859
3.4	0.9348	**7.4**	0.9935	**3.4**	0.8173	**8.8**	0.9877	**3.4**	0.6660	**10.8**	0.9871
3.5	0.9386	**7.5**	0.9938	**3.5**	0.8262	**9.0**	0.9889	**3.5**	0.6792	**11.0**	0.9883
3.6	0.9422	**7.6**	0.9942	**3.6**	0.8347	**9.2**	0.9899	**3.6**	0.6920	**11.2**	0.9893
3.7	0.9456	**7.7**	0.9945	**3.7**	0.8428	**9.4**	0.9909	**3.7**	0.7043	**11.4**	0.9903
3.8	0.9487	**7.8**	0.9948	**3.8**	0.8504	**9.6**	0.9918	**3.8**	0.7161	**11.6**	0.9911
3.9	0.9517	**7.9**	0.9951	**3.9**	0.8577	**9.8**	0.9926	**3.9**	0.7275	**11.8**	0.9919
4.0	0.9545	**8.0**	0.9953	**4.0**	0.8647	**10.0**	0.9933	**4.0**	0.7385	**12.0**	0.9926

Probabilities for the χ^2 distribution

v =	4	5	6	7	8	9	10	11	12	13	14
x											
0.5	0.0265	0.0079	0.0022	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	0.0902	0.0374	0.0144	0.0052	0.0018	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000
1.5	0.1734	0.0869	0.0405	0.0177	0.0073	0.0029	0.0011	0.0004	0.0001	0.0000	0.0000
2.0	0.2642	0.1509	0.0803	0.0402	0.0190	0.0085	0.0037	0.0015	0.0006	0.0002	0.0001
2.5	0.3554	0.2235	0.1315	0.0729	0.0383	0.0191	0.0091	0.0042	0.0018	0.0008	0.0003
3.0	0.4422	0.3000	0.1912	0.1150	0.0656	0.0357	0.0186	0.0093	0.0045	0.0021	0.0009
3.5	0.5221	0.3766	0.2560	0.1648	0.1008	0.0589	0.0329	0.0177	0.0091	0.0046	0.0022
4.0	0.5940	0.4506	0.3233	0.2202	0.1429	0.0886	0.0527	0.0301	0.0166	0.0088	0.0045
4.5	0.6575	0.5201	0.3907	0.2793	0.1906	0.1245	0.0780	0.0471	0.0274	0.0154	0.0084
5.0	0.7127	0.5841	0.4562	0.3400	0.2424	0.1657	0.1088	0.0688	0.0420	0.0248	0.0142
5.5	0.7603	0.6421	0.5185	0.4008	0.2970	0.2113	0.1446	0.0954	0.0608	0.0375	0.0224
6.0	0.8009	0.6938	0.5768	0.4603	0.3528	0.2601	0.1847	0.1266	0.0839	0.0538	0.0335
6.5	0.8352	0.7394	0.6304	0.5173	0.4086	0.3110	0.2283	0.1620	0.1112	0.0739	0.0477
7.0	0.8641	0.7794	0.6792	0.5711	0.4634	0.3629	0.2746	0.2009	0.1424	0.0978	0.0653
7.5	0.8883	0.8140	0.7229	0.6213	0.5162	0.4148	0.3225	0.2427	0.1771	0.1254	0.0863
8.0	0.9084	0.8438	0.7619	0.6674	0.5665	0.4659	0.3712	0.2867	0.2149	0.1564	0.1107
8.5	0.9251	0.8693	0.7963	0.7094	0.6138	0.5154	0.4199	0.3321	0.2551	0.1904	0.1383
9.0	0.9389	0.8909	0.8264	0.7473	0.6577	0.5627	0.4679	0.3781	0.2971	0.2271	0.1689
9.5	0.9503	0.9093	0.8527	0.7813	0.6981	0.6075	0.5146	0.4242	0.3403	0.2658	0.2022
10.0	0.9596	0.9248	0.8753	0.8114	0.7350	0.6495	0.5595	0.4696	0.3840	0.3061	0.2378
10.5	0.9672	0.9378	0.8949	0.8380	0.7683	0.6885	0.6022	0.5140	0.4278	0.3474	0.2752
11.0	0.9734	0.9486	0.9116	0.8614	0.7983	0.7243	0.6425	0.5567	0.4711	0.3892	0.3140
11.5	0.9785	0.9577	0.9259	0.8818	0.8251	0.7570	0.6801	0.5976	0.5134	0.4310	0.3536
12.0	0.9826	0.9652	0.9380	0.8994	0.8488	0.7867	0.7149	0.6364	0.5543	0.4724	0.3937
12.5	0.9860	0.9715	0.9483	0.9147	0.8697	0.8134	0.7470	0.6727	0.5936	0.5129	0.4338
13.0	0.9887	0.9766	0.9570	0.9279	0.8882	0.8374	0.7763	0.7067	0.6310	0.5522	0.4735
13.5	0.9909	0.9809	0.9643	0.9392	0.9042	0.8587	0.8030	0.7381	0.6662	0.5900	0.5124
14.0	0.9927	0.9844	0.9704	0.9488	0.9182	0.8777	0.8270	0.7670	0.6993	0.6262	0.5503
14.5	0.9941	0.9873	0.9755	0.9570	0.9304	0.8944	0.8486	0.7935	.7301	0.6604	0.5868
15.0	0.9953	0.9896	0.9797	0.9640	0.9409	0.9091	0.8679	0.8175	0.7586	0.6926	0.6218
15.5	0.9962	0.9916	0.9833	0.9699	0.9499	0.9219	0.8851	0.8393	0.7848	0.7228	0.6551
16.0	0.9970	0.9932	0.9862	0.9749	0.9576	0.9331	0.9004	0.8589	0.8088	0.7509	0.6866
16.5	0.9976	0.9944	0.9887	0.9791	0.9642	0.9429	0.9138	0.8764	0.8306	0.7768	0.7162
17.0	0.9981	0.9955	0.9907	0.9826	0.9699	0.9513	0.9256	0.8921	0.8504	0.8007	0.7438
17.5	0.9985	0.9964	0.9924	0.9856	0.9747	0.9586	0.9360	0.9061	0.8683	0.8226	0.7695
18.0	0.9988	0.9971	0.9938	0.9880	0.9788	0.9648	0.9450	0.9184	0.8843	0.8425	0.7932
18.5	0.9990	0.9976	0.9949	0.9901	0.9822	0.9702	0.9529	0.9293	0.8987	0.8606	0.8151
19.0	0.9992	0.9981	0.9958	0.9918	0.9851	0.9748	0.9597	0.9389	0.9115	0.8769	0.8351
19.5	0.9994	0.9984	0.9966	0.9932	0.9876	0.9787	0.9656	0.9473	0.9228	0.8916	0.8533
20	0.9995	0.9988	0.9972	0.9944	0.9897	0.9821	0.9707	0.9547	0.9329	0.9048	0.8699
21	0.9997	0.9992	0.9982	0.9962	0.9929	0.9873	0.9789	0.9666	0.9496	0.9271	0.8984
22	0.9998	0.9995	0.9988	0.9975	0.9951	0.9911	0.9849	0.9756	0.9625	0.9446	0.9214
23	0.9999	0.9997	0.9992	0.9983	0.9966	0.9938	0.9893	0.9823	0.9723	0.9583	0.9397
24	0.9999	0.9998	0.9995	0.9989	0.9977	0.9957	0.9924	0.9873	0.9797	0.9689	0.9542
25	0.9999	0.9999	0.9997	0.9992	0.9984	0.9970	0.9947	0.9909	0.9852	0.9769	0.9654
26	1.0000	0.9999	0.9998	0.9995	0.9989	0.9980	0.9963	0.9935	0.9893	0.9830	0.9741
27	1.0000	0.9999	0.9999	0.9997	0.9993	0.9986	0.9974	0.9954	0.9923	0.9876	0.9807
28	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9982	0.9968	0.9945	0.9910	0.9858
29	1.0000	1.0000	0.9999	0.9999	0.9997	0.9994	0.9988	0.9977	0.9961	0.9935	0.9895
30	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9972	0.9953	0.9924

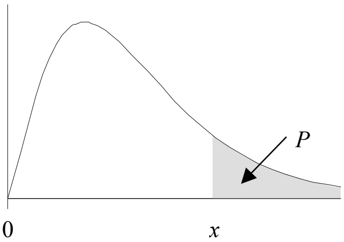
Probabilities for the χ^2 distribution

***v* =**	15	16	17	18	19	20	21	22	23	24	25
x											
3	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0079	0.0042	0.0022	0.0011	0.0006	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000
6	0.0203	0.0119	0.0068	0.0038	0.0021	0.0011	0.0006	0.0003	0.0001	0.0001	0.0000
7	0.0424	0.0267	0.0165	0.0099	0.0058	0.0033	0.0019	0.0010	0.0005	0.0003	0.0001
8	0.0762	0.0511	0.0335	0.0214	0.0133	0.0081	0.0049	0.0028	0.0016	0.0009	0.0005
9	0.1225	0.0866	0.0597	0.0403	0.0265	0.0171	0.0108	0.0067	0.0040	0.0024	0.0014
10	0.1803	0.1334	0.0964	0.0681	0.0471	0.0318	0.0211	0.0137	0.0087	0.0055	0.0033
11	0.2474	0.1905	0.1434	0.1056	0.0762	0.0538	0.0372	0.0253	0.0168	0.0110	0.0071
12	0.3210	0.2560	0.1999	0.1528	0.1144	0.0839	0.0604	0.0426	0.0295	0.0201	0.0134
13	0.3977	0.3272	0.2638	0.2084	0.1614	0.1226	0.0914	0.0668	0.0480	0.0339	0.0235
14	0.4745	0.4013	0.3329	0.2709	0.2163	0.1695	0.1304	0.0985	0.0731	0.0533	0.0383
15	0.5486	0.4754	0.4045	0.3380	0.2774	0.2236	0.1770	0.1378	0.1054	0.0792	0.0586
16	0.6179	0.5470	0.4762	0.4075	0.3427	0.2834	0.2303	0.1841	0.1447	0.1119	0.0852
17	0.6811	0.6144	0.5456	0.4769	0.4101	0.3470	0.2889	0.2366	0.1907	0.1513	0.1182
18	0.7373	0.6761	0.6112	0.5443	0.4776	0.4126	0.3510	0.2940	0.2425	0.1970	0.1576
19	0.7863	0.7313	0.6715	0.6082	0.5432	0.4782	0.4149	0.3547	0.2988	0.2480	0.2029
20	0.8281	0.7798	0.7258	0.6672	0.6054	0.5421	0.4787	0.4170	0.3581	0.3032	0.2532
21	0.8632	0.8215	0.7737	0.7206	0.6632	0.6029	0.5411	0.4793	0.4189	0.3613	0.3074
22	0.8922	0.8568	0.8153	0.7680	0.7157	0.6595	0.6005	0.5401	0.4797	0.4207	0.3643
23	0.9159	0.8863	0.8507	0.8094	0.7627	0.7112	0.6560	0.5983	0.5392	0.4802	0.4224
24	0.9349	0.9105	0.8806	0.8450	0.8038	0.7576	0.7069	0.6528	0.5962	0.5384	0.4806
25	0.9501	0.9302	0.9053	0.8751	0.8395	0.7986	0.7528	0.7029	0.6497	0.5942	0.5376
26	0.9620	0.9460	0.9255	0.9002	0.8698	0.8342	0.7936	0.7483	0.6991	0.6468	0.5924
27	0.9713	0.9585	0.9419	0.9210	0.8953	0.8647	0.8291	0.7888	0.7440	0.6955	0.6441
28	0.9784	0.9684	0.9551	0.9379	0.9166	0.8906	0.8598	0.8243	0.7842	0.7400	0.6921
29	0.9839	0.9761	0.9655	0.9516	0.9340	0.9122	0.8860	0.8551	0.8197	0.7799	0.7361
30	0.9881	0.9820	0.9737	0.9626	0.9482	0.9301	0.9080	0.8815	0.8506	0.8152	0.7757
31	0.9912	0.9865	0.9800	0.9712	0.9596	0.9448	0.9263	0.9039	0.8772	0.8462	0.8110
32	0.9936	0.9900	0.9850	0.9780	0.9687	0.9567	0.9414	0.9226	0.8999	0.8730	0.8420
33	0.9953	0.9926	0.9887	0.9833	0.9760	0.9663	0.9538	0.9381	0.9189	0.8959	0.8689
34	0.9966	0.9946	0.9916	0.9874	0.9816	0.9739	0.9638	0.9509	0.9348	0.9153	0.8921
35	0.9975	0.9960	0.9938	0.9905	0.9860	0.9799	0.9718	0.9613	0.9480	0.9316	0.9118
36	0.9982	0.9971	0.9954	0.9929	0.9894	0.9846	0.9781	0.9696	0.9587	0.9451	0.9284
37	0.9987	0.9979	0.9966	0.9948	0.9921	0.9883	0.9832	0.9763	0.9675	0.9562	0.9423
38	0.9991	0.9985	0.9975	0.9961	0.9941	0.9911	0.9871	0.9817	0.9745	0.9653	0.9537
39	0.9994	0.9989	0.9982	0.9972	0.9956	0.9933	0.9902	0.9859	0.9802	0.9727	0.9632
40	0.9995	0.9992	0.9987	0.9979	0.9967	0.9950	0.9926	0.9892	0.9846	0.9786	0.9708
41	0.9997	0.9994	0.9991	0.9985	0.9976	0.9963	0.9944	0.9918	0.9882	0.9833	0.9770
42	0.9998	0.9996	0.9993	0.9989	0.9982	0.9972	0.9958	0.9937	0.9909	0.9871	0.9820
43	0.9998	0.9997	0.9995	0.9992	0.9987	0.9980	0.9969	0.9953	0.9931	0.9901	0.9860
44	0.9999	0.9998	0.9997	0.9994	0.9991	0.9985	0.9977	0.9965	0.9947	0.9924	0.9892
45	0.9999	0.9999	0.9998	0.9996	0.9993	0.9989	0.9983	0.9973	0.9960	0.9942	0.9916
46	0.9999	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9980	0.9970	0.9956	0.9936
47	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9985	0.9978	0.9967	0.9951
48	1.0000	1.0000	0.9999	0.9998	0.9997	0.9996	0.9993	0.9989	0.9983	0.9975	0.9963
49	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9992	0.9988	0.9981	0.9972
50	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9979

Percentage Points for the χ^2 distribution

This table gives percentage points *x* defined by the equation

$$P = \frac{1}{2^{\frac{1}{2}v}\Gamma\left(\frac{1}{2}v\right)}\int_x^\infty t^{\frac{1}{2}v-1}e^{-\frac{1}{2}t}\,dt$$



(The above shape applies only for $v \geq 3$. When $v < 3$, the mode is at the origin.)

***P* =**	**99.95%**	**99.9%**	**99.5%**	**99%**	**97.5%**	**95%**	**90%**	**80%**	**70%**	**60%**
v										
1	3.927E–07	1.571E–06	3.927E–05	1.571E–04	9.821E–04	0.003932	0.01579	0.06418	0.1485	0.2750
2	0.001000	0.002001	0.01003	0.02010	0.05064	0.1026	0.2107	0.4463	0.7133	1.022
3	0.01528	0.02430	0.07172	0.1148	0.2158	0.3518	0.5844	1.005	1.424	1.869
4	0.06392	0.09080	0.2070	0.2971	0.4844	0.7107	1.064	1.649	2.195	2.753
5	0.1581	0.2102	0.4118	0.5543	0.8312	1.145	1.610	2.343	3.000	3.656
6	0.2994	0.3810	0.6757	0.8721	1.237	1.635	2.204	3.070	3.828	4.570
7	0.4849	0.5985	0.9893	1.239	1.690	2.167	2.833	3.822	4.671	5.493
8	0.7104	0.8571	1.344	1.647	2.180	2.733	3.490	4.594	5.527	6.423
9	0.9718	1.152	1.735	2.088	2.700	3.325	4.168	5.380	6.393	7.357
10	1.265	1.479	2.156	2.558	3.247	3.940	4.865	6.179	7.267	8.295
11	1.587	1.834	2.603	3.053	3.816	4.575	5.578	6.989	8.148	9.237
12	1.935	2.214	3.074	3.571	4.404	5.226	6.304	7.807	9.034	10.18
13	2.305	2.617	3.565	4.107	5.009	5.892	7.041	8.634	9.926	11.13
14	2.697	3.041	4.075	4.660	5.629	6.571	7.790	9.467	10.82	12.08
15	3.107	3.483	4.601	5.229	6.262	7.261	8.547	10.31	11.72	13.03
16	3.536	3.942	5.142	5.812	6.908	7.962	9.312	11.15	12.62	13.98
17	3.980	4.416	5.697	6.408	7.564	8.672	10.09	12.00	13.53	14.94
18	4.439	4.905	6.265	7.015	8.231	9.390	10.86	12.86	14.44	15.89
19	4.913	5.407	6.844	7.633	8.907	10.12	11.65	13.72	15.35	16.85
20	5.398	5.921	7.434	8.260	9.591	10.85	12.44	14.58	16.27	17.81
21	5.895	6.447	8.034	8.897	10.28	11.59	13.24	15.44	17.18	18.77
22	6.404	6.983	8.643	9.542	10.98	12.34	14.04	16.31	18.10	19.73
23	6.924	7.529	9.260	10.20	11.69	13.09	14.85	17.19	19.02	20.69
24	7.453	8.085	9.886	10.86	12.40	13.85	15.66	18.06	19.94	21.65
25	7.991	8.649	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62
26	8.537	9.222	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58
27	9.093	9.803	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54
28	9.656	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44
32	11.98	12.81	15.13	16.36	18.29	20.07	22.27	25.15	27.37	29.38
34	13.18	14.06	16.50	17.79	19.81	21.66	23.95	26.94	29.24	31.31
36	14.40	15.32	17.89	19.23	21.34	23.27	25.64	28.73	31.12	33.25
38	15.64	16.61	19.29	20.69	22.88	24.88	27.34	30.54	32.99	35.19
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62
70	37.47	39.04	43.28	45.44	48.76	51.74	55.33	59.90	63.35	66.40
80	44.79	46.52	51.17	53.54	57.15	60.39	64.28	69.21	72.92	76.19
90	52.28	54.16	59.20	61.75	65.65	69.13	73.29	78.56	82.51	85.99
100	59.89	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81

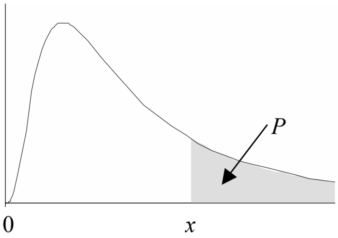
Percentage Points for the χ^2 distribution

***P* =**	**50%**	**40%**	**30%**	**20%**	**10%**	**5%**	**2.5%**	**1%**	**0.5%**	**0.1%**	**0.05%**
ν											
1	0.4549	0.7083	1.074	1.642	2.706	3.841	5.024	6.635	7.879	10.83	12.12
2	1.386	1.833	2.408	3.219	4.605	5.991	7.378	9.210	10.60	13.82	15.20
3	2.366	2.946	3.665	4.642	6.251	7.815	9.348	11.34	12.84	16.27	17.73
4	3.357	4.045	4.878	5.989	7.779	9.488	11.14	13.28	14.86	18.47	20.00
5	4.351	5.132	6.064	7.289	9.236	11.07	12.83	15.09	16.75	20.51	22.11
6	5.348	6.211	7.231	8.558	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	6.346	7.283	8.383	9.803	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	7.344	8.351	9.524	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	8.343	9.414	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	9.342	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	10.34	11.53	12.90	14.63	17.28	19.68	21.92	24.73	26.76	31.26	33.14
12	11.34	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	12.34	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	13.34	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	14.34	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	15.34	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	16.34	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	17.34	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	18.34	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	19.34	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	20.34	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	21.34	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	22.34	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	23.34	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	24.34	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	25.34	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	26.34	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.65	55.48	57.86
28	27.34	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	28.34	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	29.34	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
32	31.34	33.38	35.66	38.47	42.58	46.19	49.48	53.49	56.33	62.49	64.99
34	33.34	35.44	37.80	40.68	44.90	48.60	51.97	56.06	58.96	65.25	67.80
36	35.34	37.50	39.92	42.88	47.21	51.00	54.44	58.62	61.58	67.98	70.59
38	37.34	39.56	42.05	45.08	49.51	53.38	56.90	61.16	64.18	70.70	73.35
40	39.34	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.10
50	49.33	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	59.33	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.7
70	69.33	72.36	75.69	79.71	85.53	90.53	95.02	100.4	104.2	112.3	115.6
80	79.33	82.57	86.12	90.41	96.58	101.9	106.6	112.3	116.3	124.8	128.3
90	89.33	92.76	96.52	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
100	99.33	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

Percentage Points for the *F* distribution

The function tabulated is x defined for the specified percentage points P by the equation

$$P = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{1}{2}\nu_1\right)\Gamma\left(\frac{1}{2}\nu_2\right)} \nu_1^{\frac{1}{2}\nu_1} \nu_2^{\frac{1}{2}\nu_2} \int_x^{\infty} \frac{t^{\frac{1}{2}\nu_1 - 1}}{\left(\nu_2+\nu_1 t\right)^{\frac{1}{2}(\nu_1+\nu_2)}} dt$$



(The above shape applies only for $\nu_1 \geq 3$. When $\nu_1 < 3$, the mode is at the origin.)

10% Points for the *F* distribution

$\nu_1 =$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**24**	**∞**
ν_2													
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	62.00	63.33
2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392	9.408	9.450	9.491
3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.240	5.230	5.216	5.176	5.134
4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.936	3.920	3.896	3.831	3.761
5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.316	3.297	3.268	3.191	3.105
6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937	2.905	2.818	2.722
7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703	2.668	2.575	2.471
8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538	2.502	2.404	2.293
9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.440	2.416	2.379	2.277	2.159
10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323	2.284	2.178	2.055
11	3.225	2.860	2.660	2.536	2.451	2.389	2.342	2.304	2.274	2.248	2.209	2.100	1.972
12	3.177	2.807	2.606	2.480	2.394	2.331	2.283	2.245	2.214	2.188	2.147	2.036	1.904
13	3.136	2.763	2.560	2.434	2.347	2.283	2.234	2.195	2.164	2.138	2.097	1.983	1.846
14	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095	2.054	1.938	1.797
15	3.073	2.695	2.490	2.361	2.273	2.208	2.158	2.119	2.086	2.059	2.017	1.899	1.755
16	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028	1.985	1.866	1.718
17	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001	1.958	1.836	1.686
18	3.007	2.624	2.416	2.286	2.196	2.130	2.079	2.038	2.005	1.977	1.933	1.810	1.657
19	2.990	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956	1.912	1.787	1.631
20	2.975	2.589	2.380	2.249	2.158	2.091	2.040	1.999	1.965	1.937	1.892	1.767	1.607
21	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.920	1.875	1.748	1.586
22	2.949	2.561	2.351	2.219	2.128	2.060	2.008	1.967	1.933	1.904	1.859	1.731	1.567
23	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.890	1.845	1.716	1.549
24	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877	1.832	1.702	1.533
25	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866	1.820	1.689	1.518
26	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855	1.809	1.677	1.504
27	2.901	2.511	2.299	2.165	2.073	2.005	1.952	1.909	1.874	1.845	1.799	1.666	1.491
28	2.894	2.503	2.291	2.157	2.064	1.996	1.943	1.900	1.865	1.836	1.790	1.656	1.478
29	2.887	2.495	2.283	2.149	2.057	1.988	1.935	1.892	1.857	1.827	1.781	1.647	1.467
30	2.881	2.489	2.276	2.142	2.049	1.980	1.927	1.884	1.849	1.819	1.773	1.638	1.456
32	2.869	2.477	2.263	2.129	2.036	1.967	1.913	1.870	1.835	1.805	1.758	1.622	1.437
34	2.859	2.466	2.252	2.118	2.024	1.955	1.901	1.858	1.822	1.793	1.745	1.608	1.420
36	2.850	2.456	2.243	2.108	2.014	1.945	1.891	1.847	1.811	1.781	1.734	1.595	1.404
38	2.842	2.448	2.234	2.099	2.005	1.935	1.881	1.838	1.802	1.772	1.724	1.584	1.390
40	2.835	2.440	2.226	2.091	1.997	1.927	1.873	1.829	1.793	1.763	1.715	1.574	1.377
60	2.791	2.393	2.177	2.041	1.946	1.875	1.819	1.775	1.738	1.707	1.657	1.511	1.292
120	2.748	2.347	2.130	1.992	1.896	1.824	1.767	1.722	1.684	1.652	1.601	1.447	1.193
∞	2.706	2.303	2.084	1.945	1.847	1.774	1.717	1.670	1.632	1.599	1.546	1.383	1.000

5% Points for the *F* distribution

$v_1 =$	1	2	3	4	5	6	7	8	9	10	12	24	∞
v_2													
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.785	8.745	8.638	8.527
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.609	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	1.901	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	1.887	1.622
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.189	2.142	2.070	1.864	1.594
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.170	2.123	2.050	1.843	1.569
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.153	2.106	2.033	1.824	1.547
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.138	2.091	2.017	1.808	1.527
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077	2.003	1.793	1.509
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993	1.917	1.700	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.910	1.834	1.608	1.254
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.752	1.517	1.000

2½% Points for the *F* distribution

$v_1 =$ v_2	1	2	3	4	5	6	7	8	9	10	12	24	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.6	963.3	968.6	976.7	997.3	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.46	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.12	13.90
4	12.22	10.65	9.979	9.605	9.364	9.197	9.074	8.980	8.905	8.844	8.751	8.511	8.257
5	10.01	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.681	6.619	6.525	6.278	6.015
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.366	5.117	4.849
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.666	4.415	4.142
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.200	3.947	3.670
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.868	3.614	3.333
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.621	3.365	3.080
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.430	3.173	2.883
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.277	3.019	2.725
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.153	2.893	2.596
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147	3.050	2.789	2.487
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	2.963	2.701	2.395
16	6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	3.049	2.986	2.889	2.625	2.316
17	6.042	4.619	4.011	3.665	3.438	3.277	3.156	3.061	2.985	2.922	2.825	2.560	2.248
18	5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.929	2.866	2.769	2.503	2.187
19	5.922	4.508	3.903	3.559	3.333	3.172	3.051	2.956	2.880	2.817	2.720	2.452	2.133
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.676	2.408	2.085
21	5.827	4.420	3.819	3.475	3.250	3.090	2.969	2.874	2.798	2.735	2.637	2.368	2.042
22	5.786	4.383	3.783	3.440	3.215	3.055	2.934	2.839	2.763	2.700	2.602	2.332	2.003
23	5.750	4.349	3.750	3.408	3.183	3.023	2.902	2.808	2.731	2.668	2.570	2.299	1.968
24	5.717	4.319	3.721	3.379	3.155	2.995	2.874	2.779	2.703	2.640	2.541	2.269	1.935
25	5.686	4.291	3.694	3.353	3.129	2.969	2.848	2.753	2.677	2.613	2.515	2.242	1.906
26	5.659	4.265	3.670	3.329	3.105	2.945	2.824	2.729	2.653	2.590	2.491	2.217	1.878
27	5.633	4.242	3.647	3.307	3.083	2.923	2.802	2.707	2.631	2.568	2.469	2.195	1.853
28	5.610	4.221	3.626	3.286	3.063	2.903	2.782	2.687	2.611	2.547	2.448	2.174	1.829
29	5.588	4.201	3.607	3.267	3.044	2.884	2.763	2.669	2.592	2.529	2.430	2.154	1.807
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.575	2.511	2.412	2.136	1.787
32	5.531	4.149	3.557	3.218	2.995	2.836	2.715	2.620	2.543	2.480	2.381	2.103	1.750
34	5.499	4.120	3.529	3.191	2.968	2.808	2.688	2.593	2.516	2.453	2.353	2.075	1.717
36	5.471	4.094	3.505	3.167	2.944	2.785	2.664	2.569	2.492	2.429	2.329	2.049	1.687
38	5.446	4.071	3.483	3.145	2.923	2.763	2.643	2.548	2.471	2.407	2.307	2.027	1.661
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.452	2.388	2.288	2.007	1.637
60	5.286	3.925	3.343	3.008	2.786	2.627	2.507	2.412	2.334	2.270	2.169	1.882	1.482
120	5.152	3.805	3.227	2.894	2.674	2.515	2.395	2.299	2.222	2.157	2.055	1.760	1.311
∞	5.024	3.689	3.116	2.786	2.567	2.408	2.288	2.192	2.114	2.048	1.945	1.640	1.000

1% Points for the *F* distribution

$\nu_1 =$ ν_2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**24**	**∞**
1	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6107	6234	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.42	99.46	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.60	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	13.93	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.888	9.466	9.021
6	13.75	10.92	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.718	7.313	6.880
7	12.25	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.469	6.074	5.650
8	11.26	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.667	5.279	4.859
9	10.56	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.111	4.729	4.311
10	10.04	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.706	4.327	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.397	4.021	3.603
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.155	3.780	3.361
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	3.960	3.587	3.165
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.800	3.427	3.004
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.666	3.294	2.869
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691	3.553	3.181	2.753
17	8.400	6.112	5.185	4.669	4.336	4.101	3.927	3.791	3.682	3.593	3.455	3.083	2.653
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.371	2.999	2.566
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434	3.297	2.925	2.489
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.231	2.859	2.421
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310	3.173	2.801	2.360
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.121	2.749	2.306
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211	3.074	2.702	2.256
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.032	2.659	2.211
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.993	2.620	2.170
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094	2.958	2.585	2.132
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.926	2.552	2.097
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032	2.896	2.522	2.064
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005	2.868	2.495	2.034
30	7.562	5.390	4.510	4.018	3.699	3.473	3.305	3.173	3.067	2.979	2.843	2.469	2.006
32	7.499	5.336	4.459	3.969	3.652	3.427	3.258	3.127	3.021	2.934	2.798	2.423	1.956
34	7.444	5.289	4.416	3.927	3.611	3.386	3.218	3.087	2.981	2.894	2.758	2.383	1.911
36	7.396	5.248	4.377	3.890	3.574	3.351	3.183	3.052	2.946	2.859	2.723	2.347	1.872
38	7.353	5.211	4.343	3.858	3.542	3.319	3.152	3.021	2.915	2.828	2.692	2.316	1.837
40	7.314	5.178	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.665	2.288	1.805
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632	2.496	2.115	1.601
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472	2.336	1.950	1.381
∞	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.185	1.791	1.000

Probabilities for the Poisson distribution

The function tabulated is $P\left(X \le x\right) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}$.

$x =$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	$= x$
μ															μ
0.05	0.95123	0.99879	0.99998												**0.05**
0.10	0.90484	0.99532	0.99985												**0.10**
0.15	0.86071	0.98981	0.99950	0.99998											**0.15**
0.20	0.81873	0.98248	0.99885	0.99994											**0.20**
0.30	0.74082	0.96306	0.99640	0.99973	0.99998										**0.30**
0.40	0.67032	0.93845	0.99207	0.99922	0.99994										**0.40**
0.50	0.60653	0.90980	0.98561	0.99825	0.99983	0.99999									**0.50**
0.60	0.54881	0.87810	0.97688	0.99664	0.99961	0.99996									**0.60**
0.70	0.49659	0.84420	0.96586	0.99425	0.99921	0.99991	0.99999								**0.70**
0.80	0.44933	0.80879	0.95258	0.99092	0.99859	0.99982	0.99998								**0.80**
0.90	0.40657	0.77248	0.93714	0.98654	0.99766	0.99966	0.99996								**0.90**
1.00	0.36788	0.73576	0.91970	0.98101	0.99634	0.99941	0.99992	0.99999							**1.00**
1.10	0.33287	0.69903	0.90042	0.97426	0.99456	0.99903	0.99985	0.99998							**1.10**
1.20	0.30119	0.66263	0.87949	0.96623	0.99225	0.99850	0.99975	0.99996							**1.20**
1.30	0.27253	0.62682	0.85711	0.95690	0.98934	0.99777	0.99960	0.99994	0.99999						**1.30**
1.40	0.24660	0.59183	0.83350	0.94627	0.98575	0.99680	0.99938	0.99989	0.99998						**1.40**
1.50	0.22313	0.55783	0.80885	0.93436	0.98142	0.99554	0.99907	0.99983	0.99997						**1.50**
1.60	0.20190	0.52493	0.78336	0.92119	0.97632	0.99396	0.99866	0.99974	0.99995	0.99999					**1.60**
1.70	0.18268	0.49325	0.75722	0.90681	0.97039	0.99200	0.99812	0.99961	0.99993	0.99999					**1.70**
1.80	0.16530	0.46284	0.73062	0.89129	0.96359	0.98962	0.99743	0.99944	0.99989	0.99998					**1.80**
1.90	0.14957	0.43375	0.70372	0.87470	0.95592	0.98678	0.99655	0.99921	0.99984	0.99997	0.99999				**1.90**
2.00	0.13534	0.40601	0.67668	0.85712	0.94735	0.98344	0.99547	0.99890	0.99976	0.99995	0.99999				**2.00**
2.10	0.12246	0.37961	0.64963	0.83864	0.93787	0.97955	0.99414	0.99851	0.99966	0.99993	0.99999				**2.10**
2.20	0.11080	0.35457	0.62271	0.81935	0.92750	0.97509	0.99254	0.99802	0.99953	0.99990	0.99998				**2.20**
2.30	0.10026	0.33085	0.59604	0.79935	0.91625	0.97002	0.99064	0.99741	0.99936	0.99986	0.99997	0.99999			**2.30**
2.40	0.09072	0.30844	0.56971	0.77872	0.90413	0.96433	0.98841	0.99666	0.99914	0.99980	0.99996	0.99999			**2.40**
2.50	0.08208	0.28730	0.54381	0.75758	0.89118	0.95798	0.98581	0.99575	0.99886	0.99972	0.99994	0.99999			**2.50**
2.60	0.07427	0.26738	0.51843	0.73600	0.87742	0.95096	0.98283	0.99467	0.99851	0.99962	0.99991	0.99998			**2.60**
2.70	0.06721	0.24866	0.49362	0.71409	0.86291	0.94327	0.97943	0.99338	0.99809	0.99950	0.99988	0.99997	0.99999		**2.70**

All 1.00000

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

$x =$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**$= x$**
μ															μ
2.80	0.06081	0.23108	0.46945	0.69194	0.84768	0.93489	0.97559	0.99187	0.99757	0.99934	0.99984	0.99996	0.99999	1.00000	**2.80**
2.90	0.05502	0.21459	0.44596	0.66962	0.83178	0.92583	0.97128	0.99012	0.99694	0.99914	0.99978	0.99995	0.99999	1.00000	**2.90**
3.00	0.04979	0.19915	0.42319	0.64723	0.81526	0.91608	0.96649	0.98810	0.99620	0.99890	0.99971	0.99993	0.99998	1.00000	**3.00**
3.10	0.04505	0.18470	0.40116	0.62484	0.79819	0.90567	0.96120	0.98579	0.99532	0.99860	0.99962	0.99990	0.99998	1.00000	**3.10**
3.20	0.04076	0.17120	0.37990	0.60252	0.78061	0.89459	0.95538	0.98317	0.99429	0.99824	0.99950	0.99987	0.99997	0.99999	**3.20**
3.30	0.03688	0.15860	0.35943	0.58034	0.76259	0.88288	0.94903	0.98022	0.99309	0.99781	0.99936	0.99983	0.99996	0.99999	**3.30**
3.40	0.03337	0.14684	0.33974	0.55836	0.74418	0.87054	0.94215	0.97693	0.99171	0.99729	0.99919	0.99978	0.99994	0.99999	**3.40**
3.50	0.03020	0.13589	0.32085	0.53663	0.72544	0.85761	0.93471	0.97326	0.99013	0.99669	0.99898	0.99971	0.99992	0.99998	**3.50**
3.60	0.02732	0.12569	0.30275	0.51522	0.70644	0.84412	0.92673	0.96921	0.98833	0.99598	0.99873	0.99963	0.99990	0.99997	**3.60**
3.70	0.02472	0.11620	0.28543	0.49415	0.68722	0.83009	0.91819	0.96476	0.98630	0.99515	0.99843	0.99953	0.99987	0.99997	**3.70**
3.80	0.02237	0.10738	0.26890	0.47348	0.66784	0.81556	0.90911	0.95989	0.98402	0.99420	0.99807	0.99941	0.99983	0.99996	**3.80**
3.90	0.02024	0.09919	0.25313	0.45325	0.64837	0.80056	0.89948	0.95460	0.98147	0.99311	0.99765	0.99926	0.99978	0.99994	**3.90**
4.00	0.01832	0.09158	0.23810	0.43347	0.62884	0.78513	0.88933	0.94887	0.97864	0.99187	0.99716	0.99908	0.99973	0.99992	**4.00**
4.10	0.01657	0.08452	0.22381	0.41418	0.60931	0.76931	0.87865	0.94269	0.97551	0.99046	0.99659	0.99887	0.99966	0.99990	**4.10**
4.20	0.01500	0.07798	0.21024	0.39540	0.58983	0.75314	0.86746	0.93606	0.97207	0.98887	0.99593	0.99863	0.99957	0.99987	**4.20**
4.30	0.01357	0.07191	0.19735	0.37715	0.57044	0.73666	0.85579	0.92897	0.96830	0.98709	0.99518	0.99833	0.99947	0.99984	**4.30**
4.40	0.01228	0.06630	0.18514	0.35945	0.55118	0.71991	0.84365	0.92142	0.96420	0.98511	0.99431	0.99799	0.99934	0.99980	**4.40**
4.50	0.01111	0.06110	0.17358	0.34230	0.53210	0.70293	0.83105	0.91341	0.95974	0.98291	0.99333	0.99760	0.99919	0.99975	**4.50**
4.60	0.01005	0.05629	0.16264	0.32571	0.51323	0.68576	0.81803	0.90495	0.95493	0.98047	0.99222	0.99714	0.99902	0.99969	**4.60**
4.70	0.00910	0.05184	0.15230	0.30968	0.49461	0.66844	0.80461	0.89603	0.94974	0.97779	0.99098	0.99661	0.99882	0.99961	**4.70**
4.80	0.00823	0.04773	0.14254	0.29423	0.47626	0.65101	0.79080	0.88667	0.94418	0.97486	0.98958	0.99601	0.99858	0.99953	**4.80**
4.90	0.00745	0.04393	0.13333	0.27934	0.45821	0.63350	0.77665	0.87686	0.93824	0.97166	0.98803	0.99532	0.99830	0.99942	**4.90**
5.00	0.00674	0.04043	0.12465	0.26503	0.44049	0.61596	0.76218	0.86663	0.93191	0.96817	0.98630	0.99455	0.99798	0.99930	**5.00**
5.10	0.00610	0.03719	0.11648	0.25127	0.42313	0.59842	0.74742	0.85598	0.92518	0.96440	0.98440	0.99367	0.99761	0.99916	**5.10**
5.20	0.00552	0.03420	0.10879	0.23807	0.40613	0.58091	0.73239	0.84492	0.91806	0.96033	0.98230	0.99269	0.99719	0.99899	**5.20**

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

$x =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	$= x$
μ															μ
5.30	0.00499	0.03145	0.10155	0.22541	0.38952	0.56347	0.71713	0.83348	0.91055	0.95594	0.98000	0.99159	0.99671	0.99880	**5.30**
5.40	0.00452	0.02891	0.09476	0.21329	0.37331	0.54613	0.70167	0.82166	0.90265	0.95125	0.97749	0.99037	0.99617	0.99857	**5.40**
5.50	0.00409	0.02656	0.08838	0.20170	0.35752	0.52892	0.68604	0.80949	0.89436	0.94622	0.97475	0.98901	0.99555	0.99831	**5.50**
5.60	0.00370	0.02441	0.08239	0.19062	0.34215	0.51186	0.67026	0.79698	0.88568	0.94087	0.97178	0.98751	0.99486	0.99802	**5.60**
5.70	0.00335	0.02242	0.07677	0.18005	0.32721	0.49498	0.65437	0.78415	0.87662	0.93518	0.96856	0.98586	0.99408	0.99768	**5.70**
5.80	0.00303	0.02059	0.07151	0.16996	0.31272	0.47831	0.63839	0.77103	0.86719	0.92916	0.96510	0.98405	0.99321	0.99730	**5.80**
5.90	0.00274	0.01890	0.06658	0.16035	0.29866	0.46187	0.62236	0.75763	0.85739	0.92279	0.96137	0.98207	0.99224	0.99686	**5.90**
6.00	0.00248	0.01735	0.06197	0.15120	0.28506	0.44568	0.60630	0.74398	0.84724	0.91608	0.95738	0.97991	0.99117	0.99637	**6.00**
6.10	0.00224	0.01592	0.05765	0.14250	0.27189	0.42975	0.59024	0.73010	0.83674	0.90902	0.95311	0.97756	0.98999	0.99582	**6.10**
6.20	0.00203	0.01461	0.05362	0.13423	0.25918	0.41411	0.57421	0.71602	0.82591	0.90162	0.94856	0.97502	0.98868	0.99520	**6.20**
6.30	0.00184	0.01341	0.04985	0.12637	0.24690	0.39877	0.55823	0.70175	0.81477	0.89388	0.94372	0.97227	0.98725	0.99451	**6.30**
6.40	0.00166	0.01230	0.04632	0.11892	0.23507	0.38374	0.54233	0.68732	0.80331	0.88580	0.93859	0.96930	0.98568	0.99375	**6.40**
6.50	0.00150	0.01128	0.04304	0.11185	0.22367	0.36904	0.52652	0.67276	0.79157	0.87738	0.93316	0.96612	0.98397	0.99290	**6.50**
6.60	0.00136	0.01034	0.03997	0.10515	0.21270	0.35467	0.51084	0.65808	0.77956	0.86864	0.92743	0.96271	0.98211	0.99196	**6.60**
6.70	0.00123	0.00948	0.03711	0.09881	0.20216	0.34065	0.49530	0.64332	0.76728	0.85957	0.92140	0.95906	0.98009	0.99093	**6.70**
6.80	0.00111	0.00869	0.03444	0.09281	0.19203	0.32698	0.47992	0.62849	0.75477	0.85018	0.91507	0.95517	0.97790	0.98979	**6.80**
6.90	0.00101	0.00796	0.03195	0.08713	0.18231	0.31366	0.46472	0.61361	0.74203	0.84049	0.90843	0.95104	0.97554	0.98855	**6.90**
7.00	0.00091	0.00730	0.02964	0.08177	0.17299	0.30071	0.44971	0.59871	0.72909	0.83050	0.90148	0.94665	0.97300	0.98719	**7.00**
7.25	0.00071	0.00586	0.02452	0.06963	0.15138	0.26992	0.41316	0.56152	0.69596	0.80427	0.88279	0.93454	0.96581	0.98324	**7.25**
7.50	0.00055	0.00470	0.02026	0.05915	0.13206	0.24144	0.37815	0.52464	0.66197	0.77641	0.86224	0.92076	0.95733	0.97844	**7.50**
7.75	0.00043	0.00377	0.01670	0.05012	0.11487	0.21522	0.34485	0.48837	0.62740	0.74712	0.83990	0.90527	0.94749	0.97266	**7.75**
8.00	0.00034	0.00302	0.01375	0.04238	0.09963	0.19124	0.31337	0.45296	0.59255	0.71662	0.81589	0.88808	0.93620	0.96582	**8.00**
8.25	0.00026	0.00242	0.01131	0.03576	0.08619	0.16939	0.28380	0.41864	0.55770	0.68516	0.79032	0.86919	0.92341	0.95782	**8.25**
8.50	0.00020	0.00193	0.00928	0.03011	0.07436	0.14960	0.25618	0.38560	0.52311	0.65297	0.76336	0.84866	0.90908	0.94859	**8.50**
8.75	0.00016	0.00154	0.00761	0.02530	0.06401	0.13174	0.23051	0.35398	0.48902	0.62031	0.73519	0.82657	0.89320	0.93805	**8.75**

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}.$$

$x =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	$= x$
μ															μ
9.00	0.00012	0.00123	0.00623	0.02123	0.05496	0.11569	0.20678	0.32390	0.45565	0.58741	0.70599	0.80301	0.87577	0.92615	**9.00**
9.25	0.00010	0.00099	0.00510	0.01777	0.04709	0.10133	0.18495	0.29544	0.42320	0.55451	0.67597	0.77810	0.85683	0.91285	**9.25**
9.50	0.00007	0.00079	0.00416	0.01486	0.04026	0.08853	0.16495	0.26866	0.39182	0.52183	0.64533	0.75199	0.83643	0.89814	**9.50**
9.75	0.00006	0.00063	0.00340	0.01240	0.03435	0.07716	0.14671	0.24359	0.36166	0.48957	0.61428	0.72483	0.81464	0.88200	**9.75**
10.00	0.00005	0.00050	0.00277	0.01034	0.02925	0.06709	0.13014	0.22022	0.33282	0.45793	0.58304	0.69678	0.79156	0.86446	**10.00**
10.25	0.00004	0.00040	0.00226	0.00860	0.02486	0.05820	0.11515	0.19854	0.30538	0.42707	0.55179	0.66802	0.76729	0.84556	**10.25**
10.50	0.00003	0.00032	0.00183	0.00715	0.02109	0.05038	0.10163	0.17851	0.27941	0.39713	0.52074	0.63873	0.74196	0.82535	**10.50**
10.75	0.00002	0.00025	0.00149	0.00593	0.01786	0.04352	0.08949	0.16008	0.25494	0.36825	0.49005	0.60908	0.71572	0.80390	**10.75**
11.00	0.00002	0.00020	0.00121	0.00492	0.01510	0.03752	0.07861	0.14319	0.23199	0.34051	0.45989	0.57927	0.68870	0.78129	**11.00**
11.25	0.00001	0.00016	0.00098	0.00407	0.01275	0.03228	0.06891	0.12777	0.21054	0.31401	0.43041	0.54945	0.66105	0.75763	**11.25**
11.50	0.00001	0.00013	0.00080	0.00336	0.01075	0.02773	0.06027	0.11373	0.19059	0.28879	0.40173	0.51980	0.63295	0.73304	**11.50**
11.75	0.00001	0.00010	0.00065	0.00278	0.00904	0.02377	0.05260	0.10101	0.17210	0.26492	0.37397	0.49047	0.60453	0.70763	**11.75**
12.00	0.00001	0.00008	0.00052	0.00229	0.00760	0.02034	0.04582	0.08950	0.15503	0.24239	0.34723	0.46160	0.57597	0.68154	**12.00**
12.25	0.00000	0.00006	0.00042	0.00189	0.00638	0.01738	0.03984	0.07914	0.13932	0.22123	0.32158	0.43332	0.54740	0.65489	**12.25**
12.50	0.00000	0.00005	0.00034	0.00155	0.00535	0.01482	0.03457	0.06983	0.12492	0.20143	0.29707	0.40576	0.51898	0.62784	**12.50**
12.75	0.00000	0.00004	0.00028	0.00128	0.00447	0.01262	0.02994	0.06148	0.11175	0.18297	0.27377	0.37901	0.49083	0.60051	**12.75**
13.00	0.00000	0.00003	0.00022	0.00105	0.00374	0.01073	0.02589	0.05403	0.09976	0.16581	0.25168	0.35316	0.46310	0.57304	**13.00**
13.25	0.00000	0.00003	0.00018	0.00086	0.00312	0.00911	0.02234	0.04739	0.08886	0.14993	0.23083	0.32829	0.43590	0.54558	**13.25**
13.50	0.00000	0.00002	0.00014	0.00071	0.00260	0.00773	0.01925	0.04148	0.07900	0.13526	0.21123	0.30445	0.40933	0.51825	**13.50**
13.75	0.00000	0.00002	0.00012	0.00058	0.00217	0.00654	0.01656	0.03625	0.07008	0.12177	0.19285	0.28169	0.38349	0.49116	**13.75**
14.00	0.00000	0.00001	0.00009	0.00047	0.00181	0.00553	0.01423	0.03162	0.06206	0.10940	0.17568	0.26004	0.35846	0.46445	**14.00**
14.25	0.00000	0.00001	0.00008	0.00039	0.00150	0.00467	0.01220	0.02753	0.05484	0.09808	0.15970	0.23952	0.33430	0.43820	**14.25**
14.50	0.00000	0.00001	0.00006	0.00032	0.00125	0.00394	0.01045	0.02394	0.04838	0.08776	0.14486	0.22013	0.31108	0.41253	**14.50**
14.75	0.00000	0.00001	0.00005	0.00026	0.00103	0.00332	0.00894	0.02077	0.04260	0.07837	0.13113	0.20188	0.28884	0.38751	**14.75**
15.00	0.00000	0.00000	0.00004	0.00021	0.00086	0.00279	0.00763	0.01800	0.03745	0.06985	0.11846	0.18475	0.26761	0.36322	**15.00**

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

x =	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**= x**
μ															**μ**
15.50			0.00003	0.00014	0.00059	0.00197	0.00554	0.01346	0.02879	0.05519	0.09612	0.15378	0.22827	0.31708	**15.50**
16.00			0.00002	0.00009	0.00040	0.00138	0.00401	0.01000	0.02199	0.04330	0.07740	0.12699	0.19312	0.27451	**16.00**
16.50			0.00001	0.00006	0.00027	0.00097	0.00288	0.00739	0.01669	0.03374	0.06187	0.10407	0.16210	0.23574	**16.50**
17.00			0.00001	0.00004	0.00018	0.00067	0.00206	0.00543	0.01260	0.02612	0.04912	0.08467	0.13502	0.20087	**17.00**
17.50			0.00000	0.00003	0.00012	0.00047	0.00147	0.00397	0.00945	0.02010	0.03875	0.06840	0.11165	0.16987	**17.50**
18.00				0.00002	0.00008	0.00032	0.00104	0.00289	0.00706	0.01538	0.03037	0.05489	0.09167	0.14260	**18.00**
18.50				0.00001	0.00006	0.00022	0.00074	0.00210	0.00524	0.01170	0.02366	0.04376	0.07475	0.11886	**18.50**
19.00				0.00001	0.00004	0.00015	0.00052	0.00151	0.00387	0.00886	0.01832	0.03467	0.06056	0.09840	**19.00**
19.50					0.00003	0.00011	0.00036	0.00109	0.00285	0.00667	0.01411	0.02731	0.04875	0.08092	**19.50**
20.00					0.00002	0.00007	0.00026	0.00078	0.00209	0.00500	0.01081	0.02139	0.03901	0.06613	**20.00**
	All 0.00000														
20.50					0.00001	0.00005	0.00018	0.00056	0.00152	0.00373	0.00824	0.01666	0.03103	0.05371	**20.50**
21.00					0.00001	0.00003	0.00012	0.00039	0.00111	0.00277	0.00625	0.01290	0.02455	0.04336	**21.00**
21.50						0.00002	0.00009	0.00028	0.00080	0.00204	0.00472	0.00995	0.01931	0.03481	**21.50**
22.00						0.00002	0.00006	0.00020	0.00058	0.00150	0.00355	0.00763	0.01512	0.02778	**22.00**
22.50						0.00001	0.00004	0.00014	0.00041	0.00110	0.00265	0.00583	0.01177	0.02206	**22.50**
23.00						0.00001	0.00003	0.00010	0.00030	0.00081	0.00198	0.00443	0.00912	0.01743	**23.00**
23.50							0.00002	0.00007	0.00021	0.00059	0.00147	0.00335	0.00704	0.01370	**23.50**
24.00							0.00001	0.00005	0.00015	0.00043	0.00108	0.00252	0.00540	0.01072	**24.00**
24.50							0.00001	0.00003	0.00011	0.00031	0.00080	0.00189	0.00413	0.00834	**24.50**
25.00							0.00001	0.00002	0.00008	0.00022	0.00059	0.00142	0.00314	0.00647	**25.00**

Probabilities for the Poisson distribution

$$P\left(X \le x\right) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

x =	14	15	16	17	18	19	20	21	22	23	24	25	= *x*
μ													μ
3.30													**3.30**
3.40													**3.40**
3.50													**3.50**
3.60	0.99999												**3.60**
3.70	0.99999												**3.70**
3.80	0.99999												**3.80**
3.90	0.99999					**All 1.00000**							**3.90**
4.00	0.99998												**4.00**
4.10	0.99997	0.99999											**4.10**
4.20	0.99997	0.99999											**4.20**
4.30	0.99996	0.99999											**4.30**
4.40	0.99994	0.99998											**4.40**
4.50	0.99993	0.99998	0.99999										**4.50**
4.60	0.99991	0.99997	0.99999										**4.60**
4.70	0.99988	0.99997	0.99999										**4.70**
4.80	0.99985	0.99996	0.99999										**4.80**
4.90	0.99982	0.99995	0.99998										**4.90**
5.00	0.99977	0.99993	0.99998	0.99999									**5.00**
5.10	0.99972	0.99991	0.99997	0.99999									**5.10**
5.20	0.99966	0.99989	0.99997	0.99999									**5.20**
5.30	0.99959	0.99987	0.99996	0.99999									**5.30**
5.40	0.99950	0.99984	0.99995	0.99999									**5.40**
5.50	0.99940	0.99980	0.99994	0.99998	0.99999								**5.50**
5.60	0.99928	0.99976	0.99992	0.99998	0.99999								**5.60**
5.70	0.99915	0.99970	0.99990	0.99997	0.99999								**5.70**
5.80	0.99899	0.99964	0.99988	0.99996	0.99999								**5.80**
5.90	0.99881	0.99957	0.99986	0.99995	0.99999								**5.90**
6.00	0.99860	0.99949	0.99983	0.99994	0.99998	0.99999							**6.00**
6.10	0.99836	0.99939	0.99979	0.99993	0.99998	0.99999							**6.10**
6.20	0.99809	0.99928	0.99975	0.99991	0.99997	0.99999							**6.20**

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

x =	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**= x**
μ													**μ**
6.30	0.99778	0.99916	0.99970	0.99990	0.99997	0.99999							**6.30**
6.40	0.99744	0.99901	0.99964	0.99987	0.99996	0.99999							**6.40**
6.50	0.99704	0.99884	0.99957	0.99985	0.99995	0.99998							**6.50**
6.60	0.99661	0.99865	0.99949	0.99982	0.99994	0.99998	0.99999						**6.60**
6.70	0.99611	0.99843	0.99940	0.99978	0.99993	0.99998	0.99999						**6.70**
6.80	0.99557	0.99818	0.99930	0.99974	0.99991	0.99997	0.99999						**6.80**
6.90	0.99496	0.99791	0.99918	0.99969	0.99989	0.99996	0.99999						**6.90**
7.00	0.99428	0.99759	0.99904	0.99964	0.99987	0.99996	0.99999						**7.00**
7.25	0.99227	0.99664	0.99862	0.99946	0.99980	0.99993	0.99998	0.99999					**7.25**
7.50	0.98974	0.99539	0.99804	0.99921	0.99970	0.99989	0.99996	0.99999					**7.50**
7.75	0.98659	0.99379	0.99728	0.99887	0.99955	0.99983	0.99994	0.99998	0.99999				**7.75**
8.00	0.98274	0.99177	0.99628	0.99841	0.99935	0.99975	0.99991	0.99997	0.99999				**8.00**
8.25	0.97810	0.98925	0.99500	0.99779	0.99907	0.99963	0.99986	0.99995	0.99998	0.99999			**8.25**
8.50	0.97257	0.98617	0.99339	0.99700	0.99870	0.99947	0.99979	0.99992	0.99997	0.99999			**8.50**
8.75	0.96608	0.98243	0.99137	0.99597	0.99821	0.99924	0.99969	0.99988	0.99996	0.99998	0.99999		**8.75**
9.00	0.95853	0.97796	0.98889	0.99468	0.99757	0.99894	0.99956	0.99983	0.99993	0.99998	0.99999		**9.00**
9.25	0.94986	0.97269	0.98588	0.99306	0.99675	0.99855	0.99938	0.99975	0.99990	0.99996	0.99999		**9.25**
9.50	0.94001	0.96653	0.98227	0.99107	0.99572	0.99804	0.99914	0.99964	0.99985	0.99994	0.99998	0.99999	**9.50**
9.75	0.92891	0.95941	0.97799	0.98864	0.99442	0.99738	0.99882	0.99949	0.99979	0.99992	0.99997	0.99999	**9.75**
10.00	0.91654	0.95126	0.97296	0.98572	0.99281	0.99655	0.99841	0.99930	0.99970	0.99988	0.99995	0.99998	**10.00**
10.25	0.90287	0.94203	0.96712	0.98224	0.99085	0.99550	0.99788	0.99905	0.99959	0.99983	0.99993	0.99997	**10.25**
10.50	0.88789	0.93167	0.96039	0.97814	0.98849	0.99421	0.99721	0.99871	0.99943	0.99976	0.99990	0.99996	**10.50**
10.75	0.87160	0.92013	0.95273	0.97335	0.98566	0.99263	0.99637	0.99829	0.99922	0.99966	0.99986	0.99994	**10.75**
11.00	0.85404	0.90740	0.94408	0.96781	0.98231	0.99071	0.99533	0.99775	0.99896	0.99954	0.99980	0.99992	**11.00**
11.25	0.83524	0.89345	0.93438	0.96146	0.97839	0.98841	0.99405	0.99707	0.99861	0.99937	0.99972	0.99988	**11.25**
11.50	0.81526	0.87829	0.92360	0.95425	0.97383	0.98568	0.99250	0.99623	0.99818	0.99915	0.99962	0.99984	**11.50**
11.75	0.79416	0.86194	0.91172	0.94612	0.96858	0.98247	0.99063	0.99519	0.99763	0.99888	0.99949	0.99977	**11.75**
12.00	0.77202	0.84442	0.89871	0.93703	0.96258	0.97872	0.98840	0.99393	0.99695	0.99853	0.99931	0.99969	**12.00**
12.25	0.74895	0.82576	0.88457	0.92695	0.95579	0.97438	0.98577	0.99242	0.99612	0.99809	0.99909	0.99958	**12.25**
12.50	0.72503	0.80603	0.86931	0.91584	0.94815	0.96941	0.98269	0.99060	0.99509	0.99754	0.99881	0.99944	**12.50**

All 1.00000

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

$x =$	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**$= x$**
μ													μ
12.75	0.70039	0.78529	0.85294	0.90368	0.93962	0.96374	0.97911	0.98845	0.99386	0.99686	0.99845	0.99926	**12.75**
13.00	0.67513	0.76361	0.83549	0.89046	0.93017	0.95733	0.97499	0.98592	0.99238	0.99603	0.99801	0.99903	**13.00**
13.25	0.64938	0.74108	0.81701	0.87619	0.91976	0.95014	0.97027	0.98297	0.99062	0.99502	0.99746	0.99875	**13.25**
13.50	0.62327	0.71779	0.79755	0.86088	0.90838	0.94213	0.96491	0.97955	0.98854	0.99382	0.99678	0.99838	**13.50**
13.75	0.59691	0.69385	0.77716	0.84454	0.89601	0.93326	0.95886	0.97563	0.98611	0.99238	0.99597	0.99794	**13.75**
14.00	0.57044	0.66936	0.75592	0.82720	0.88264	0.92350	0.95209	0.97116	0.98329	0.99067	0.99498	0.99739	**14.00**
14.25	0.54396	0.64443	0.73391	0.80891	0.86829	0.91282	0.94455	0.96608	0.98003	0.98867	0.99380	0.99673	**14.25**
14.50	0.51760	0.61916	0.71121	0.78972	0.85296	0.90122	0.93622	0.96038	0.97630	0.98634	0.99241	0.99592	**14.50**
14.75	0.49146	0.59368	0.68791	0.76968	0.83668	0.88869	0.92705	0.95399	0.97206	0.98364	0.99076	0.99496	**14.75**
15.00	0.46565	0.56809	0.66412	0.74886	0.81947	0.87522	0.91703	0.94689	0.96726	0.98054	0.98884	0.99382	**15.00**
15.50	0.41541	0.51701	0.61544	0.70518	0.78246	0.84551	0.89437	0.93043	0.95584	0.97296	0.98402	0.99087	**15.50**
16.00	0.36753	0.46674	0.56596	0.65934	0.74235	0.81225	0.86817	0.91077	0.94176	0.96331	0.97768	0.98688	**16.00**
16.50	0.32254	0.41802	0.51648	0.61205	0.69965	0.77572	0.83848	0.88780	0.92478	0.95131	0.96955	0.98159	**16.50**
17.00	0.28083	0.37145	0.46774	0.56402	0.65496	0.73632	0.80548	0.86147	0.90473	0.93670	0.95935	0.97476	**17.00**
17.50	0.24264	0.32754	0.42040	0.51600	0.60893	0.69453	0.76943	0.83185	0.88150	0.91928	0.94682	0.96611	**17.50**
18.00	0.20808	0.28665	0.37505	0.46865	0.56224	0.65092	0.73072	0.79912	0.85509	0.89889	0.93174	0.95539	**18.00**
18.50	0.17714	0.24903	0.33214	0.42259	0.51555	0.60607	0.68979	0.76355	0.82558	0.87547	0.91392	0.94238	**18.50**
19.00	0.14975	0.21479	0.29203	0.37836	0.46948	0.56061	0.64717	0.72550	0.79314	0.84902	0.89325	0.92687	**19.00**
19.50	0.12573	0.18398	0.25497	0.33639	0.42461	0.51514	0.60342	0.68538	0.75804	0.81963	0.86968	0.90872	**19.50**
20.00	0.10486	0.15651	0.22107	0.29703	0.38142	0.47026	0.55909	0.64370	0.72061	0.78749	0.84323	0.88782	**20.00**
20.50	0.08690	0.13227	0.19040	0.26050	0.34034	0.42648	0.51477	0.60095	0.68127	0.75285	0.81399	0.86413	**20.50**
21.00	0.07157	0.11107	0.16292	0.22696	0.30168	0.38426	0.47097	0.55769	0.64046	0.71603	0.78216	0.83770	**21.00**
21.50	0.05860	0.09269	0.13852	0.19647	0.26568	0.34401	0.42821	0.51442	0.59866	0.67741	0.74796	0.80863	**21.50**
22.00	0.04769	0.07689	0.11704	0.16900	0.23250	0.30603	0.38691	0.47164	0.55638	0.63742	0.71172	0.77710	**22.00**
22.50	0.03860	0.06341	0.09830	0.14447	0.20219	0.27054	0.34744	0.42983	0.51409	0.59652	0.67379	0.74334	**22.50**
23.00	0.03107	0.05200	0.08208	0.12277	0.17477	0.23771	0.31010	0.38938	0.47227	0.55515	0.63458	0.70766	**23.00**
23.50	0.02488	0.04241	0.06814	0.10372	0.15017	0.20761	0.27512	0.35065	0.43134	0.51378	0.59451	0.67039	**23.50**
24.00	0.01983	0.03440	0.05626	0.08713	0.12828	0.18026	0.24264	0.31393	0.39170	0.47285	0.55400	0.63191	**24.00**
24.50	0.01572	0.02776	0.04620	0.07278	0.10896	0.15561	0.21276	0.27943	0.35367	0.43276	0.51350	0.59262	**24.50**
25.00	0.01240	0.02229	0.03775	0.06048	0.09204	0.13357	0.18549	0.24730	0.31753	0.39388	0.47340	0.55292	**25.00**

Probabilities for the Poisson distribution

$$P(X \le x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

x =	26	27	28	29	30	31	32	33	34	35	36	37	= *x*
μ													μ
9.00													**9.00**
9.25													**9.25**
9.50													**9.50**
9.75													**9.75**
10.00	0.99999				**All 1.00000**								**10.00**
10.25	0.99999												**10.25**
10.50	0.99999	0.99999											**10.50**
10.75	0.99998	0.99999											**10.75**
11.00	0.99997	0.99999											**11.00**
11.25	0.99995	0.99998	0.99999										**11.25**
11.50	0.99993	0.99997	0.99999										**11.50**
11.75	0.99990	0.99996	0.99998	0.99999									**11.75**
12.00	0.99987	0.99994	0.99998	0.99999									**12.00**
12.25	0.99982	0.99992	0.99997	0.99999	0.99999								**12.25**
12.50	0.99975	0.99989	0.99995	0.99998	0.99999								**12.50**
12.75	0.99966	0.99985	0.99994	0.99997	0.99999								**12.75**
13.00	0.99955	0.99980	0.99991	0.99996	0.99998	0.99999							**13.00**
13.25	0.99940	0.99972	0.99988	0.99995	0.99998	0.99999							**13.25**
13.50	0.99922	0.99963	0.99983	0.99993	0.99997	0.99999	0.99999						**13.50**
13.75	0.99898	0.99951	0.99978	0.99990	0.99996	0.99998	0.99999						**13.75**
14.00	0.99869	0.99936	0.99970	0.99986	0.99994	0.99997	0.99999						**14.00**
14.25	0.99833	0.99918	0.99961	0.99982	0.99992	0.99996	0.99998	0.99999					**14.25**
14.50	0.99789	0.99894	0.99948	0.99976	0.99989	0.99995	0.99998	0.99999					**14.50**
14.75	0.99734	0.99865	0.99933	0.99968	0.99985	0.99993	0.99997	0.99999	0.99999				**14.75**
15.00	0.99669	0.99828	0.99914	0.99958	0.99980	0.99991	0.99996	0.99998	0.99999				**15.00**
15.50	0.99496	0.99731	0.99861	0.99930	0.99966	0.99984	0.99993	0.99997	0.99999	0.99999			**15.50**
16.00	0.99254	0.99589	0.99781	0.99887	0.99943	0.99972	0.99987	0.99994	0.99997	0.99999			**16.00**
16.50	0.98923	0.99390	0.99665	0.99822	0.99908	0.99954	0.99978	0.99989	0.99995	0.99998	0.99999		**16.50**
17.00	0.98483	0.99117	0.99502	0.99727	0.99855	0.99925	0.99963	0.99982	0.99991	0.99996	0.99998	0.99999	**17.00**
17.50	0.97908	0.98750	0.99275	0.99593	0.99778	0.99882	0.99939	0.99970	0.99985	0.99993	0.99997	0.99999	**17.50**

Probabilities for the Poisson distribution

$$P\left(X \le x\right) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

x **=**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**=** ***x***
μ													**μ**
18.00	0.97177	0.98268	0.98970	0.99406	0.99667	0.99819	0.99904	0.99951	0.99975	0.99988	0.99994	0.99997	**18.00**
18.50	0.96263	0.97650	0.98567	0.99152	0.99512	0.99728	0.99852	0.99922	0.99960	0.99980	0.99990	0.99995	**18.50**
19.00	0.95144	0.96873	0.98046	0.98815	0.99302	0.99600	0.99777	0.99879	0.99936	0.99967	0.99984	0.99992	**19.00**
19.50	0.93800	0.95914	0.97387	0.98377	0.99021	0.99425	0.99672	0.99818	0.99902	0.99948	0.99973	0.99987	**19.50**
20.00	0.92211	0.94752	0.96567	0.97818	0.98653	0.99191	0.99527	0.99731	0.99851	0.99920	0.99958	0.99978	**20.00**
20.50	0.90366	0.93368	0.95565	0.97119	0.98180	0.98882	0.99332	0.99611	0.99780	0.99878	0.99934	0.99966	**20.50**
21.00	0.88257	0.91746	0.94363	0.96258	0.97585	0.98483	0.99073	0.99448	0.99680	0.99819	0.99900	0.99946	**21.00**
21.50	0.85880	0.89875	0.92943	0.95217	0.96847	0.97978	0.98737	0.99232	0.99545	0.99737	0.99852	0.99919	**21.50**
22.00	0.83242	0.87750	0.91291	0.93978	0.95949	0.97347	0.98308	0.98949	0.99364	0.99624	0.99784	0.99879	**22.00**
22.50	0.80353	0.85368	0.89399	0.92526	0.94871	0.96573	0.97770	0.98586	0.99126	0.99473	0.99690	0.99822	**22.50**
23.00	0.77230	0.82737	0.87260	0.90848	0.93598	0.95639	0.97106	0.98128	0.98819	0.99274	0.99564	0.99745	**23.00**
23.50	0.73897	0.79866	0.84876	0.88936	0.92117	0.94527	0.96298	0.97559	0.98430	0.99015	0.99397	0.99640	**23.50**
24.00	0.70382	0.76774	0.82253	0.86788	0.90415	0.93224	0.95330	0.96862	0.97943	0.98684	0.99179	0.99499	**24.00**
24.50	0.66717	0.73483	0.79402	0.84403	0.88487	0.91715	0.94187	0.96021	0.97343	0.98269	0.98899	0.99316	**24.50**
25.00	0.62939	0.70019	0.76340	0.81790	0.86331	0.89993	0.92854	0.95022	0.96616	0.97754	0.98545	0.99079	**25.00**

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^{t}}{t!}.$$

$x =$	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**$= x$**
μ													**μ**
15.50													**15.50**
16.00													**16.00**
16.50													**16.50**
17.00													**17.00**
17.50	0.99999					**All 1.00000**							**17.50**
18.00	0.99999	0.99999											**18.00**
18.50	0.99998	0.99999											**18.50**
19.00	0.99996	0.99998	0.99999										**19.00**
19.50	0.99993	0.99997	0.99999	0.99999									**19.50**
20.00	0.99989	0.99995	0.99997	0.99999	0.99999								**20.00**
20.50	0.99982	0.99991	0.99996	0.99998	0.99999								**20.50**
21.00	0.99972	0.99986	0.99993	0.99996	0.99998	0.99999							**21.00**
21.50	0.99956	0.99977	0.99988	0.99994	0.99997	0.99999	0.99999						**21.50**
22.00	0.99933	0.99964	0.99981	0.99990	0.99995	0.99998	0.99999	0.99999					**22.00**
22.50	0.99900	0.99945	0.99971	0.99985	0.99992	0.99996	0.99998	0.99999					**22.50**
23.00	0.99854	0.99918	0.99955	0.99976	0.99988	0.99994	0.99997	0.99998	0.99999				**23.00**
23.50	0.99790	0.99880	0.99933	0.99963	0.99981	0.99990	0.99995	0.99997	0.99999	0.99999			**23.50**
24.00	0.99702	0.99827	0.99901	0.99945	0.99970	0.99984	0.99992	0.99996	0.99998	0.99999	0.99999		**24.00**
24.50	0.99585	0.99754	0.99857	0.99919	0.99955	0.99976	0.99987	0.99993	0.99997	0.99998	0.99999		**24.50**
25.00	0.99430	0.99656	0.99796	0.99882	0.99933	0.99963	0.99980	0.99989	0.99994	0.99997	0.99999	0.99999	**25.00**

Probabilities for the Binomial distribution

The function tabulated is $P(X \le x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$.

	***p* =**	**0.01**	**0.05**	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**	**0.95**	**0.99**
n	***x***															
2	0	0.9801	0.9025	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100	0.0025	0.0001
2	1	0.9999	0.9975	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900	0.0975	0.0199
3	0	0.9703	0.8574	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0156	0.0080	0.0010	0.0001	0.0000
3	1	0.9997	0.9928	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1563	0.1040	0.0280	0.0073	0.0003
3	2	1.0000	0.9999	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.5781	0.4880	0.2710	0.1426	0.0297
4	0	0.9606	0.8145	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0039	0.0016	0.0001	0.0000	0.0000
4	1	0.9994	0.9860	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0508	0.0272	0.0037	0.0005	0.0000
4	2	1.0000	0.9995	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.2617	0.1808	0.0523	0.0140	0.0006
4	3	1.0000	1.0000	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.6836	0.5904	0.3439	0.1855	0.0394
5	0	0.9510	0.7738	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0010	0.0003	0.0000	0.0000	0.0000
5	1	0.9990	0.9774	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0156	0.0067	0.0005	0.0000	0.0000
5	2	1.0000	0.9988	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.1035	0.0579	0.0086	0.0012	0.0000
5	3	1.0000	1.0000	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.3672	0.2627	0.0815	0.0226	0.0010
5	4	1.0000	1.0000	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.7627	0.6723	0.4095	0.2262	0.0490
6	0	0.9415	0.7351	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
6	1	0.9985	0.9672	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0046	0.0016	0.0001	0.0000	0.0000
6	2	1.0000	0.9978	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0376	0.0170	0.0013	0.0001	0.0000
6	3	1.0000	0.9999	0.9987	0.9830	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.1694	0.0989	0.0159	0.0022	0.0000
6	4	1.0000	1.0000	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.4661	0.3446	0.1143	0.0328	0.0015
6	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.8220	0.7379	0.4686	0.2649	0.0585
7	0	0.9321	0.6983	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
7	1	0.9980	0.9556	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0013	0.0004	0.0000	0.0000	0.0000
7	2	1.0000	0.9962	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0129	0.0047	0.0002	0.0000	0.0000
7	3	1.0000	0.9998	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0706	0.0333	0.0027	0.0002	0.0000
7	4	1.0000	1.0000	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.2436	0.1480	0.0257	0.0038	0.0000
7	5	1.0000	1.0000	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.5551	0.4233	0.1497	0.0444	0.0020
7	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.8665	0.7903	0.5217	0.3017	0.0679

Probabilities for the Binomial distribution

The function tabulated is $P(X \le x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$.

	p =	0.01	0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95	0.99
n	x															
8	0	0.9227	0.6634	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
8	1	0.9973	0.9428	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000
8	2	0.9999	0.9942	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0042	0.0012	0.0000	0.0000	0.0000
8	3	1.0000	0.9996	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0273	0.0104	0.0004	0.0000	0.0000
8	4	1.0000	1.0000	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.1138	0.0563	0.0050	0.0004	0.0000
8	5	1.0000	1.0000	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.3215	0.2031	0.0381	0.0058	0.0001
8	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.6329	0.4967	0.1869	0.0572	0.0027
8	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8999	0.8322	0.5695	0.3366	0.0773
9	0	0.9135	0.6302	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1	0.9966	0.9288	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
9	2	0.9999	0.9916	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0013	0.0003	0.0000	0.0000	0.0000
9	3	1.0000	0.9994	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0100	0.0031	0.0001	0.0000	0.0000
9	4	1.0000	1.0000	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0489	0.0196	0.0009	0.0000	0.0000
9	5	1.0000	1.0000	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.1657	0.0856	0.0083	0.0006	0.0000
9	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.3993	0.2618	0.0530	0.0084	0.0001
9	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.6997	0.5638	0.2252	0.0712	0.0034
9	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.9249	0.8658	0.6126	0.3698	0.0865
10	0	0.9044	0.5987	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1	0.9957	0.9139	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
10	2	0.9999	0.9885	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000
10	3	1.0000	0.9990	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0035	0.0009	0.0000	0.0000	0.0000
10	4	1.0000	0.9999	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0197	0.0064	0.0001	0.0000	0.0000
10	5	1.0000	1.0000	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0781	0.0328	0.0016	0.0001	0.0000
10	6	1.0000	1.0000	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.2241	0.1209	0.0128	0.0010	0.0000
10	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.4744	0.3222	0.0702	0.0115	0.0001
10	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.7560	0.6242	0.2639	0.0861	0.0043
10	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9990	0.9940	0.9718	0.9437	0.8926	0.6513	0.4013	0.0956

Probabilities for the Binomial distribution

The function tabulated is $P(X \le x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$.

	p =	**0.01**	**0.05**	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**	**0.95**	**0.99**
n	***x***															
12	0	0.8864	0.5404	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1	0.9938	0.8816	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	2	0.9998	0.9804	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
12	3	1.0000	0.9978	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000
12	4	1.0000	0.9998	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0028	0.0006	0.0000	0.0000	0.0000
12	5	1.0000	1.0000	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0143	0.0039	0.0001	0.0000	0.0000
12	6	1.0000	1.0000	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0544	0.0194	0.0005	0.0000	0.0000
12	7	1.0000	1.0000	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.1576	0.0726	0.0043	0.0002	0.0000
12	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.3512	0.2054	0.0256	0.0022	0.0000
12	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.6093	0.4417	0.1109	0.0196	0.0002
12	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9968	0.9804	0.9150	0.8416	0.7251	0.3410	0.1184	0.0062
12	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9862	0.9683	0.9313	0.7176	0.4596	0.1136
20	0	0.8179	0.3585	0.1216	0.0115	0.0032	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	1	0.9831	0.7358	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	2	0.9990	0.9245	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	3	1.0000	0.9841	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	4	1.0000	0.9974	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	5	1.0000	0.9997	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	6	1.0000	1.0000	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
20	7	1.0000	1.0000	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
20	8	1.0000	1.0000	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
20	9	1.0000	1.0000	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
20	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0139	0.0026	0.0000	0.0000	0.0000
20	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0409	0.0100	0.0001	0.0000	0.0000
20	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.1018	0.0321	0.0004	0.0000	0.0000
20	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.2142	0.0867	0.0024	0.0000	0.0000
20	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9984	0.9793	0.8744	0.5836	0.3828	0.1958	0.0113	0.0003	0.0000
20	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9941	0.9490	0.7625	0.5852	0.3704	0.0432	0.0026	0.0000
20	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987	0.9840	0.8929	0.7748	0.5886	0.1330	0.0159	0.0000
20	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9964	0.9645	0.9087	0.7939	0.3231	0.0755	0.0010
20	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9924	0.9757	0.9308	0.6083	0.2642	0.0169
20	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992	0.9968	0.9885	0.8784	0.6415	0.1821

Critical values for the Grouping of Signs test

n_2

n_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																									
2																									
3									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
5				1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2
6			1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3
7			1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
8			1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	4	4
9			1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4
10		1	1	1	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
11		1	1	1	2	2	2	2	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5
12		1	1	1	2	2	2	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5
13		1	1	1	2	2	2	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6
14		1	1	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6
15		1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	6
16		1	1	2	2	2	3	3	3	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7
17		1	1	2	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7
18		1	1	2	2	3	3	3	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7	7
19		1	1	2	2	3	3	3	4	4	4	5	5	5	6	6	6	6	6	7	7	7	7	7	8
20		1	1	2	2	3	3	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8
21		1	1	2	2	3	3	4	4	4	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8
22		1	1	2	2	3	3	4	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	8
23		1	1	2	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	8	9
24		1	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	9	9
25		1	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	8	9	9	9

The table shows the greatest integer x for which $\sum_{t=1}^{x} \binom{n_1-1}{t-1}\binom{n_2+1}{t} \Big/ \binom{n_1+n_2}{n_1} < 0.05$.

Pseudorandom values from $U(0,1)$

1	2	3	4	5	6	7	8	9	10
0.587	0.155	0.999	0.122	0.659	0.975	0.059	0.567	0.651	0.686
0.030	0.447	0.048	0.201	0.931	0.071	0.033	0.388	0.849	0.033
0.048	0.224	0.359	0.463	0.710	0.861	0.972	0.543	0.550	0.248
0.593	0.478	0.929	0.301	0.688	0.750	0.211	0.911	0.479	0.046
0.165	0.113	0.695	0.513	0.711	0.402	0.121	0.843	0.951	0.229
0.788	0.493	0.329	0.160	0.708	0.309	0.878	0.650	0.279	0.617
0.714	0.980	0.946	0.530	0.973	0.440	0.728	0.652	0.303	0.398
0.265	0.320	0.065	0.573	0.708	0.682	0.014	0.128	0.113	0.938
0.712	0.524	0.747	0.136	0.004	0.165	0.070	0.431	0.201	0.965
0.630	0.933	0.863	0.802	0.642	0.625	0.244	0.961	0.458	0.127
0.569	0.813	0.341	0.055	0.483	0.756	0.186	0.273	0.443	0.618
0.766	0.449	0.026	0.276	0.977	0.410	0.102	0.695	0.487	0.640
0.638	0.335	0.466	0.808	0.907	0.162	0.355	0.333	0.529	0.390
0.984	0.575	0.300	0.836	0.276	0.638	0.674	0.625	0.885	0.451
0.721	0.857	0.303	0.076	0.124	0.688	0.455	0.536	0.842	0.533
0.028	0.271	0.245	0.290	0.534	0.924	0.093	0.724	0.651	0.422
0.726	0.399	0.474	0.221	0.898	0.838	0.723	0.139	0.219	0.711
0.218	0.240	0.036	0.206	0.582	0.203	0.676	0.371	0.791	0.069
0.792	0.704	0.959	0.615	0.440	0.311	0.994	0.785	0.041	0.737
0.656	0.285	0.886	0.954	0.846	0.595	0.215	0.484	0.158	0.435

Pseudorandom values from $N(0,1)$

1	2	3	4	5	6	7	8	9	10
–0.603	0.825	1.166	1.880	1.261	2.542	0.312	0.611	0.286	0.223
1.469	0.282	–1.250	–1.176	–0.064	0.860	–1.505	–0.828	–0.965	–0.166
–2.199	0.169	0.278	0.580	–0.875	0.373	–0.132	–0.153	–1.322	2.340
1.863	–1.302	0.260	–1.023	0.114	–0.904	0.500	–0.255	0.283	0.291
0.076	0.373	–0.448	0.998	0.149	1.987	–0.405	0.324	0.112	–1.367
–0.667	–0.589	0.080	1.007	1.548	1.204	1.886	–0.080	0.341	–0.808
0.495	–1.693	0.647	0.172	1.143	–1.519	–2.557	1.351	–0.466	0.494
–0.161	0.990	–1.348	2.047	0.167	0.599	–0.530	1.244	0.278	0.627
1.105	0.851	–1.012	0.891	0.256	0.297	1.267	–0.053	–1.776	1.392
0.800	–0.867	0.229	–0.534	–0.602	1.685	–1.210	–0.986	0.979	0.810
–0.738	0.765	–2.068	–0.660	2.704	0.161	0.790	–0.284	–1.041	–0.852
–0.489	–0.250	–0.917	–2.549	–1.879	0.156	–1.451	–0.158	–2.252	–0.309
0.170	–1.623	0.442	–0.253	–0.786	–0.468	0.435	1.544	–1.014	–1.187
–1.301	–0.901	0.810	–0.244	0.524	–0.622	–0.785	–0.949	–0.923	0.510
0.059	–1.489	0.235	–0.230	1.262	0.751	–0.377	0.631	0.520	1.508
0.599	0.196	–1.785	–0.899	–1.347	–0.227	1.027	0.704	1.943	–0.902
0.329	–1.008	0.834	1.079	–0.101	–0.322	–0.315	–0.254	–0.711	–0.285
–0.229	0.446	0.086	0.024	0.555	–0.360	0.111	0.589	–0.325	–0.056
–0.987	–0.214	0.925	–0.656	1.991	1.030	–0.961	–0.078	1.023	–0.070
0.805	–0.359	–1.179	0.324	–0.208	–0.632	1.170	–0.432	0.716	–1.801