MTH6127 **Metric Spaces and Topology**

Course work 12 **12 April, 2024**

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1. Show that a finite union of compact subsets of X is compact.
2. Show that a discrete metric space is compact if and only if it is finite.
3. Apply the definition of compactness to show that the half-open interval $(0, 1]$ is not compact.
4. Consider the map $f : [0, 1) \to S^1$ given by

$$f(t) = (\cos 2\pi t, \sin 2\pi t).$$

   Here $S^1 \subset \mathbb{C}$ is the circle $S^1 = \{z \in \mathbb{C}; |z| = 1\}$.

   Show that f is continuous, bijective but is not a homeomorphism.
5. Show that the spaces $[0, 1)$ and S^1 are not homeomorphic.
6. Let $p : X \to Y$ be a continuous map with the property that there exists a continuous map $f : Y \to X$ such that $p \circ f$ equals the identity map of Y. Show that p is a quotient map.
7. Define an equivalence relation on the plane $X = \mathbb{R}^2$ as follows:

$$(x_1, y_1) \sim (x_2, y_2), \quad \text{if} \quad x_1 + y_1^2 = x_2 + y_2^2.$$

   Show that the quotient space $X^* = \mathbb{R}^2/\sim$ is homeomorphic to $\mathbb{R}$.
8. Define an equivalence relation on the plane $X = \mathbb{R}^2$ as follows:

$$(x_1, y_1) \sim (x_2, y_2), \quad \text{if} \quad x_1^2 + y_1^2 = x_2^2 + y_2^2.$$

   Show that the quotient space $X^* = \mathbb{R}^2/\sim$ is homeomorphic to $[0, \infty)$.