MTH6127 **Metric Spaces and Topology**

Course work 8 **15 March 2024**

1. Let X be a topological space; let A be a subspace of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

2. Let Y be a subspace of X. If U is open in Y and Y is open in X then U is open in X.

3. Let Y be the subset $[0, 1) \cup \{2\}$ of $\mathbb{R}$. Show that in the subspace topology on Y the single point $\{2\}$ is closed and open. Besides, show that the set $[0, 1)$ is closed and open in Y.

4. Show that if Y is a subspace of X and A is a subspace of Y, then the topology A inherits as a subspace of Y is the same as the topology it inherits as a subspace of X.

5. Show that the set $\{(x, y); x \geq 0,\ y \geq 0\} \subset \mathbb{R}^2$ is closed.

6. In the finite complement topology on a set X, the closed sets consist of X itself and all finite subsets of X.

7. Consider the following subset $Y = [0, 1] \cup (2, 3)$ of the real line $\mathbb{R}$. Show that both sets $[0, 1]$ and $(2, 3)$ are open and closed in the subspace topology of Y.