MTH6127 **Metric Spaces and Topology**

Course work 7 **8 March 2024**

1. Decide if the mapping $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \frac{1}{2} \cos x$ is a contraction.

2. Apply the Contraction Mapping Theorem to solve numerically the equation

$$x = \frac{1}{2} \cos x.$$

3. Compute the iterations $x_n = f(x_{n-1})$ with two different initial conditions. Compare the results.

4. Consider $\mathbb{R}^2$ with the d_1-metric, i.e. $d_1(v, v') = |x - x'| + |y - y'|$ where $v = (x, y)$ and $v' = (x', y')$. Is this metric space complete? Justify your answer.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(v) = (\frac{1}{3}y, \frac{1}{3}(x + 1))$,where $v = (x, y)$. Show that f is a contraction with respect to the d_1-metric.

6. Find the fixed point of f .

7. Assume that a Cauchy sequence (x_n) in a metric space (X, d) contains a subsequence (x_{n_i}) which converges to a point $x_0 \in X$. Show that the whole sequence (x_n) converges to x_0 as well.