MTH6127 **Metric Spaces and Topology**

Course work 5 **23 February 2024**

1. Consider the Hamming metric on Σ^n in the alphabet $\Sigma = \{0, 1, 2\}$. What is the cardinality of the closed ball $B[w; 1]$?

2. Show that two open intervals $(a, b) \subset \mathbb{R}$ and $(a', b') \subset \mathbb{R}$ are isometric if and only if they have the same length, i.e. $b - a = b' - a'$.

3. Let $X = \mathbb{R}$ with the standard metric. Which of the following sets are dense in X?

   (a) The set A of rational numbers shifted by π, i.e. the set of numbers of the form $x = r + \pi$, where $r \in \mathbb{Q}$.

   (b) The set B of rational multiples of $\sqrt{2}$, i.e. the set of numbers of the form $x = r \cdot \sqrt{2}$ where $r \in \mathbb{Q}$.

   (c) The set C of rational numbers whose decimal representation does not contain the digit "7".

4. Let (X, d) be a metric space. Let $Y \subset X$ be a finite subset. Prove that Y is closed.

5. Let $(V, || \cdot ||)$ be a normed space. Prove that the set $F = \{x \in V; ||x|| = 1\}$ is closed but not open.

6. Which of the following sets viewed with the metric induced from $\mathbb{R}$ are complete:

   (a) $(0, 1)$,

   (b) $(0, \infty)$,

   (c) $[0, \infty)$,

   (d) $\mathbb{R} - \mathbb{Z}$,

   (e) $\mathbb{Z}$,

   (f) The Cantor set C,

   (g) $\mathbb{Q}$.