MTH6127 **Metric Spaces and Topology**

Course work 2 **1 February 2024**

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1. For the metric $d_{L^1}(f,g)$ defined by

$$d_{L^1}(f,g) = \int_a^b |f(x) - g(x)|dx,$$

where $f, g \in C[a,b]$, compute the distance $d_{L^1}(f,g)$ between $f(x) = e^x$ and $g(x) = 2$ where $[a,b] = [0,5]$.

2. Let $X = \mathbb{R}^m$. For any $x = (x_1, ..., x_m)$, $y = (y_1, ..., y_m) \in X$, we set

$$d_\infty(x,y) := \max_k \{|x_k - y_k|\}.$$

Prove that d_∞ defines a metric on X.

3. Let (X,d) be a metric space. Define two new functions d_a and d_b on $X \times X$ by

$$d_a(x,y) := \min\{d(x,y), 1\}, \quad d_b(x,y) := \frac{d(x,y)}{1 + d(x,y)}, \quad \text{for} \quad x, y \in X.$$

Prove that d_a and d_b are also metrics on X.

4. We define *"the Jungle metric"* d_J on $X = \mathbb{R}^2$ by

$$d_J(x,y) := \begin{cases} |x_2 - y_2| & \text{if } x_1 = y_1, \\ |x_2| + |x_1 - y_1| + |y_2| & \text{otherwise.} \end{cases}$$

("climb down from the tree, walk to another one, climb up the tree"). Prove that d_J defines a metric on X.