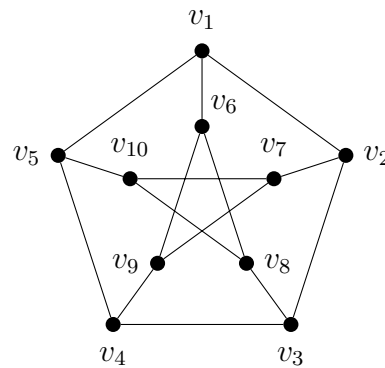


You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

- In the graph shown below, find (a) a shortest  $v_1-v_8$ -path, (b) a longest  $v_1-v_8$ -path, (c) a shortest cycle, and (d) a longest cycle. Explain in each case why the path or cycle you have found has minimum or maximum length.



**Note:** We have not yet discussed any algorithms for finding shortest and longest paths and cycles. You may therefore have to look for them by trial and error, and explain at the end why you are done.

- Show that any digraph that contains a closed directed walk of length at least one contains a directed cycle.
  - What is the analogous statement for graphs? Give a proof or a counterexample for this statement.
- Find all unlabeled trees with six vertices. You may want to start by considering the sequence  $d_1, d_2, \dots, d_6$  of degrees of the vertices in such a tree, and using what you know about this sequence.