# **MTH5113** (2023/24): Problem Sheet 1

All coursework should be submitted individually.

- Problems marked "[**Marked**]" should be submitted and will be marked.

Please submit the completed problem on QMPlus:

- At the portal for **Coursework Submission 1**.

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**(1)** *(Warm-up)* Compute each of the following. *Make sure each of your answers is the right type (scalar, vector, tangent vector) of object!*

**(a)** $-2 \cdot (-1, 1, 7)_{(2,0,3)} + 3 \cdot (3, -2, 4)_{(2,0,3)}$.

**(b)** $(17, -\frac{11}{3})_{(\pi,e)} \cdot (-\frac{2}{5}, \frac{3}{4})_{(\pi,e)}$.

**(c)** $(1, 1, 1)_{(-1,0,-1)} \times (-1, -1, 1)_{(-1,0,-1)}$.

**(2)** *(Warm-up)* Simplify (i.e. compute) each of the following expressions, and then plot it on either a Cartesian plane or in 3-dimensional space:

**(a)** $3 \cdot (-1, 4)_{(0,1)} - (-2, 10)_{(0,1)}$.

**(b)** $(1, 0, 1)_{(0,1,0)} \times (-1, 1, -1)_{(0,1,0)}$.

**(3)** [**Tutorial**] The *lemniscate of Gerono* can be represented by the vector-valued function

$$\lambda : \mathbb{R} \to \mathbb{R}^2, \qquad \lambda(t) = (\cos t, \sin t \cos t).$$

**(a)** Compute $\lambda(t)$, for

$$t = 0, \qquad t = \frac{\pi}{4}, \qquad t = \frac{\pi}{2}, \qquad t = \frac{3\pi}{4}, \qquad \dots, \qquad t = \frac{7\pi}{4}, \qquad t = 2\pi.$$

**(b)** Plot, on a single Cartesian plane, the values $\lambda(t)$ obtained in part (a).

**(c)** On the same diagram as in part (b), sketch the image of $\lambda$.

**(d)** Find computer software or a website that can plot parametric curves, and use it to plot the image of $\lambda$. (See the *Additional Resources* section on the *QMPlus* page.)

**(4) [Marked]** Consider the following vector-valued function:

$$\gamma : (-\pi, 0) \times (-1, 1) \to \mathbb{R}^3, \qquad \gamma(u, v) = \{\cosh(v)\cos(u), v, \cosh(v)\sin(u)\}.$$

**(a)** Sketch the values of $\gamma$ obtained by holding $v = v_0$ constant and varying $u$, where (i) $v_0 = \pm\frac{1}{4}$, (ii) $v_0 = \pm\frac{1}{2}$, and (iii) $v_0 = \pm\frac{3}{4}$ (*you should draw six curves*).

**(b)** Sketch the values of $\gamma$ obtained by holding $u = u_0$ constant and varying $v$, where (i) $u_0 = -\frac{\pi}{4}$, (ii) $u_0 = -\frac{\pi}{2}$, and (iii) $u_0 = -\frac{3\pi}{4}$.

**(c)** Sketch the full image of $\gamma$.

**(5) [Tutorial]** Consider the following function:

$$\mathbf{f} : \mathbb{R} \times (0, 1) \to \mathbb{R}^3, \qquad \mathbf{f}(u, v) = (v\cos u, v\sin u, 1 - v).$$

**(a)** Sketch the values of $\mathbf{f}$ obtained by holding $u = u_0$ constant and varying $v$, where (i) $u_0 = 0$, (ii) $u_0 = \frac{2\pi}{3}$, and (iii) $u_0 = -\frac{2\pi}{3}$.

**(b)** Sketch the values of $\mathbf{f}$ obtained by holding $v = v_0$ constant and varying $u$, where (i) $v_0 = \frac{1}{4}$, (ii) $v_0 = \frac{1}{2}$, and (iii) $v_0 = \frac{3}{4}$.

**(c)** Sketch the full image of $\mathbf{f}$.

**(d)** Find computer software or a website that can plot parametric surfaces, and use it to plot the image of $\mathbf{f}$. (See the *Additional Resources* section on the *QMPlus* page.)

**(6)** *(You're doing it wrong!)* Mr Error is new to *MTH5113* and has no idea what is going on! Each of the following expressions that Mr Error wrote, while in his confused state, has something very wrong with it—explain the mistake in each expression!

**(a)** $-2 \cdot (1, 2)_{(-1,0)} + 7 \cdot (-2, 4)_{(0,0)}$.

**(b)** $(2, 7)_{(0,0)} \times (-1, 6)_{(0,0)}$.

**(c)** $(2, 4, -1)_{(-1,2,3)} + (-3, 1, 1)_{(-1,2,3)} \cdot (1, -1, -2)_{(-1,2,3)}$.

**(7)** *(Plotting curves)* For each of the following vector-valued functions $\gamma$: (i) plot at least three values of $\gamma$, and (ii) sketch the image of $\gamma$.

**(a)** $\gamma : \mathbb{R} \to \mathbb{R}^2$, where $\gamma(t) = (t^3, t)$.

**(b)** $\gamma : \mathbb{R} \to \mathbb{R}^3$, where $\gamma(t) = (t, \cos(2\pi t), \sin(2\pi t))$. *(It can be a bit tricky to plot a 3-dimensional picture on paper, and you do not need to be too precise here. Do the best you can—the main point is to capture the qualitative behaviour of $\gamma$.)*

**(8)** *(Plotting surfaces)* For each of the following vector-valued functions $\sigma$: (i) sketch at least three paths obtained by holding $u$ constant and varying $v$, (ii) sketch at least three paths obtained by holding $v$ constant and varying $u$, and (iii) sketch the image of $\sigma$.

**(a)** $\sigma : \mathbb{R}^2 \to \mathbb{R}^3$, where $\sigma(u, v) = (u, v, u^2 + v^2)$.

**(b)** $\sigma : \mathbb{R}^2 \to \mathbb{R}^3$, where $\sigma(u, v) = ((2 + \cos u)\cos v, (2 + \cos u)\sin v, \sin u)$.

**(9)** *(Fun with Continuity)* Let I and C be the open interval and the unit circle

$$I = (0, 1), \qquad C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}.$$

**(a)** Give an informal argument (i.e. a formal proof is not required) as to why there cannot exist an injective and continuous function $g : C \to I$.

**(b)** Does there exist an injective and continuous function $\mathbf{h} : I \to C$?