MTH5105 Differential and Integral Analysis 2010-2011

Exercises 8

There are two sections. Questions in Section 1 will be used for feedback. Questions in Section 2 are voluntary but highly recommended. Starred questions are more difficult than unstarred ones.

1 Exercises for Feedback

1) Let the sequence of functions $g_n : \mathbb{R} \to \mathbb{R}$ ($n \in \mathbb{N}$) be given by

$$g_n(x) = \frac{x}{1 + nx^2} .$$

   (a) Compute $g(x) = \lim_{n\to\infty} g_n(x)$.

   (b) Show that g_n converges to g uniformly.

   (c) Compute $h(x) = \lim_{n\to\infty} g'_n(x)$.

   (d) Does $g'(x) = h(x)$ hold?

   (e) Why does Theorem 9.5 not apply here?

2 Extra Exercises

2) For $x \in \mathbb{R}$, compute $f(x) = \sum_{n=1}^{\infty} \frac{x}{(1+x^2)^n}$. Show that the convergence is not uniform.

3) (a) Show that the following sequences of functions converge uniformly on the given intervals.

$$\text{(i)} \quad u_n(x) = (1-x)x^n , \qquad [0,1] ;$$

$$\text{(ii)} \quad v_n(x) = \frac{x^2}{1+nx^2} , \qquad \mathbb{R} .$$

   (b) Which of the following sequences of functions converge uniformly to $s(x) = 1$ on the interval $[0, 1]$?

$$\text{(i)} \quad f_n(x) = (1 + x/n)^2 ,$$

$$\text{(ii)} \quad g_n(x) = 1 + x^n(1-x)^n ,$$

$$\text{(iii)} \quad h_n(x) = 1 - x^n(1-x^n) .$$

*4) Let $f_n : \mathbb{R} \to \mathbb{R}$ be a sequence of continuous functions converging uniformly to a function f. Show that if $\lim_{n\to\infty} x_n = x$ then

$$\lim_{n\to\infty} f_n(x_n) = f(x) .$$

The deadline is 5.00pm (strict) on Monday 28th March. Please hand in your coursework to the orange coursework box on the second floor. Coursework will be returned during the exercise class immediately following the deadline.

Thomas Prellberg, March 2011