MATH 5105 Differential and Integral Analysis
Assignment 2

1. Assume that γ is continuous on $[c, d]$ and differentiable on (c, d).

   (a) Then prove if γ is non-decreasing (monotone increasing) then $\gamma' \geq 0$.

   Proof. Let $x, y \in [c, d]$. We want to show $\gamma'(x) \geq 0 \forall x \in (c, d)$. We have that for any $x < y$ or $y < x$,

   $$0 \leq \frac{\gamma(y) - \gamma(x)}{y - x}$$

   Therefore if we let $y \to x$ and we get

   $$0 \leq \gamma'(x).$$

   $\square$

   (b) If γ is strictly increasing, is it true that $\gamma' > 0$? Either give a counterexample and show where the proof of (a) fails or prove your statement.

   Proof. If γ is strictly increasing, it not true that $\gamma' > 0$. Consider the function $\gamma(x) = x^3$. γ is clearly strictly increasing but $\gamma'(0) = 0$. The proof above fails because

   $$0 < \frac{\gamma(y) - \gamma(x)}{y - x}$$

   is not preserved into the limit. $\square$