Queen Mary
University of London

MTH793P Advanced Machine Learning, Semester B, 2021/22

Additional Practice Problems

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1. Complete the following matrix to have minimal rank:

$$\begin{pmatrix} 1 & ? & 2 & 2 \\ ? & ? & 0 & 1 \\ 0 & ? & 1 & ? \end{pmatrix}$$

2. Let $X = \begin{pmatrix} 6 & 4 & 4 \\ 4 & 6 & -4 \end{pmatrix}$. Find (by hand) a rank-1 matrix $\hat{M} \in \mathbb{R}^{2\times 3}$ that satisfies

$$\|\hat{M} - X\|_{\text{fro}} \leq \|M - X\|_{\text{fro}}$$

for all $M \in \mathbb{R}^{2\times 3}$.

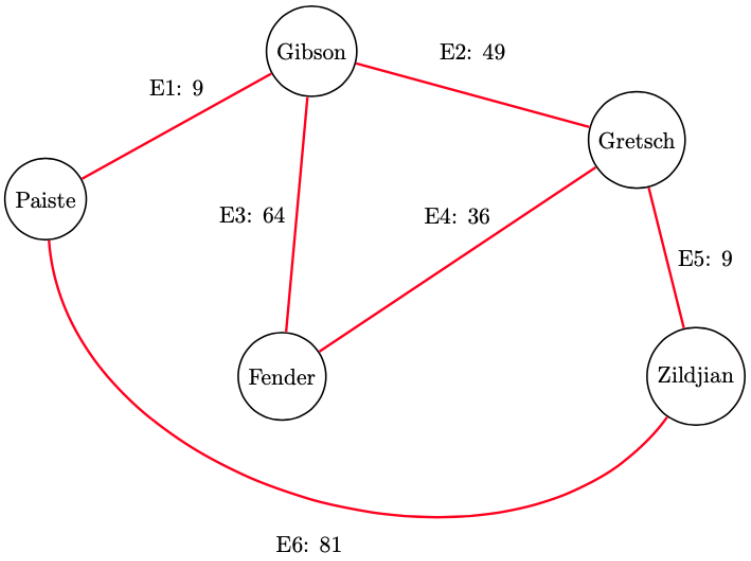
3. Let

$$X = \begin{pmatrix} -2 & 4 & 17 & -6 \\ 1 & 40 & 1 & 3 \\ 4 & -8 & 4 & 12 \end{pmatrix}.$$

Find a decomposition $X = L + E$ where L is a low-rank matrix (rank-1 in this case) and E is sparse.

4. Write down the incidence matrix and the Laplacian for the weighted undirected graph below. Order the columns of the incidence matrix alphabetically according to the vertex name and the rows according to the edge numbering ($E_1, E_2, \ldots$).

5. We want to use the graph in Question 4. to determine whether a node in the graph belongs to the class "guitars" or the class "cymbals". Suppose that we are in a semi-supervised setting, where the node "Fender" is already labelled 1 (class "guitars") and the node "Zildjian" is labelled 0 (class "cymbals"). Determine the labels for all remaining nodes, and classify each node.


Gibson
Gretsch
Paiste
Fender
Zildjian
E1: 9
E2: 49
E3: 64
E4: 36
E5: 9
E6: 81