

Annotated
w/ solutions



Foundation Level Examination by course unit 2015

SEF026: Essential Foundation Mathematical Skills

Duration: 2 hours

Date and time: 2 June 2015, 10.00 am

You should attempt ALL questions. To pass the examination, 18 correct answers are required.

Record each answer by ticking the corresponding box on the answer sheet provided.
Do not use pencil or red ink.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it shall be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiner(s): Prof C-H. Chu

Question 1. Compute the remainder of the following division: $2015 \div 7$.

- [a] 4
- [b] 2
- [c] 3
- [d] 0
- [e] not in the list

$2015 = 287 \times 7 + 6$

Question 2. How many prime numbers are there between 1 and 23, including both?

- [a] 5
- [b] 7
- [c] 6
- [d] 8
- [e] not in the list.

- 2, 3, 5, 7, 11, 13, 17, 19, 23

$3^6 \cdot 7$ $3 \cdot 7 \cdot 11 \cdot 13 \Rightarrow 9$

Question 3. Determine the greatest common divisor of 5103 and 3003.

- [a] 7
- [b] 9
- [c] 3
- [d] 27
- [e] not in the list.

$3 \cdot 7 = 21$

Question 4. Factor 2220 into primes.

- [a] $2^3 \cdot 3^2 \cdot 5^2 \cdot 7$
- [b] $2^2 \cdot 3 \cdot 5 \cdot 37$
- [c] $2 \cdot 5^2 \cdot 127$
- [d] $2 \cdot 3^2 \cdot 5 \cdot 7$
- [e] not in the list

$12 \cdot 5 = 2^3 \cdot 3 \cdot 5$
 $12 \cdot 6 = 2^3 \cdot 3^2$
 $\Rightarrow 2^3 \cdot 3^2 \cdot 5 = 360$

Question 5. Determine the least common multiple x of 60 and 72.

- [a] $120 \leq x < 200$
- [b] $350 \leq x < 400$
- [c] $300 \leq x < 350$
- [d] $900 \leq x < 1000$
- [e] not in the list.

Question 6. How many of the following equalities are correct?

$2.31 \times 10^6 \stackrel{x}{=} 231000$, $0.06 \times 10^0 \stackrel{x}{=} 0.6$,
 $2005 \times 10^{-3} \stackrel{\checkmark}{=} 2.005$, $2345 \div 10^{-3} \stackrel{x}{=} 2.345$.

- [a] 4
- [b] 2
- [c] 1
- [d] 3
- [e] not in the list.

Question 7. Sort the following numbers by decreasing magnitude

$$x = 601 \times 10^{-4} \quad y = 0.0512, \quad z = 0.0059, \quad w = 0.06.$$

0.0601

Smaller 57

[a] $w > x > z > y$ [b] $w > x > y > z$

[e] not in the list.

[c] $x > w > y > z$ [d] $w > z > x > y$

Question 8. Sort the following fractions by increasing magnitude

$$\frac{5}{8}, \quad \frac{8}{13}, \quad \frac{13}{21}$$

$$\frac{8}{13} < \frac{13}{21} < \frac{5}{8}$$

[a] $\frac{13}{21} < \frac{5}{8} < \frac{8}{13}$ [b] $\frac{5}{8} < \frac{13}{21} < \frac{8}{13}$

[e] not in the list.

$$8.21 < 13.13$$

168 169

[c] $\frac{8}{13} < \frac{5}{8} < \frac{13}{21}$ [d] $\frac{13}{21} < \frac{8}{13} < \frac{5}{8}$

Question 9. Estimate $x = 3.2 \times 10^{-2} + 16 \times 10^{-3} - 3 \times 10^{-9}$.

$$1.6 \times 10^{-2}$$

[a] $4.8 \times 10^{-2} < x < 4.9 \times 10^{-2}$ [b] $4.7 \times 10^{-2} < x < 4.8 \times 10^{-2}$

[e] not in the list.

[c] $3.1 \times 10^{-2} < x < 3.2 \times 10^{-2}$ [d] $3.9 \times 10^{-2} < x < 4 \times 10^{-2}$

Question 10. Determine the integer nearest to $\frac{777}{49}$.

$$15.8...$$

[a] 16 [b] 17

[e] not in the list.

[c] 15 [d] 14

Question 11. How many of the following fractions are reduced?

$$\frac{63}{56}, \quad \frac{97}{93}, \quad \frac{91}{63}, \quad \frac{39}{26}$$

x x x

[a] 0 [b] 1

[e] not in the list.

[c] 2 [d] 3

Question 12. Compute the remainder of the following division

$$(z^4 - 3z^3 + 2z - 4) \div (z^2 + 2).$$

[a] $8z$

[b] $8 - 4z$

[e] not in the list.

[c] $8z + 8$

[d] $2 - z$

$(z^2+2)(z^2-3z-2) + 8z$

Question 13. When $z^3 + 81u^2 - 9zu^2 - 9z^2$ is factored completely, one of the factors is

[a] $-9 + z$

[b] $z - 9u$

[e] not in the list.

[c] $9 + z$

[d] $z - 9u^2$

$= (81 - 9z)u^2 + z^2(z-9)$
 $= (z-9)(z^2 - 9u^2)$
 $= (z+3u)(z-9)(z-3u)$

Question 14. Compute the quotient of the following division

$$(a^4 - a^3 + a + 1) \div (-a - 2).$$

[a] $-a^3 + a^2 + 2a + 5$

[b] $-a^3 + 3a^2 - 6a + 11$

[e] not in the list.

[c] $-a^3 - a^2 - 2a - 5$

[d] $-a^3 + 3a^2 + 6a - 13$

$-(a+2)(11 - 6a + 3a^2 - a^3) + 23$

Question 15. Simplify $\frac{2}{a^2 - 9} - \frac{3}{a^2 + a - 12} - \frac{1}{a + 4}$.

[a] $\frac{a^2 - a - 10}{(a + 4)(a + 3)(a - 3)}$

[b] $\frac{26 - a - a^2}{(a + 4)(a + 3)(a - 3)}$

$= \frac{2}{(a+3)(a-3)} - \frac{3}{(a+3)(a+4)} - \frac{1}{a+4}$

[c] $\frac{8 - a - a^2}{(a + 4)(a + 3)(a - 3)}$

[d] $\frac{-a^2 + 5a + 26}{(a + 4)(a + 3)(a - 3)}$

[e] not in the list.

Question 16. Simplify $\frac{6x}{4x^2 - 1} - \frac{4x - 1}{1 - 2x}$.

[a] $\frac{8x^2 + 12x + 1}{(1 + 2x)(1 - 2x)}$

[b] $\frac{8x^2 + 8x - 1}{(2x - 1)(2x + 1)}$

[e] not in the list.

[c] $\frac{8x^2 + 1}{(1 + 2x)(1 - 2x)}$

[d] $\frac{8x^2 + 8x - 1}{(1 - 2x)(1 + 2x)}$

$\frac{6x + (4x^2 - 1)(1 + 2x)}{(2x+1)(2x-1)}$

Question 17. Simplify $\left(\frac{1/y}{-z^2}\right)^2 \left(\frac{x}{zy^{-3}}\right)^{-3}$.

[a] $\frac{1}{x^3y^7z}$ [b] $\frac{y^7}{x^3z}$

[c] $\frac{x^3y^7}{z}$ [d] $\frac{1}{x^3y^{11}z}$

[e] not in the list.

even # \rightarrow pos answer
look at each power

$= -x(9x^2 - 18x + 1)$

Question 18. Given $f(x) = \frac{1}{x}(-x^2 + 18x^3 - 9x^4)$, compute $f\left(-\frac{1}{3}\right)$.

[a] $-\frac{4}{3}$ [b] $\frac{22}{9}$

[c] $\frac{8}{3}$ [d] 2

[e] not in the list.

Question 19. Solve the simultaneous equations $3 = -3x + y$ and $-x = 4y + x + 1$.

[a] $x = \frac{11}{10}, y = \frac{3}{10}$ [b] $x = -\frac{13}{14}, y = \frac{3}{14}$

[c] $x = \frac{13}{10}, y = \frac{9}{10}$ [d] $x = -\frac{9}{14}, y = \frac{11}{14}$

[e] not in the list.

$4y + 2x + 1 = 0$

Question 20. Solve the equation $\frac{x}{3} = -\frac{1-x}{5} + \frac{1}{2}x$.

[a] $\frac{-6}{19}$ [b] -6

[c] $\frac{6}{31}$ [d] $\frac{6}{11}$

[e] not in the list.

(multiply by 30)

Question 21. Solve the inequality $-\frac{3(1-2t)}{6} < -\frac{4+6t}{2}$.

[a] $t < \frac{5}{4}$ [b] $t < -\frac{5}{4}$

[c] $t > -\frac{5}{8}$ [d] $t > -\frac{5}{4}$

[e] not in the list.

$=$ has $t = -\frac{3}{8}$

Question 22. Solve the simultaneous equations

$$5x + 4y = x + y - 1 \quad \text{and} \quad -y = 2(2 - x).$$

[a] $x = -\frac{11}{2}, y = 7$ [b] $x = \frac{11}{10}, y = -\frac{9}{5}$

[c] $x = -\frac{13}{2}, y = -9$ [d] $x = \frac{13}{10}, y = -\frac{7}{5}$

[e] not in the list.

Question 23. Simplify $\frac{\sqrt{15}}{1-\sqrt{6}} + \sqrt{\frac{3}{5}}$.

[a] $\frac{\sqrt{18}}{\sqrt{5}}$

[b] $\sqrt{\frac{90}{5}}$

[c] $\frac{2\sqrt{3}}{5}$

[d] $\frac{\sqrt{15}}{5}$

Handwritten solution:

$$= \frac{\sqrt{15}}{1-\sqrt{6}} + \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{5}\sqrt{15} + \sqrt{3}(1-\sqrt{6})}{(1-\sqrt{6})\sqrt{5}}$$

$$= -3\sqrt{\frac{2}{5}}$$

[e] not in the list.

Handwritten note: or estimate that its negative

Handwritten note: take care $(a^2 - \frac{b^2}{4})$

Question 24. Simplify

$$\left[\left(\frac{b}{4} - 2a \right) \left(-\frac{b}{3} \right) - \left(a + \frac{b}{2} \right) \left(a - \frac{b}{2} \right) + \left(a - \frac{b}{3} \right)^2 \right] \div \left(-\frac{b}{6} \right).$$

[a] $-\frac{8}{3}b$

[b] $-\frac{8}{3}b + 8a$

[c] $-\frac{5}{3}b$

[d] $-\frac{5}{3}b - 8a$

[e] not in the list.

Question 25. Solve the equation $3 + 7x^2 = (2 - x)^2$.

[a] $\frac{-2 \pm \sqrt{10}}{6}$

[b] $\frac{4 \pm \sqrt{40}}{12}$

[c] $\pm \frac{\sqrt{6}}{6}$

[d] $\frac{2 \pm \sqrt{10}}{6}$

[e] not in the list

Handwritten solution:

$$= 4 + 7x^2 - 4x$$
 Simplify and use formula.

End of Paper.